UE OXBRIDGE-PREP

TEST OF MATHEMATICS FOR UNIVERSITY ASSESSMENT

TMUA

PAST PAPERS

2016-2023

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On-line Complete Solutions

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Introduction

"TMUA Past Papers" is presented by UE International Education (ueie.com), which is designed as a companion to the TMUA Standard Course and the TMUA Question Practice. It aims to help students to prepare the Test of Mathematics for University Admissions. It is also a useful reference for teachers who are teaching TMUA.

All questions in this collection are reproduced from the official past papers released by the University of Cambridge, with a few typos from the source files corrected. The 2024 Edition collects a total of 320 TMUA questions from 2016 to 2023.

In addition, subscribed users can access three more on-line TMUA mock papers, which are made up by our professional teachers based on the latest research on TMUA questions.

How to Access Full Solutions

Although this document is free for everyone to use, the detailed solutions to all questions are **only available for subscribed users** who have purchased one of the following products of the UE Oxbridge-Prep series (click on the link to learn more):

TMUA Standard Course

TMUA Question Practice

At least one of the official solution, hand-written solution or video solution is provided for each question. Hand-written solutions are provided if official solutions are unavailable. There are video solutions for some questions.

All solutions can be accessed **ON-LINE ONLY**.

TMUA Score Conversions

You may look up TMUA score conversions through the following page:

Cambridge TMUA Score Conversions

Statistics of Solutions

Year	Number of Questions	Official Solutions	Handwritten Solutions	Video Solutions
2016	**40**	40	6	6
2017	**40**	40	3	3
2018	**40**	40	2	2
2019	**40**	40	5	5
2020	**40**	0	40	3
2021	**40**	20	20	0
2022	**40**	0	40	0
2023	**40**	0	40	0
Total	**320**	180 (56%)	196 (61%)	19 (5.9%)

简介

《TMUA 历年真题集》由优易国际教育（ueie.com）出品，是 TMUA 标准课程和 TMUA 刷题训练的配套资料之一。其主要用途是帮助学生提高备考 TMUA 数学考试的效率，以及为教授 TMUA 考试的同行老师提供参考。

真题集中的所有真题均由剑桥大学官方发布的真题重新排版制作而成，并修订了源文件中的若干印刷错误。2024 版收录了 2016 年至 2023 年共 320 道 TMUA 真题。

此外，我们还为付费订阅用户提供三套线上 TMUA 模考题。这些模考题是由我们的专业教师团队依据近几年 TMUA 考试命题趋势而命制的。

真题解析在哪里可以看到

所有用户均可免费使用真题集，但所有题目的解析仅向购买以下任意优易牛剑备考系列产品之一的**付费用户开放**：

TMUA 标准课

TMUA 刷题训练

所有真题都有详细解析，解析形式为官方解析、手写解析或视频讲解中的一种或多种。如果没有官方解析，则提供手写解析。部分题目提供视频讲解。

所有解析均**只能在线查看**。

TMUA 分数转换

你可以通过下方页面查询 TMUA 分数转换关系：

剑桥 TMUA 分数转换

解析数量统计

年份	真题数量	官方解析题量	手写解析题量	视频講解题量
2016	**40**	40	6	6
2017	**40**	40	3	3
2018	**40**	40	2	2
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簡介

《TMUA 歷年真題集》由優易國際教育（ueie.com）出品，是 TMUA 標準課程和 TMUA 刷題訓練的配套資料之一。其主要用途是幫助學生提高備考 TMUA 數學考試的效率，以及為教授 TMUA 考試的同儕老師提供參考。

真題集中的所有真題均由劍橋大學官方發布的真題重新排版製作而成，並修訂了源文檔中的若干印刷錯誤。2024 版收錄了 2016 年至 2023 年共 320 道 TMUA 真題。

此外，我們還為付費訂閱用戶提供三套在線 TMUA 模擬題。這些模擬題系我們的專業教師團隊依據近幾年 TMUA 考試命題趨勢而命製的。

真題解析在哪裡可以看到

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TMUA 標準課

TMUA 刷題訓練

所有真題都有詳細解析，解析形式為官方解析、手寫解析或視訊講解中的一種或多種。如果沒有官方解析，則提供手寫解析。部分題目提供影片講解。

所有解析均只能線上查看。

TMUA 分數換算

你可以透過下方頁面查詢 TMUA 分數換算關係：

劍橋 TMUA 分數換算

解析數量統計

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Answer Keys

TMUA 2016-2023

Only keys to multiple-choice questions are provided.

Full solutions can be accessed on-line by the links or scanning the QR codes provided.

2016 Answer Keys	
Paper 1	
Q1	H
Q2	E
Q3	C
Q4	B
Q5	C
Q6	C
Q7	B
Q8	F
Q9	D
Q10	E
Q11	E
Q12	E
Q13	C
Q14	D
Q15	C
Q16	C
Q17	D
Q18	A
Q19	B
Q20	D

2016 Answer Keys	
Paper 2	
Q1	A
Q2	B
Q3	D
Q4	C
Q5	C
Q6	C
Q7	C
Q8	B
Q9	D
Q10	E
Q11	D
Q12	F
Q13	E
Q14	B
Q15	D
Q16	C
Q17	H
Q18	D
Q19	F
Q20	E

2017 Answer Keys	
Paper 1	
Q1	C
Q2	C
Q3	A
Q4	B
Q5	C
Q6	B
Q7	B
Q8	A
Q9	F
Q10	E
Q11	A
Q12	B
Q13	C
Q14	F
Q15	B
Q16	E
Q17	D
Q18	A
Q19	D
Q20	E

2017 Answer Keys	
Paper 2	
Q1	A
Q2	E
Q3	G
Q4	B
Q5	B
Q6	A
Q7	E
Q8	E
Q9	D
Q10	D
Q11	B
Q12	C
Q13	B
Q14	F
Q15	D
Q16	C
Q17	F
Q18	B
Q19	E
Q20	B

2018 Answer Keys	
Paper 1	
Q1	D
Q2	C
Q3	E
Q4	G
Q5	D
Q6	E
Q7	A
Q8	D
Q9	B
Q10	E
Q11	C
Q12	F
Q13	C
Q14	B
Q15	E
Q16	F
Q17	A
Q18	B
Q19	D
Q20	E

2018 Answer Keys	
Paper 2	
Q1	C
Q2	B
Q3	F
Q4	D
Q5	A
Q6	C
Q7	B
Q8	D
Q9	F
Q10	B
Q11	A
Q12	F
Q13	F
Q14	E
Q15	G
Q16	F
Q17	C
Q18	C
Q19	C
Q20	B

2019 Answer Keys	
Paper 1	
Q1	A
Q2	A
Q3	E
Q4	C
Q5	E
Q6	C
Q7	F
Q8	E
Q9	D
Q10	F
Q11	H
Q12	C
Q13	B
Q14	B
Q15	A
Q16	C
Q17	C
Q18	B
Q19	C
Q20	E

2019 Answer Keys	
Paper 2	
Q1	E
Q2	C
Q3	A
Q4	D
Q5	B
Q6	D
Q7	B
Q8	E
Q9	D
Q10	A
Q11	A
Q12	F
Q13	D
Q14	B
Q15	B
Q16	F
Q17	D
Q18	E
Q19	E
Q20	B

2020 Answer Keys		2020 Answer Keys		2021 Answer Keys		2021 Answer Keys	
Paper 1		Paper 2		Paper 1		Paper 2	
Q1	C	Q1	E	Q1	F	Q1	D
Q2	C	Q2	F	Q2	F	Q2	E
Q3	B	Q3	C	Q3	G	Q3	C
Q4	D	Q4	G	Q4	B	Q4	C
Q5	A	Q5	A	Q5	F	Q5	B
Q6	C	Q6	A	Q6	D	Q6	D
Q7	A	Q7	H	Q7	G	Q7	B
Q8	D	Q8	E	Q8	A	Q8	C
Q9	C	Q9	F	Q9	C	Q9	C
Q10	A	Q10	F	Q10	B	Q10	E
Q11	E	Q11	G	Q11	A	Q11	C
Q12	D	Q12	D	Q12	E	Q12	B
Q13	F	Q13	D	Q13	C	Q13	A
Q14	E	Q14	A	Q14	B	Q14	C
Q15	C	Q15	D	Q15	C	Q15	B
Q16	C	Q16	D	Q16	B	Q16	E
Q17	A	Q17	E	Q17	A	Q17	F
Q18	A	Q18	G	Q18	B	Q18	C
Q19	E	Q19	E	Q19	B	Q19	F
Q20	C	Q20	C	Q20	D	Q20	E

2022 Answer Keys	
Paper 1	
Q1	C
Q2	D
Q3	F
Q4	C
Q5	H
Q6	F
Q7	E
Q8	B
Q9	E
Q10	C
Q11	A
Q12	D
Q13	A
Q14	D
Q15	H
Q16	B
Q17	D
Q18	B
Q19	F
Q20	B

2022 Answer Keys	
Paper 2	
Q1	B
Q2	E
Q3	C
Q4	B
Q5	F
Q6	C
Q7	F
Q8	C
Q9	A
Q10	G
Q11	C
Q12	F
Q13	E
Q14	D
Q15	F
Q16	D
Q17	E
Q18	E
Q19	B
Q20	E

2023 Answer Keys	
Paper 1	
Q1	F
Q2	A
Q3	C
Q4	C
Q5	F
Q6	E
Q7	F
Q8	B
Q9	E
Q10	B
Q11	B
Q12	F
Q13	F
Q14	A
Q15	F
Q16	E
Q17	E
Q18	E
Q19	D
Q20	F

2023 Answer Keys	
Paper 2	
Q1	H
Q2	F
Q3	C
Q4	G
Q5	A
Q6	F
Q7	E
Q8	D
Q9	D
Q10	A
Q11	G
Q12	C
Q13	C
Q14	F
Q15	B
Q16	B
Q17	F
Q18	D
Q19	H
Q20	D

TMUA 2016 S1

TMUA 2016 S1

On-line Exam

Scan the QR code or click on the link to take an on-line exam.

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TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.

There is no formulae booklet for this test.

[TMUA, 2016S1Q1]

It is given that the expansion of $(ax+b)^3$ is $8x^3 - px^2 + 18x - 3\sqrt{3}$, where a, b and p are real constants.

What is the value of p?

(A) $-12\sqrt{3}$

(B) $-6\sqrt{3}$

(C) $-4\sqrt{3}$

(D) $-\sqrt{3}$

(E) $\sqrt{3}$

(F) $4\sqrt{3}$

(G) $6\sqrt{3}$

(H) $12\sqrt{3}$

[TMUA, 2016S1Q2]

The expression $3x^3 + 13x^2 + 8x + a$, where a is a constant, has $(x+2)$ as a factor.

Which one of the following is a complete factorisation of the expression?

(A) $(x+2)(x-1)(3x-2)$

(B) $(x+2)(x+1)(3x-2)$

(C) $(x+2)(x+1)(3x+2)$

(D) $(x+2)(x-3)(3x+2)$

(E) $(x+2)(x+3)(3x-2)$

(F) $(x+2)(x+3)(3x+2)$

[TMUA, 2016S1Q3]

A line is drawn normal to the curve $y = \frac{2}{x^2}$ at the point on the curve where $x = 1$.

This line cuts the x-axis at P and the y-axis at Q.

The length of PQ is

(A) $\frac{3\sqrt{5}}{2}$

(B) $\frac{3\sqrt{17}}{4}$

(C) $\frac{7\sqrt{17}}{4}$

(D) $\frac{35}{4}$

(E) $\frac{35\sqrt{5}}{2}$

(F) $\frac{3\sqrt{17}}{2}$

[TMUA, 2016S1Q4]

The sequence a_n is defined by the rule:

$$a_n = (-1)^n - (-1)^{n-1} + (-1)^{n+2} \quad \text{for} \quad n \geq 1.$$

Find the value of

$$\sum_{n=1}^{39} a_n \, .$$

(A) -39

(B) -3

(C) -1

(D) 0

(E) 1

(F) 3

(G) 39

[TMUA, 2016S1Q5]

What is the total area enclosed between the curve $y = x^2 - 1$, the x-axis and the lines $x = -2$ and $x = 2$?

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) 4

(D) $\frac{16}{3}$

(E) 12

(F) 16

[TMUA, 2016S1Q6]

P, Q, and R are each mixtures of red and white paint.

The percentage by volume of red paint in P is 30%.

The percentage by volume of red paint in Q is 20%.

The mixtures P, Q, and R are combined in the proportion $12 : 5 : 3$ respectively.

If the resulting mixture contains 25% by volume of red paint, what percentage by volume of mixture R is red paint?

(A) 25%

(B) 23%

(C) $13\frac{1}{3}\%$

(D) $19\frac{1}{2}\%$

(E) $9\frac{3}{4}\%$

(F) It is impossible to achieve this result.

[TMUA, 2016S1Q7]

60% of a sports club's members are women and the remainder are men.

This sports club offers the opportunity to play tennis or cricket. Every member plays exactly one of the two sports.

$\frac{2}{5}$ of the male members of the club play cricket;

$\frac{2}{3}$ of the cricketing members of the club are women.

What is the probability that a member of the club, chosen at random, is a woman who plays tennis?

(A) $\frac{1}{5}$

(B) $\frac{7}{25}$

(C) $\frac{1}{3}$

(D) $\frac{11}{25}$

(E) $\frac{3}{5}$

[TMUA, 2016S1Q8]

Find the maximum angle x in the range $0° \le x \le 360°$ which satisfies the equation

$$\cos^2(2x) + \sqrt{3}\sin(2x) - \frac{7}{4} = 0$$

(A) 30°

(B) 60°

(C) 120°

(D) 150°

(E) 210°

(F) 240°

(G) 300°

(H) 330°

[TMUA, 2016S1Q9]

The line segment joining the points $(3, 3)$ and $(7, 5)$ is a diameter of a circle.

This circle is translated by 3 units in the negative x-direction, then reflected in the x-axis, and then enlarged by a scale factor of 4 about the centre of the resulting circle.

The equation of the final circle is

(A) $(x-2)^2 + (y-4)^2 = 320$

(B) $(x-2)^2 + (y+4)^2 = 320$

(C) $(x-2)^2 + (y-4)^2 = 80$

(D) $(x-2)^2 + (y+4)^2 = 80$

(E) $(x-2)^2 + (y-4)^2 = 20$

(F) $(x-2)^2 + (y+4)^2 = 20$

[TMUA, 2016S1Q10]

How many solutions does the equation $x \tan x = 1$ have in the interval $-2\pi \leq x \leq 2\pi$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

[TMUA, 2016S1Q11]

The real roots of the equation $4^{2x} + 12 = 2^{2x+3}$ are p and q, where $p > q$.

The values of $p - q$ can be expressed as

(A) $\frac{3}{4}$

(B) 1

(C) 4

(D) $-\frac{1}{2} + \log_{10}\frac{3}{2}$

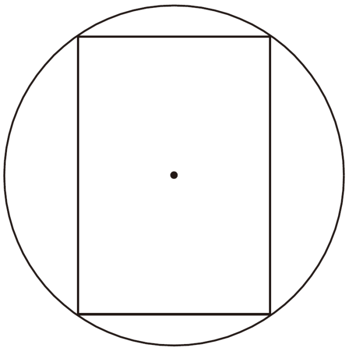
(E) $\frac{\log_{10} 3}{\log_{10} 4}$

(F) $\frac{\log_{10} 3}{\log_{10} 2}$

[TMUA, 2016S1Q12]

A right circular cylinder is contained within a sphere of radius 5 cm in such a way that the whole of the circumferences of both ends of the cylinder are in contact with the sphere.

The diagram shows a planar cross section through the centre of the sphere and cylinder.



[diagram not to scale]

Find, in cubic centimetres, the maximum possible volume of the cylinder.

(A) 250π

(B) 500π

(C) 1000π

(D) $\frac{250\sqrt{3}}{3}\pi$

(E) $\frac{500\sqrt{3}}{9}\pi$

(F) $\frac{1000\sqrt{3}}{9}\pi$

[TMUA, 2016S1Q13]

How many real roots does the equation $3x^5 - 10x^3 - 120x + 30 = 0$ have?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

[TMUA, 2016S1Q14]

The terms of an infinite series S are formed by adding together the corresponding terms in two infinite geometric series, T and U.

The first term of T and the first term of U are each 4.

In order, the first three terms of the combined series S are 8, 3, and $\frac{5}{4}$.

What is the sum to infinity of S?

(A) $\frac{32}{5}$

(B) $\frac{20}{3}$

(C) $\frac{64}{5}$

(D) $\frac{40}{3}$

(E) 16

(F) 32

[TMUA, 2016S1Q15]

The least possible value of the gradient of the curve $y = (2x + a)(x - 2a)^2$ at the point where $x = 1$, as a varies, is

(A) $-\frac{49}{4}$

(B) -8

(C) $-\frac{25}{4}$

(D) $\frac{7}{4}$

(E) $\frac{47}{16}$

[TMUA, 2016S1Q16]

Given the simultaneous equations

$$\log_{10} 2 + \log_{10}(y - 1) = 2\log_{10} x$$
$$\log_{10}(y + 3 - 3x) = 0$$

the values of y are

(A) $\frac{5}{2} \pm \frac{3\sqrt{5}}{2}$

(B) $3 \pm \sqrt{3}$

(C) $7 \pm 3\sqrt{3}$

(D) $3, 9$

(E) $1, 13$

[TMUA, 2016S1Q17]

It is given that

$$y = (1 + 2\cos x)\cos 2x \quad \text{for} \quad 0 < x < \pi.$$

The complete set of values of x for which y is negative is

(A) $0 < x < \frac{\pi}{4},\ \frac{2\pi}{3} < x < \frac{3\pi}{4}$

(B) $0 < x < \frac{\pi}{4},\ \frac{3\pi}{4} < x < \pi$

(C) $0 < x < \frac{2\pi}{3},\ \frac{3\pi}{4} < x < \pi$

(D) $\frac{\pi}{4} < x < \frac{2\pi}{3},\ \frac{3\pi}{4} < x < \pi$

(E) $\frac{\pi}{4} < x < \frac{2\pi}{3}$

(F) $\frac{\pi}{4} < x < \frac{3\pi}{4}$

[TMUA, 2016S1Q18]

The function $\frac{1-x}{\sqrt[3]{x^2}}$ is defined for all $x \neq 0$.

The complete set of values of x for which the function is decreasing is

(A) $x \leq -2,\ x > 0$

(B) $-2 \leq x < 0$

(C) $x \leq 1,\ x \neq 0$

(D) $x \geq 1$

(E) $-2 \leq x \leq 1,\ x \neq 0$

(F) $x \leq -2,\ x \geq 1$

[TMUA, 2016S1Q19]

The coefficient of x^3 in the expansion of $(1 + 2x + 3x^2)^6$ is equal to twice the coefficient of x^4 in the expansion of $(1 - ax^2)^5$.

Find all possible values of the constant a.

(A) $\pm 2\sqrt{2}$

(B) $\pm\sqrt{17}$

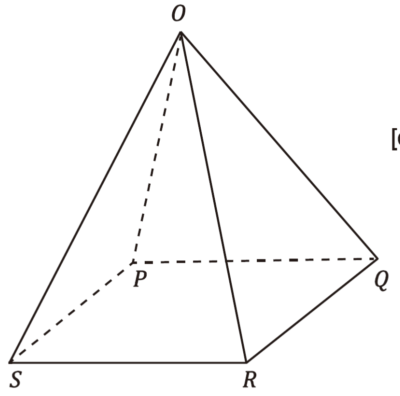
(C) $\pm\sqrt{34}$

(D) $\pm 2\sqrt{17}$

(E) There are no possible values of a.

[TMUA, 2016S1Q20]

The diagram shows a square-based pyramid with base $PQRS$ and vertex O. All the edges of the pyramid are of length 20 metres.



[diagram not to scale]

Find the shortest distance, in metres, along the outer surface of the pyramid from P to the midpoint of OR.

(A) $10\sqrt{5-2\sqrt{3}}$

(B) $10\sqrt{3}$

(C) $10\sqrt{5}$

(D) $10\sqrt{7}$

(E) $10\sqrt{5+2\sqrt{3}}$

TMUA 2016 S2

TMUA 2016 S2

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[TMUA, 2016S2Q1]

Find the value of

$$\int_1^2 \left(x^2 - \frac{4}{x^2}\right)^2 \mathrm{d}x$$

(A) $\frac{43}{15}$

(B) 3

(C) $\frac{97}{15}$

(D) $\frac{103}{15}$

(E) $\frac{163}{15}$

(F) 18

[TMUA, 2016S2Q2]

$$f(x) = \frac{(x^2+5)(2x)}{\sqrt[4]{x^3}}, \quad x > 0$$

Which one of the following is equal to $f'(x)$?

(A) $8x^{\frac{9}{4}} + \frac{40}{3}x^{\frac{1}{4}}$

(B) $\frac{9}{2}x^{\frac{5}{4}} + \frac{5}{2}x^{-\frac{3}{4}}$

(C) $8x^{\frac{9}{4}} + \frac{40}{3}x^{-\frac{1}{4}}$

(D) $\frac{8}{13}x^{\frac{13}{4}} + 8x^{\frac{5}{4}}$

[TMUA, 2016S2Q3]

What is the value, in radians, of the largest angle x in the range $0 \leq x \leq 2\pi$ that satisfies the equation $8\sin^2 x + 4\cos^2 x = 7$?

(A) $\frac{2\pi}{3}$

(B) $\frac{5\pi}{6}$

(C) $\frac{4\pi}{3}$

(D) $\frac{5\pi}{3}$

(E) $\frac{7\pi}{4}$

(F) $\frac{11\pi}{6}$

[TMUA, 2016S2Q4]

Five sealed urns, labelled P, Q, R, S, and T, each contain the **same** (non-zero) number of balls. The following statements are attached to the urns.

Urn P This urn contains one or four balls.

Urn Q This urn contains two or four balls.

Urn R This urn contains more than two balls and fewer than five balls.

Urn S This urn contains one or two balls.

Urn T This urn contains fewer than three balls.

Exactly one of the urns has a true statement attached to it.

Which urn is it?

(A) Urn P

(B) Urn Q

(C) Urn R

(D) Urn S

(E) Urn T

[TMUA, 2016S2Q5]

Consider the statement:

($*$) A whole number n is prime if it is 1 less or 5 less than a multiple of 6.

How many counterexamples to ($*$) are there in the range $0 < n < 50$?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

[TMUA, 2016S2Q6]

The sequence of functions $f_1(x), f_2(x), f_3(x), \ldots$ is defined as follows:

$$f_1(x) = x^{10}$$
$$f_{n+1}(x) = xf_n'(x) \quad \text{for} \quad n \geq 1$$

where $f_n'(x) = \frac{\mathrm{d}f_n(x)}{\mathrm{d}x}$.

Find the value of

$$\sum_{n=1}^{20} f_n(x)\,.$$

(A) $\frac{x^{10}(x^{20}-1)}{x-1}$

(B) $\frac{x^{10}(x^{21}-1)}{x-1}$

(C) $\left(\frac{10^{20}-1}{9}\right)x^{10}$

(D) $\left(\frac{10^{21}-1}{9}\right)x^{10}$

(E) $\left(\frac{(10x)^{20}-1}{10x-1}\right)x^{10}$

(F) $\left(\frac{(10x)^{21}-1}{10x-1}\right)x^{10}$

(G) $x^{10} + x^9 + x^8 + \cdots + x + 1$

(H) $x^{10} + 10x^9 + (10 \times 9)x^8 + \cdots + (10 \times 9 \times \ldots \times 2)x + (10 \times 9 \times \ldots \times 2 \times 1)$

[TMUA, 2016S2Q7]

The four real numbers a, b, c and d are all greater than 1.

Suppose that they satisfy the equation $\log_c d = (\log_a b)^2$.

Use some of the lines given to construct a proof that, in this case, it follows that

$$(*) \qquad \log_b d = (\log_a b)(\log_a c).$$

(1) Let $x = \log_a b$ and $y = \log_a c$

(2) $d = (c^x)^2$

(3) $d = c^{(x^2)}$

(4) $d = b^{xy}$

(5) $d = (a^y)^{(x^2)}$

(6) $d = \left((a^y)x\right)^2$

(7) $d = (a^x)^{xy}$

(8) $d = a^{(y^{2x})}$

(9) $d = a^{(x^2 y)}$

(A) (1). Then (2), so (6), so (8), so (7), and therefore (4), hence $(*)$ as required.

(B) (1). Then (2), so (7), so (8), so (6), and therefore (4), hence $(*)$ as required.

(C) (1). Then (3), so (5), so (9), so (7), and therefore (4), hence $(*)$ as required.

(D) (1). Then (3), so (7), so (9), so (5), and therefore (4), hence $(*)$ as required.

(E) (1). Then (4), so (5), so (9), so (7), and therefore (3), hence $(*)$ as required.

(F) (1). Then (4), so (6), so (8), so (7), and therefore (2), hence $(*)$ as required.

(G) (1). Then (4), so (7), so (8), so (6), and therefore (2), hence $(*)$ as required.

(H) (1). Then (4), so (7), so (9), so (5), and therefore (3), hence $(*)$ as required.

[TMUA, 2016S2Q8]

A region is defined by the inequalities $x + y > 6$ and $x - y > -4$.

Consider the three statements:

1. $x > 1$
2. $y > 5$
3. $(x + y)(x - y) > -24$

Which of the above statements is/are true for **every** point in the region?

(A) none

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1, 2 and 3

[TMUA, 2016S2Q9]

Triangles ABC and XYZ have the **same area**.

Which of these extra conditions, taken independently, would **imply** that they are congruent?

(1) $AB = XY$ **and** $BC = YZ$

(2) $AB = XY$ **and** $\angle ABC = \angle XYZ$

(3) $\angle ABC = \angle XYZ$ **and** $\angle BCA = \angle YZX$

(A) Condition (1): Does not imply congruent; Condition (2): Does not imply congruent; Condition (3): Does not imply congruent.

(B) Condition (1): Does not imply congruent; Condition (2): Does not imply congruent; Condition (3): Implies congruent.

(C) Condition (1): Does not imply congruent; Condition (2): Implies congruent; Condition (3): Does not imply congruent.

(D) Condition (1): Does not imply congruent; Condition (2): Implies congruent; Condition (3): Implies congruent.

(E) Condition (1): Implies congruent; Condition (2): Does not imply congruent; Condition (3): Does not imply congruent.

(F) Condition (1): Implies congruent; Condition (2): Does not imply congruent; Condition (3): Implies congruent.

(G) Condition (1): Implies congruent; Condition (2): Implies congruent; Condition (3): Does not imply congruent.

(H) Condition (1): Implies congruent; Condition (2): Implies congruent; Condition (3): Implies congruent.

[TMUA, 2016S2Q10]

In this equation x and y are non-zero real numbers.

Which one of the following is **sufficient** to conclude that $x < y$?

(A) $x^4 < y^4$

(B) $y^4 < x^4$

(C) $x^{-1} < y^{-1}$

(D) $y^{-1} < x^{-1}$

(E) $x^{\frac{3}{5}} < y^{\frac{3}{5}}$

(F) $y^{\frac{3}{5}} < x^{\frac{3}{5}}$

[TMUA, 2016S2Q11]

$f(x)$ is a polynomial with real coefficients.

The equation $f(x) = 0$ has exactly two real roots, $x = -p$ and $x = p$, where $p > 0$.

Consider the following three statements:

1 $f'(x) = 0$ for exactly one value of x between $-p$ and p.

2 The area between the curve $y = f(x)$, the x-axis and the lines $x = -p$ and $x = p$ is given by $2\int_0^p f(x)\,\mathrm{d}x$.

3 The graph of $y = -f(-x)$ intersects the x-axis at the points $x = -p$ and $x = p$ only.

Which of the above statements **must** be true?

(A) none

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1, 2 and 3

[TMUA, 2016S2Q12]

The first term of an arithmetic sequence is a and the common difference is d.

The sum of the first n terms is denoted by S_n.

If $S_8 > 3S_6$, what can be deduced about the sign of a and the sign of d?

(A) both a and d are negative

(B) a is positive, d is negative

(C) a is negative, d is positive

(D) a is negative, but the sign of d cannot be deduced

(E) d is negative, but the sign of a cannot be deduced

(F) neither the sign of a nor the sign of d can be deduced

[TMUA, 2016S2Q13]

In this question a, b and c are positive integers.

The following is an attempted proof of the false statement:

If a divides bc, then a divides b or a divides c.

['a divides bc' means 'a is a factor of bc']

Which line contains the error in this proof?

1. The statement is equivalent to 'if a does not divide b and a does not divide c then a does not divide bc'.
2. Suppose a does not divide b and a does not divide c. Then the remainder when dividing b by a is r, where $0 < r < a$, and the remainder when dividing c by a is s, where $0 < s < a$.
3. So $b = ax + r$ and $c = ay + s$ for some integers x and y.
4. Thus $bc = a(axy + xs + yr) + rs$.
5. So the remainder when dividing bc by a is rs.
6. Since $r > 0$ and $s > 0$, it follows that $rs > 0$.
7. Hence a does not divide bc.

(A) Line 1

(B) Line 2

(C) Line 3

(D) Line 4

(E) Line 5

(F) Line 6

[TMUA, 2016S2Q14]

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d and e are real numbers.

Suppose $f(x) = 1$ has p distinct real solutions, $f(x) = 2$ has q distinct real solutions $f(x) = 3$ has r distinct real solutions, and $f(x) = 4$ has s distinct real solutions.

Which one of the following is **not** possible?

(A) $p = 1, q = 2, r = 4$ and $s = 3$

(B) $p = 1, q = 3, r = 2$ and $s = 4$

(C) $p = 1, q = 4, r = 3$ and $s = 2$

(D) $p = 2, q = 4, r = 3$ and $s = 1$

(E) $p = 4, q = 3, r = 2$ and $s = 1$

[TMUA, 2016S2Q15]

Consider the quadratic $f(x) = x^2 - 2px + q$ and the statement:

(∗) $f(x) = 0$ has two real roots whose difference is greater than 2 and less than 4.

Which one of the following statements is true **if and only if** (∗) is true?

(A) $q < p^2 < q + 4$

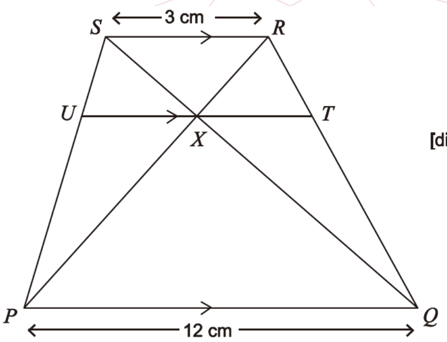
(B) $\sqrt{q+1} < p < \sqrt{q+4}$

(C) $q - 3 \le p^2 - 4 \le q$

(D) $q < p^2 - 1 < q + 3$

(E) $q - 2 < p^2 - 3 < q + 2$

[TMUA, 2016S2Q16]



[diagram not to scale]

In the figure, $PQRS$ is a trapezium with PQ parallel to SR.

The diagonals of the trapezium meet at X.

U lies on SP and T lies on RQ such that UT is a line segment through X parallel to PQ.

The length of PQ is 12 cm and the length of SR is 3 cm.

What, in centimetres is the length of UT?

(A) 4.2

(B) 4.5

(C) 4.8

(D) 5.25

(E) 6

[TMUA, 2016S2Q17]

Consider these simultaneous equations where c is a constant:

$$y = 3\sin x + 2$$
$$y = x + c$$

Which of the following statements is/are true?

1 For some value of c: there is exactly one solution with $0 \le x \le \pi$ **and** there is at least one solution with $-\pi < x < 0$.

2 For some value of c: there is exactly one solution with $0 \le x \le \pi$ **and** there are no solutions with $-\pi < x < 0$.

3 For some value of c: there is exactly one solution with $0 \le x \le \pi$ **and** there are no solutions with $x > \pi$.

(A) none

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1, 2 and 3

[TMUA, 2016S2Q18]

Considering this statement about a function $f(x)$:

(∗) if $\left(f(x)\right)^2 \le 1$ for all $-1 \le x \le 1$ then $\int_{-1}^{1} \left(f(x)\right)^2 \, dx \le \int_{-1}^{1} f(x) \, dx$.

Which one of the following functions provides a **counterexample** to (∗)?

(A) $f(x) = x + \frac{1}{2}$

(B) $f(x) = x - \frac{1}{2}$

(C) $f(x) = x + x^3$

(D) $f(x) = x - x^3$

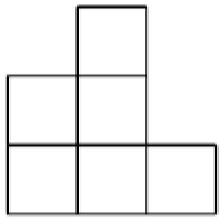
(E) $f(x) = x^2 + x^4$

(F) $f(x) = x^2 - x^4$

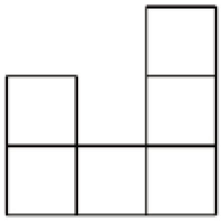
[TMUA, 2016S2Q19]

Some identical unit cubes are used to construct a three-dimensional object by gluing them together face to face.

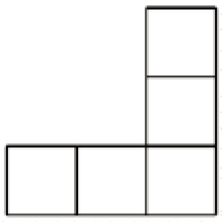
Sketches of this object are made by looking at it from the right-hand side from the front and from above. These sketches are called the side elevation the front elevation and the plan view respectively.



This is the side elevation of the object.



This is the front elevation of the object.



This is the plan view of the object.

How many cubes were used to construct the object?

(A) exactly 6

(B) either 6 or 7

(C) exactly 7

(D) either 7 or 8

(E) exactly 8

(F) either 8 or 9

(G) exactly 9

[TMUA, 2016S2Q20]

Each interior angle of a regular polygon with n sides is $\frac{3}{4}$ of each interior angle of a second regular polygon with m sides.

How many pairs of positive integers n and m are there for which this statement is true?

(A) none

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

(H) infinitely many

TMUA 2017 S1

TMUA 2017 S1
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, data of birth, and full name.

Calculators and dictionaries must **NOT** be used.

There is no formulae booklet for this test.

[TMUA, 2017S1Q1]

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - \frac{2-3x}{x^3}, \quad x \neq 0$$

and $y = 5$ when $x = 1$, find y in terms of x.

(A) $y = \frac{1}{3}x^3 + x^{-2} - 3x^{-1} + 6\frac{2}{3}$

(B) $y = x^3 + \frac{1}{2}x^{-2} - 3x^{-1} + 6\frac{1}{2}$

(C) $y = x^3 + x^{-2} - 3x^{-1} + 6$

(D) $y = x^3 + x^{-2} - x^{-1} + 4$

(E) $y = x^3 + 2x^{-2} - x^{-1} + 3$

(F) $y = 3x^3 + x^{-2} - x^{-1} + 2$

[TMUA, 2017S1Q2]

The function f is given by

$$f(x) = \left(\frac{2}{x} - \frac{1}{2x^2}\right)^2 \quad (x \neq 0)$$

What is the value of $f''(1)$?

(A) -3

(B) -1

(C) 5

(D) 17

(E) 29

(F) 80

[TMUA, 2017S1Q3]

A line l has equation $y = 6 - 2x$.

A second line is perpendicular to l and passes through the point $(-6, 0)$.

Find the area of the region enclosed by the two lines and the x-axis.

(A) $16\frac{1}{5}$

(B) 18

(C) $21\frac{3}{5}$

(D) 27

(E) $40\frac{1}{2}$

[TMUA, 2017S1Q4]

When $(3x^2 + 8x - 3)$ is multiplied by $(px - 1)$ and the resulting product is divided by $(x + 1)$, the remainder is 24.

What is the value of p?

(A) -4

(B) 2

(C) 4

(D) $\frac{8}{7}$

(E) $\frac{11}{4}$

[TMUA, 2017S1Q5]

S is the complete set of values of x which satisfy **both** the inequalities

$$x^2 - 8x + 12 < 0 \quad \text{and} \quad 2x + 1 > 9.$$

The set S can also be represented as a single inequality.

Which one of the following single inequalities represents the set S ?

(A) $(x^2 - 8x + 12)(2x + 1) < 0$

(B) $(x^2 - 8x + 12)(2x + 1) > 0$

(C) $x^2 - 10x + 24 < 0$

(D) $x^2 - 10x + 24 > 0$

(E) $x^2 - 6x + 8 < 0$

(F) $x^2 - 6x + 8 > 0$

(G) $x < 2$

(H) $x > 6$

[TMUA, 2017S1Q6]

A tangent to the circle $x^2 + y^2 = 144$ passes through the point $(20, 0)$ and crosses the positive y-axis.

What is the value of y at the point where the tangent meets the y-axis?

(A) 12

(B) 15

(C) $\frac{49}{3}$

(D) 20

(E) $\frac{64}{3}$

(F) $\frac{80}{3}$

[TMUA, 2017S1Q7]

The first three terms of an arithmetic progression are p, q and p^2 respectively, where $p < 0$.

The first three terms of a geometric progression are p, p^2 and q respectively.

Find the sum of the first 10 terms of the arithmetic progression.

(A) $\frac{23}{8}$

(B) $\frac{95}{8}$

(C) $\frac{115}{8}$

(D) $\frac{185}{8}$

[TMUA, 2017S1Q8]

Find the complete set of values of x, with $0 \leq x \leq \pi$, for which

$$(1 - 2\sin x)\cos x \geq 0$$

(A) $0 \leq x \leq \frac{\pi}{6}$, $\frac{\pi}{2} \leq x \leq \frac{5\pi}{6}$

(B) $0 \leq x \leq \frac{\pi}{6}$, $\frac{5\pi}{6} \leq x \leq \pi$

(C) $\frac{\pi}{6} \leq x \leq \frac{\pi}{2}$, $\frac{5\pi}{6} \leq x \leq \pi$

(D) $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$

[TMUA, 2017S1Q9]

A circle has equation $x^2 + y^2 - 18x - 22y + 178 = 0$.

A regular hexagon is drawn inside this circle so that the vertices of the hexagon touch the circle.

What is the area of the hexagon?

(A) 6

(B) $6\sqrt{3}$

(C) 18

(D) $18\sqrt{3}$

(E) 36

(F) $36\sqrt{3}$

(G) 48

(H) $48\sqrt{3}$

[TMUA, 2017S1Q10]

A curve C has equation $y = f(x)$ where

$$f(x) = p^3 - 6p^2x + 3px^2 - x^3$$

and p is real.

The gradient of the normal to the curve C at the point where $x = -1$ is M.

What is the greatest possible value of M as p varies?

(A) $-\frac{3}{2}$

(B) $-\frac{2}{3}$

(C) $-\frac{1}{2}$

(D) $\frac{1}{4}$

(E) $\frac{2}{3}$

(F) $\frac{3}{2}$

[TMUA, 2017S1Q11]

The sequence x_n is defined by the rules

$$x_1 = 7$$
$$x_{n+1} = \frac{23x_n - 53}{5x_n + 1}$$

The first three terms in the sequence are 7, 3, 1.

What is the value of x_{100}?

(A) -5

(B) 0

(C) 1

(D) 3

(E) 7

[TMUA, 2017S1Q12]

The polynomial function $f(x)$ is such that $f(x) > 0$ for all value of x.

Given $\int_2^4 f(x)\,\mathrm{d}x = A$, which one of the following statements **must** be correct?

(A) $\int_0^2 [f(x+2)+1]\,\mathrm{d}x = A+1$

(B) $\int_0^2 [f(x+2)+1]\,\mathrm{d}x = A+2$

(C) $\int_2^4 [f(x+2)+1]\,\mathrm{d}x = A+1$

(D) $\int_2^4 [f(x+2)+1]\,\mathrm{d}x = A+2$

(E) $\int_4^6 [f(x+2)+1]\,\mathrm{d}x = A+1$

(F) $\int_4^6 [f(x+2)+1]\,\mathrm{d}x = A+2$

[TMUA, 2017S1Q13]

In the expansion of $(a+bx)^5$ the coefficient of x^4 is 8 times the coefficient of x^2.

Given that a and b are non-zero **positive** integers, what is the smallest possible value of $a+b$?

(A) 3

(B) 4

(C) 5

(D) 9

(E) 13

(F) 17

[TMUA, 2017S1Q14]

The solution of the simultaneous equations

$$2^x + 3 \times 2^y = 3$$
$$2^{2x} - 9 \times 2^{2y} = 6$$

is $x = p, y = q$.

Find the value of $p - q$.

(A) $\frac{5}{12}$

(B) $\frac{7}{3}$

(C) $\log_2 \frac{5}{12}$

(D) $\log_2 \frac{7}{3}$

(E) $\log_2 9$

(F) $\log_2 15$

[TMUA, 2017S1Q15]

It is given that $f(x) = -2x^2 + 10$.

Consider the following three curves:

(1) $y = f(x)$

(2) $y = f(x+1)$

(3) the curve $y = f(x+1)$ reflected in the line $y = 6$

The trapezium rule is used to estimate the area under each of these three curves between $x = 0$ and $x = 1$.

State whether the trapezium rule gives an overestimate or underestimate for each of these areas.

(A) (1) underestimate (2) underestimate (3) underestimate

(B) (1) underestimate (2) underestimate (3) overestimate

(C) (1) underestimate (2) overestimate (3) underestimate

(D) (1) underestimate (2) overestimate (3) overestimate

(E) (1) overestimate (2) underestimate (3) underestimate

(F) (1) overestimate (2) underestimate (3) overestimate

(G) (1) overestimate (2) overestimate (3) underestimate

(H) (1) overestimate (2) overestimate (3) overestimate

[TMUA, 2017S1Q16]

The functions f and g are given by $f(x) = 3x^2 + 12x + 4$ and $g(x) = x^3 + 6x^2 + 9x - 8$.

What is the complete set of values of x for which one of the functions is increasing and the other decreasing?

(A) $x \geq -1$

(B) $x \leq -1$

(C) $-3 \leq x \leq -2,\ x \geq -1$

(D) $x \leq -2,\ x \geq -1$

(E) $x \leq -3,\ -2 \leq x \leq -1$

(F) $x \leq -3,\ x \geq -2$

(G) $-2 \leq x \leq -1$

[TMUA, 2017S1Q17]

The two functions $F(n)$ and $G(n)$ are defined as follows for positive integers n:

$$F(n) = \frac{1}{n}\int_0^n (n-x)\,\mathrm{d}x$$

$$G(n) = \sum_{r=1}^{n} F(r)$$

What is the smallest positive integer n such that $G(n) > 150$?

(A) 22

(B) 23

(C) 24

(D) 25

(E) 26

[TMUA, 2017S1Q18]

The graph of $y = \log_{10} x$ is translated in the positive y-direction by 2 units.

This translation is equivalent to a stretch of factor k parallel to the x-axis.

What is the value of k?

(A) 0.01

(B) $\log_{10} 2$

(C) 0.5

(D) 2

(E) $\log_2 10$

(F) 100

[TMUA, 2017S1Q19]

The set of solutions to the inequality $x^2 + bx + c < 0$ is the interval $p < x < q$, where b, c, p and q are real constants with $c < 0$.

In terms of p, q and c, what is the set of solutions to the inequality $x^2 + bcx + c^3 < 0$?

(A) $\frac{p}{c} < x < \frac{q}{c}$

(B) $\frac{q}{c} < x < \frac{p}{c}$

(C) $pc < x < qc$

(D) $qc < x < pc$

(E) $pc^2 < x < qc^2$

(F) $qc^2 < x < pc^2$

[TMUA, 2017S1Q20]

The lengths of the sides QR, RP and PQ in triangle PQR are a, $a + d$ and $a + 2d$ respectively, where a and d are positive and such that $3d > 2a$.

What is the full range, in degrees, of possible values for angle PRQ?

(A) $0 <$ angle $PRQ < 60$

(B) $0 <$ angle $PRQ < 120$

(C) $60 <$ angle $PRQ < 120$

(D) $60 <$ angle $PRQ < 180$

(E) $120 <$ angle $PRQ < 180$

TMUA 2017 S2

TMUA 2017 S2

On-line Exam

Scan the QR code or click on the link to take an on-line exam.

Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

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[TMUA, 2017S2Q1]

Given that $y = \frac{(1-3x)^2}{2x^{\frac{3}{2}}}$, which one of the following is a correct expression for $\frac{dy}{dx}$?

(A) $\frac{9}{4}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

(B) $\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$

(C) $\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

(D) $-\frac{9}{4}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} + \frac{3}{4}x^{-\frac{5}{2}}$

(E) $-\frac{9}{4}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

(F) $-\frac{9}{4}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$

[TMUA, 2017S2Q2]

$PQRS$ is a rectangle.

The coordinates of P and Q are $(0, 6)$ and $(1, 8)$ respectively.

The perpendicular to PQ at Q meets the x-axis at R.

What is the area of $PQRS$?

(A) $\frac{5}{2}$

(B) $4\sqrt{10}$

(C) 20

(D) $8\sqrt{10}$

(E) 40

[TMUA, 2017S2Q3]

The first term of a geometric progression is $2\sqrt{3}$ and the fourth term is $\frac{9}{4}$.

What is the sum to infinity of this geometric progression?

(A) $-2(2-\sqrt{3})$

(B) $4(2\sqrt{3}-3)$

(C) $\frac{16(8\sqrt{3}+9)}{37}$

(D) $\frac{4(2\sqrt{3}-3)}{7}$

(E) $\frac{4(2\sqrt{3}+3)}{7}$

(F) $2(2+\sqrt{3})$

(G) $4(2\sqrt{3}+3)$

[TMUA, 2017S2Q4]

The following question appeared in an examination:

Given that $\tan x = \sqrt{3}$, find the possible values of $\sin 2x$.

A student gave the following answer:

$\tan x = \sqrt{3}$ so $x = 60°$ and $2x = 120°$, therefore $\sin 2x = \frac{\sqrt{3}}{2}$.

Which one of the following statements is correct?

(A) $\frac{\sqrt{3}}{2}$ is the only possible value, and this is fully supported by the reasoning given in the student's answer.

(B) $\frac{\sqrt{3}}{2}$ is the only possible value, but the reasoning given should consider other possible values of x for which $\tan x = \sqrt{3}$.

(C) $\frac{\sqrt{3}}{2}$ is the only possible value, but the reasoning given should consider other possible values of x for which $\sin 2x = \frac{\sqrt{3}}{2}$.

(D) $\frac{\sqrt{3}}{2}$ is **not** the only possible value because the reasoning given should have considered other possible values of x for which $\tan x = \sqrt{3}$.

(E) $\frac{\sqrt{3}}{2}$ is **not** the only possible value because the reasoning given should have considered other possible values of x for which $\sin 2x = \frac{\sqrt{3}}{2}$.

[TMUA, 2017S2Q5]

Consider the following three statements:

1. $10p^2 + 1$ and $10p^2 - 1$ are both prime when p is an odd prime.
2. Every prime greater than 5 is of the form $6n + 1$ for some integer n.
3. No multiple of 7 greater than 7 is prime.

The result $91 = 7 \times 13$ can be used to provide a **counterexample** to which of the above statements?

(A) none of them

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1, 2 and 3

[TMUA, 2017S2Q6]

A sequence $u_0, u_1, u_2, \ldots$ is defined as follows:

$$u_0 = 1$$
$$u_n = \int_0^1 4xu_{n-1}\,\mathrm{d}x \quad \text{for } n \geq 1$$

What is the value of u_{1000}?

(A) 2^{1000}

(B) 4^{1000}

(C) $\frac{4}{1000!}$

(D) $\frac{4}{1001!}$

(E) $\frac{2^{1000}}{1000!}$

(F) $\frac{4^{1000}}{1000!}$

(G) $\frac{2^{1000}}{1001!}$

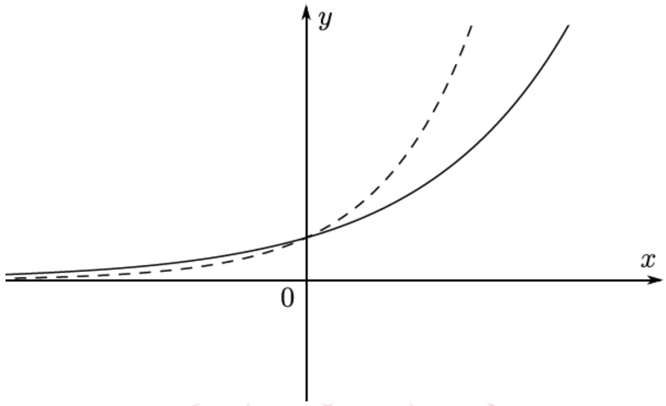
(H) $\frac{4^{1000}}{1001!}$

[TMUA, 2017S2Q7]

The graphs of two functions are shown here:

$y = a^x$ is shown with a solid line, where a is a positive real number

$y = f(x)$ is shown with a dashed line



Which of the following statements (1, 2, 3, 4) could be true?

1. $f(x) = b^x$ for some $b > a$
2. $f(x) = b^x$ for some $b < a$
3. $f(x) = a^{kx}$ for some $k > 1$
4. $f(x) = a^{kx}$ for some $k < 1$

(A) 1 only

(B) 2 only

(C) 3 only

(D) 4 only

(E) 1 and 3 only

(F) 2 and 3 only

(G) 2 and 3 only

(H) 2 and 4 only

[TMUA, 2017S2Q8]

Which one of the following numbers is smallest in value?

(A) $\log_2 7$

(B) $(2^{-3} + 2^{-2})^{-1}$

(C) $2^{\pi/3}$

(D) $\frac{1}{4(\sqrt{2}-1)^3}$

(E) $4\sin^2\left(\frac{\pi}{4}\right)$

[TMUA, 2017S2Q9]

Consider the following attempt to prove this true theorem:

Theorem: $a^3 + b^3 = c^3$ has no solutions with a, b and c positive integers.

Attempted proof:

Suppose that there are positive integers a, b and c such that $a^3 + b^3 = c^3$.

I We have $a^3 = c^3 - b^3$.

II Hence $a^3 = (c-b)(c^2 + cb + b^2)$.

III It follows that $a = c - b$ and $a^2 = c^2 + cb + b^2$, since $a \leq a^2$ and $c - b \leq c^2 + cb + b^2$.

IV Eliminating a, we have $(c-b)^2 = c^2 + cb + b^2$.

V Multiplying out, we have $c^2 - 2cb + b^2 = c^2 + cb + b^2$.

VI Hence $3cb = 0$ so one of b and c is zero.

But this is a contradiction to the original assumption that all of a, b and c are positive. It follows that the equation has no solutions.

Comment on this proof by choosing one of the following options:

(A) The proof is correct.

(B) The proof is incorrect and the first mistake occurs on line I.

(C) The proof is incorrect and the first mistake occurs on line II.

(D) The proof is incorrect and the first mistake occurs on line III.

(E) The proof is incorrect and the first mistake occurs on line IV.

(F) The proof is incorrect and the first mistake occurs on line V.

(G) The proof is incorrect and the first mistake occurs on line VI.

[TMUA, 2017S2Q10]

$f(x)$ is a function defined for all real values of x.

Which one of the following is a **sufficient** condition for $\int_1^3 f(x)\,\mathrm{d}x = 0$?

(A) $f(2) = 0$

(B) $f(1) = f(3) = 0$

(C) $f(-x) = -f(x)$ for all x

(D) $f(x+2) = -f(2-x)$ for all x

(E) $f(x-2) = -f(2-x)$ for all x

[TMUA, 2017S2Q11]

The function $f(x)$ is increasing and $f(0) = 0$.

The positive constants a and b are such that $a < b$.

The area of the region enclosed by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is denoted by R.

The function $g(x)$ is defined by $g(x) = f(x) + 2f(b)$.

Which of the following is an expression for the area enclosed by the curve $y = g(x)$, the x-axis and the lines $x = a$ and $x = b$?

(A) $R + (b-a)f(b)$

(B) $R + 2(b-a)f(b)$

(C) $R + 2f(b) - f(a)$

(D) $R + 2f(b)$

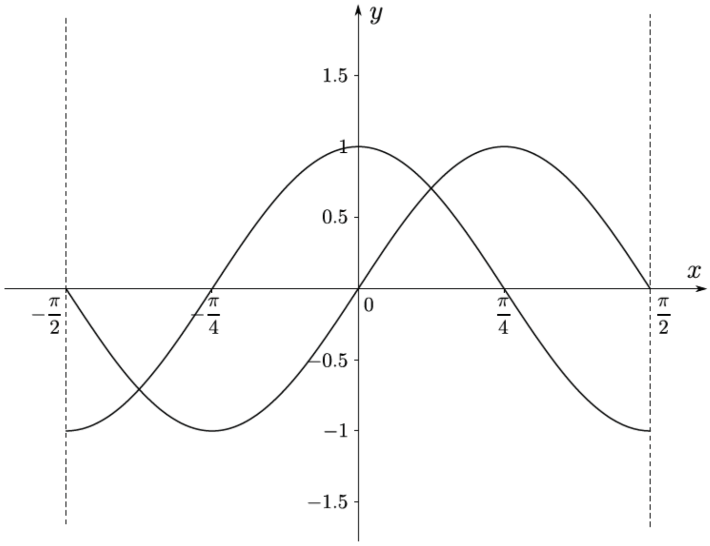
(E) $R + \left(f(b)\right)^2$

(F) $R + \left(f(b)\right)^2 - \left(f(a)\right)^2$

(G) $R + 2\left(f(b) - f(a)\right)f(b)$

[TMUA, 2017S2Q12]

The diagram shows the graphs of $y = \sin 2x$ and $y = \cos 2x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.



Which one of the following is **not** true?

(A) $\cos 2x < \sin 2x < \tan x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(B) $\cos 2x < \tan x < \sin 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(C) $\sin 2x < \cos 2x < \tan x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(D) $\sin 2x < \tan x < \cos 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(E) $\tan x < \sin 2x < \cos 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(F) $\tan x < \cos 2x < \sin 2x$ for some real number x with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

[TMUA, 2017S2Q13]

The positive real numbers $a \times 10^{-1}$, $b \times 10^{-2}$ and $c \times 10^{-1}$ are each in standard form, and

$$(a \times 10^{-3}) + (b \times 10^{-2}) = (c \times 10^{-1}).$$

Which of the following statements (I, II, III, IV) **must** be true?

I $\quad a > 9$

II $\quad b > 9$

III $\quad a < c$

IV $\quad b < c$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I and IV only

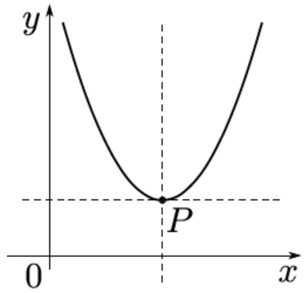
(F) II and III only

(G) II and IV only

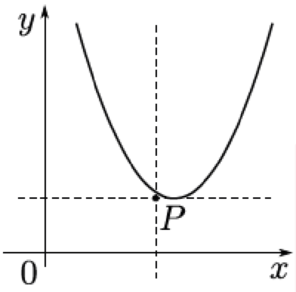
(H) I, II, III and IV

[TMUA, 2017S2Q14]

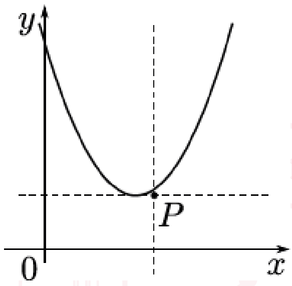
The diagram below shows the graph of $y = x^2 - 2bx + c$. The vertex of this graph is at the point P.



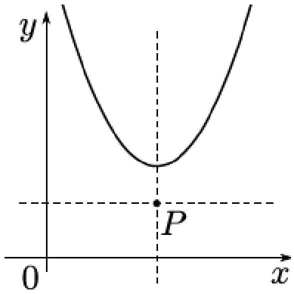
Which one of the following could be the graph of $y = x^2 - 2Bx + c$, where $B > b$?



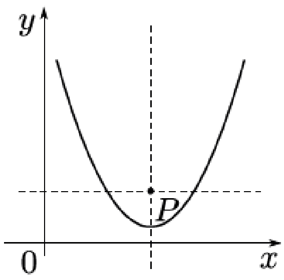
(A)



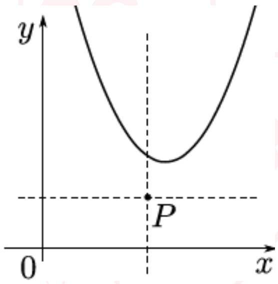
(B)



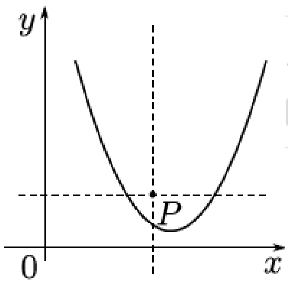
(C)



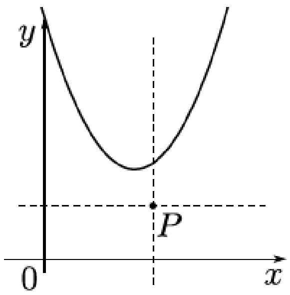
(D)



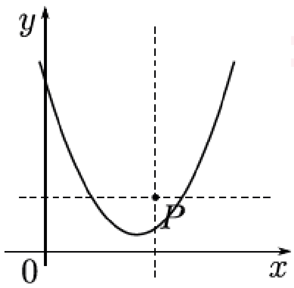
(E)



(F)



(G)



(H)

[TMUA, 2017S2Q15]

The function f is defined on the positive integers as follows:

$$f(1) = 5\text{, and for } n \geq 1: \quad \begin{cases} f(n+1) = 3f(n) + 1 & \text{if } f(n) \text{ is odd} \\ f(n+1) = \frac{1}{2}f(n) & \text{if } f(n) \text{ is even} \end{cases}$$

The function g is defined on the positive integers as follows:

$$g(1) = 3\text{, and for } n \geq 1: \quad \begin{cases} g(n+1) = g(n) + 5 & \text{if } g(n) \text{ is odd} \\ g(n+1) = \frac{1}{2}g(n) & \text{if } g(n) \text{ is even} \end{cases}$$

What is the value of $f(1000) - g(1000)$?

(A) -6

(B) -5

(C) 1

(D) 2

(E) 4

(F) 8

[TMUA, 2017S2Q16]

Consider the following statement:

(∗) **If** $f(x)$ is an integer for every integer x, **then** $f'(x)$ is an integer for every integer x.

Which one of the following is a **counterexample** to (∗)?

(A) $f(x) = \frac{x^3+x+1}{4}$

(B) $f(x) = \frac{x^4+x^2+x}{2}$

(C) $f(x) = \frac{x^4+x^3+x^2+x}{2}$

(D) $f(x) = \frac{x^4+2x^3+x^2}{4}$

[TMUA, 2017S2Q17]

A set S of whole numbers is called *stapled* **if and only if** for every whole number a which is in S there exists a prime factor of a which divides at least one other number in S.

Let T be a set of whole numbers. Which of the following is true **if and only if** T is **not** stapled?

(A) For every number a which is in T, there is no prime factor of a which divides every other number in T.

(B) For every number a which is in T, there is no prime factor of a which divides at least one other number in T.

(C) For every number a which is in T, there is a prime factor of a which does not divide any other number in T.

(D) For every number a which is in T, there is a prime factor of a which does not divide at least one other number in T.

(E) There exists a number a which is in T such that there is no prime factor of a which divides every other number in T.

(F) There exists a number a which is in T such that there is no prime factor of a which divides at least one other number in T.

(G) There exists a number a which is in T such that there is a prime factor of a which does not divide any other number in T.

(H) There exists a number a which is in T such that there is a prime factor of a which does not divide at least one other number in T.

[TMUA, 2017S2Q18]

Consider the following problem:

Solve the inequality $\left(\frac{1}{4}\right)^n < \left(\frac{1}{32}\right)^{10}$, where n is a positive integer.

A student produces the following argument:

$$\left(\frac{1}{4}\right)^n < \left(\frac{1}{32}\right)^{10}$$

(I)

$$\log_{\frac{1}{2}}\left(\frac{1}{4}\right)^n < \log_{\frac{1}{2}}\left(\frac{1}{32}\right)^{10}$$

(II)

$$n\log_{\frac{1}{2}}\left(\frac{1}{4}\right) < 10\log_{\frac{1}{2}}\left(\frac{1}{32}\right)$$

(III)

$$n < \frac{10\log_{\frac{1}{2}}(\frac{1}{32})}{\log_{\frac{1}{2}}(\frac{1}{4})}$$

(IV)

$$n < \frac{10\times 5}{2} = 25$$

(V)

$$1 \leqslant n \leqslant 24$$

Which step (if any) in the argument is invalid?

(A) There are no invalid steps; the argument is correct.

(B) Only step (I) is invalid; the rest are correct

(C) Only step (II) is invalid; the rest are correct

(D) Only step (III) is invalid; the rest are correct

(E) Only step (IV) is invalid; the rest are correct

(F) Only step (V) is invalid; the rest are correct

[TMUA, 2017S2Q19]

Which one of the following is a **sufficient** condition for the equation $x^3 - 3x^2 + a = 0$, where a is a constant, to have exactly one real root?

(A) $a > 0$

(B) $a \leq 0$

(C) $a \geq 4$

(D) $a < 4$

(E) $|a| > 4$

(F) $|a| \leq 4$

(G) $a = \frac{9}{4}$

(H) $|a| = \frac{3}{2}$

[TMUA, 2017S2Q20]

I have forgotten my 5-character computer password, but I know that it consists of the letters **a**, **b**, **c**, **d**, **e** in some order. When I enter a potential password into the computer, it tells me exactly how many of the letters are in the correct position.

When I enter **abcde**, it tells me that none of the letters are in the correct position. The same happens when I enter **cdbea** and **eadbc**.

Using the best strategy, how many **further** attempts must I make in order to **guarantee** that I can **deduce** the correct password?

(A) None: I can deduce it immediately

(B) One

(C) Two

(D) Three

(E) More than three

TMUA 2018 S1

TMUA 2018 S1
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[TMUA, 2018S1Q1]

Find the value of

$$\int_1^4 \frac{3-2x}{x\sqrt{x}}\,\mathrm{d}x.$$

(A) $-\frac{13}{2}$

(B) $-\frac{85}{16}$

(C) $-\frac{13}{8}$

(D) -1

(E) $-\frac{1}{4}$

(F) $\frac{7}{4}$

(G) 7

[TMUA, 2018S1Q2]

An arithmetic progression has first term a and common difference d.

The sum of the first 5 terms is equal to the sum of the first 8 terms.

Which one of the following expresses the relationship between a and d?

(A) $a = -\frac{38}{3}d$

(B) $a = -7d$

(C) $a = -6d$

(D) $a = 6d$

(E) $a = 7d$

(F) $a = \frac{38}{3}d$

[TMUA, 2018S1Q3]

Find the shortest distance between the two circles with equations:

$$(x+2)^2 + (y-3)^2 = 18$$
$$(x-7)^2 + (y+6)^2 = 2$$

(A) 0

(B) 4

(C) 16

(D) $2\sqrt{2}$

(E) $5\sqrt{2}$

[TMUA, 2018S1Q4]

Consider the simultaneous equations

$$3x^2 + 2xy = 4$$
$$x + y = a$$

where a is a real constant.

Find the complete set of values of a for which the equations have two distinct real solutions for x.

(A) There are no values of a

(B) $-2 < a < 2$

(C) $-1 < a < 1$

(D) $a = 0$

(E) $a < -1$ or $a > 1$

(F) $a < -2$ or $a > 2$

(G) All real values of a

[TMUA, 2018S1Q5]

The function f is defined by $f(x) = x^3 + ax^2 + bx + c$.

a, b and c take the values 1, 2 and 3 with no two of them being equal and not necessarily in this order.

The remainder when $f(x)$ is divided by $(x + 2)$ is R.

The remainder when $f(x)$ is divided by $(x + 3)$ is S.

What is the largest possible value of $R - S$?

(A) -26

(B) 5

(C) 7

(D) 17

(E) 29

[TMUA, 2018S1Q6]

Find the number of solutions of the equation

$$x \sin 2x = \cos 2x$$

with $0 \leq x \leq 2\pi$.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

[TMUA, 2018S1Q7]

The non-zero constant k is chosen so that the coefficients of x^6 in the expansions of $(1 + kx^2)^7$ and $(k + x)^{10}$ are equal.

What is the value of k?

(A) $\frac{1}{6}$

(B) 6

(C) $\frac{\sqrt{6}}{6}$

(D) $\sqrt{6}$

(E) $\frac{\sqrt{30}}{30}$

(F) $\sqrt{30}$

[TMUA, 2018S1Q8]

The sum to infinity of a geometric progression is 6.

The sum to infinity of the squares of each term in the progression is 12.

Find the sum to infinity of the cubes of each term in the progression.

(A) 8

(B) 18

(C) 24

(D) $\frac{216}{7}$

(E) 72

(F) 216

[TMUA, 2018S1Q9]

Find the complete set of values of the constant c for which the cubic equation

$$2x^3 - 3x^2 - 12x + c = 0$$

has three distinct real solutions.

(A) $-20 < c < 7$

(B) $-7 < c < 20$

(C) $c > 7$

(D) $c > -7$

(E) $c < 20$

(F) $c < -20$

[TMUA, 2018S1Q10]

x and y satisfy $|2 - x| \leq 6$ and $|y + 2| \leq 4$.

What is the greatest possible value of $|xy|$?

(A) 16

(B) 24

(C) 32

(D) 40

(E) 48

(F) There is no greatest possible value

[TMUA, 2018S1Q11]

The line $y = mx + 5$, where $m > 0$, is normal to the curve $y = 10 - x^2$ at the point (p, q).

What is the value of p ?

(A) $\frac{\sqrt{2}}{6}$

(B) $-\frac{\sqrt{2}}{6}$

(C) $\frac{3\sqrt{2}}{2}$

(D) $-\frac{3\sqrt{2}}{2}$

(E) $\sqrt{5}$

(F) $-\sqrt{5}$

[TMUA, 2018S1Q12]

A curve has equation $y = f(x)$, where

$$f(x) = x(x-p)(x-q)(r-x)$$

with $0 < p < q < r$.

You are given that:

$$\int_0^r f(x)\,\mathrm{d}x = 0$$
$$\int_0^q f(x)\,\mathrm{d}x = -2$$
$$\int_p^r f(x)\,\mathrm{d}x = -3$$

What is the total area enclosed by the curve and the x-axis for $0 \le x \le r$?

(A) 0

(B) 1

(C) 4

(D) 5

(E) 6

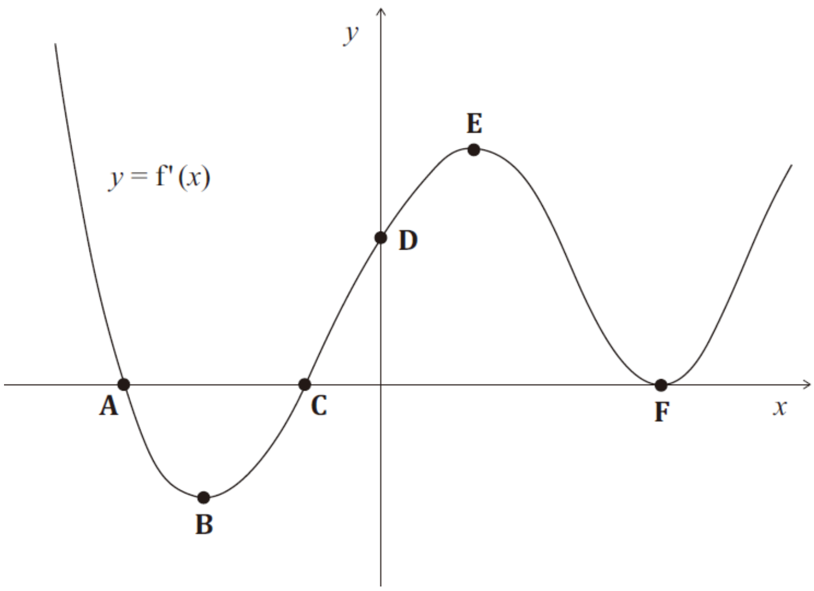
(F) 10

[TMUA, 2018S1Q13]

The function $f(x)$ has derivative $f'(x)$.

The diagram below shows the graph of $y = f'(x)$.

Which point corresponds to a local minimum of $f(x)$?



[TMUA, 2018S1Q14]

The line $y = mx + 4$ passes through the points $(3, \log_2 p)$ and $(\log_2 p\,, 4)$.

What are the possible values of p?

(A) $p = 1$ and $p = 4$

(B) $p = 1$ and $p = 16$

(C) $p = \frac{1}{4}$ and $p = 4$

(D) $p = \frac{1}{4}$ and $p = 64$

(E) $p = \frac{1}{64}$ and $p = 4$

(F) $p = \frac{1}{64}$ and $p = 16$

[TMUA, 2018S1Q15]

Find the sum of the real solutions of the equation:

$$3^x - \left(\sqrt{3}\right)^{x+4} + 20 = 0$$

(A) 1

(B) 4

(C) 9

(D) $\log_3 20$

(E) $2\log_3 20$

(F) $4\log_3 20$

[TMUA, 2018S1Q16]

The curve C has equation $y = x^2 + bx + 2$, where $b \geq 0$.

Find the value of b that minimises the distance between the origin and the stationary point of the curve C.

(A) $b = 0$

(B) $b = 1$

(C) $b = 2$

(D) $b = \frac{\sqrt{6}}{2}$

(E) $b = \sqrt{2}$

(F) $b = \sqrt{6}$

[TMUA, 2018S1Q17]

There are two sets of data: the mean of the first set is 15, and the mean of the second set is 20.

One of the pieces of data from the first set is exchanged with one of the pieces of data from the second set.

As a result, the mean of the first set of data increases from 15 to 16, and the mean of the second set of data decreases from 20 to 17.

What is the mean of the set made by combining all the data?

(A) $16\frac{1}{4}$

(B) $16\frac{1}{3}$

(C) $16\frac{1}{2}$

(D) $16\frac{2}{3}$

(E) $16\frac{3}{4}$

[TMUA, 2018S1Q18]

What is the smallest positive value of a for which the line $x = a$ is a line of symmetry of the graph of $y = \sin\left(2x - \frac{4\pi}{3}\right)$?

(A) $\frac{\pi}{12}$

(B) $\frac{5\pi}{12}$

(C) $\frac{7\pi}{12}$

(D) $\frac{11\pi}{12}$

(E) $\frac{19\pi}{12}$

[TMUA, 2018S1Q19]

A triangle ABC is to be drawn with $AB = 10$ cm, $BC = 7$ cm and the angle at A equal to θ, where θ is a certain specified angle.

Of the two possible triangles that could be drawn, the larger triangle has three times the area of the smaller one.

What is the value of $\cos\theta$?

(A) $\frac{5}{7}$

(B) $\frac{151}{200}$

(C) $\frac{2\sqrt{2}}{5}$

(D) $\frac{\sqrt{17}}{5}$

(E) $\frac{\sqrt{51}}{8}$

(F) $\frac{\sqrt{34}}{8}$

[TMUA, 2018S1Q20]

Find the value of

$$\sin^2 0° + \sin^2 1° + \sin^2 2° + \sin^2 3° + \cdots + \sin^2 87° + \sin^2 88° + \sin^2 89° + \sin^2 90°.$$

(A) 0.5

(B) 1

(C) 1.5

(D) 45

(E) 45.5

(F) 46

TMUA 2018 S2

TMUA 2018 S2
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

You also require a soft pencil and an eraser.

This paper is the second of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.

There is no formulae booklet for this test.

[TMUA, 2018S2Q1]

The function f is given, for $x > 0$, by

$$f(x) = \frac{x^3 - 4x}{2\sqrt{x}}$$

Find the values of $f'(4)$.

(A) 3

(B) 9

(C) 9.5

(D) 12

(E) 39.5

(F) 88

[TMUA, 2018S2Q2]

Find the value of the constant term in the expansion of

$$\left(x^6 - \frac{1}{x^2}\right)^{12}$$

(A) -495

(B) -220

(C) -66

(D) 66

(E) 220

(F) 495

[TMUA, 2018S2Q3]

Consider the following statement:

> A car journey consists of two parts. In the first part, the average speed is u km/h. In the second part, the average speed is v km/h. Hence the average speed for the whole journey is $\frac{1}{2}(u+v)$ km/h.

Which of the following examples of car journeys provide(s) a **counterexample** to the statement?

I In the first part of the journey, the car travels at a constant speed of 50 km/h for 100 km. In the second part of the journey, the car travels at a constant speed of 40 km/h for 100 km.

II In the first part of the journey, the car travels at a constant speed of 50 km/h for one hour. In the second part of the journey, the car travels at a constant speed of 40 km/h for one hour.

III In the first part of the journey, the car travels at a constant speed of 50 km/h for 80 km. In the second part of the journey, the car travels at a constant speed of 40 km/h for 100 km.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2018S2Q4]

The non-zero real number c is such that the equation $\cos x = c$ has two solutions for $0 < x < \frac{3}{2}\pi$.

How many solutions of the equation $\cos^2 2x = c^2$ are there in the range $0 < x < \frac{3}{2}\pi$?

(A) 2

(B) 3

(C) 4

(D) 6

(E) 7

(F) 8

[TMUA, 2018S2Q5]

The two diagonals of the quadrilateral Q are perpendicular.

Consider the following statements:

I One of the diagonals of Q is a line of symmetry of Q.

II The midpoints of the sides of Q are the vertices of a square.

Which of these statements is/are **necessarily** true for the quadrilateral Q ?

(A) neither of them

(B) I only

(C) II only

(D) I and II

[TMUA, 2018S2Q6]

Which one of the following functions provides a **counterexample** to the statement:

if $f'(x) > 0$ for all real x, **then** $f(x) > 0$ for all real x.

(A) $f(x) = x^2 + 1$

(B) $f(x) = x^2 - 1$

(C) $f(x) = x^3 + x + 1$

(D) $f(x) = 1 - x$

(E) $f(x) = 2^x$

[TMUA, 2018S2Q7]

Sequence 1 is an arithmetic progression with first term 11 and common difference 3.

Sequence 2 is an arithmetic progression with first term 2 and common difference 5.

Some numbers that appear in Sequence 1 also appear in Sequence 2. Let N be the 20th such number.

What is the remainder when N is divided by 7?

(A) 0

(B) 1

(C) 2

(D) 3

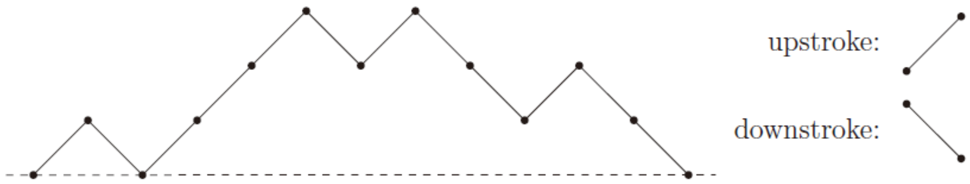
(E) 4

(F) 5

(G) 6

[TMUA, 2018S2Q8]

The diagram shows an example of a *mountain profile*.



This consists of *upstrokes* which go upwards from left to right, and *downstrokes* which go downwards from left to right. The example shown has six upstrokes and six downstrokes. The horizontal line at the bottom is known as *sea level*.

A *mountain profile of order* n consists of n upstrokes and n downstrokes, with the condition that the profile begins and ends at sea level and never goes below sea level (although it might reach sea level at any point). So the example shown is a mountain profile of order 6.

Mountain profiles can be coded by using U to indicate an upstroke and D to indicate a downstroke. The example shown has the code UDUUUDUDDUDD. A sequence of U's and D's obtained from a mountain profile in this way is known as a valid code.

Which of the following statements is/are true?

I If a valid code is written in reverse order, the result is always a valid code.

II If each U in a valid code is replaced by D and each D by U, the result is always a valid code.

III If U is added at the beginning of a valid code and D is added at the end of the code, the result is always a *valid code*.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2018S2Q9]

Consider the following attempt to solve the equation $4x\sqrt{2x-1} = 10x - 5$:

$$4x\sqrt{2x-1} = 10x - 5$$

(I)

$$4x\sqrt{2x-1} = 5(2x-1)$$

(II)

$$16x^2(2x-1) = 25(2x-1)^2$$

(III)

$$16x^2 = 25(2x-1)$$

(IV)

$$16x^2 - 50x + 25 = 0$$

(V)

$$(8x-5)(2x-5) = 0$$

(VI)

The solutions of the original equation are $x = \frac{5}{8}$ and $x = \frac{5}{2}$.

Which one of the following is true?

(A) The solution is correct.

(B) Only one of $x = \frac{5}{8}$ and $x = \frac{5}{2}$ is correct and the error arises as a result of step (II).

(C) Only one of $x = \frac{5}{8}$ and $x = \frac{5}{2}$ is correct and the error arises as a result of step (III).

(D) Only one of $x = \frac{5}{8}$ and $x = \frac{5}{2}$ is correct and the error arises as a result of step (IV).

(E) There is another value of x that satisfies the original equation and the error arises as a result of step (II).

(F) There is another value of x that satisfies the original equation and the error arises as a result of step (III).

(G) There is another value of x that satisfies the original equation and the error arises as a result of step (IV).

[TMUA, 2018S2Q10]

The function $f(x)$ is defined for all real numbers.

Consider the following three conditions, where a is a real constant:

I $\quad f(a-x) = f(a+x)$ for all real x.

II $\quad f(2a-x) = f(x)$ for all real x.

III $\quad f(a-x) = f(x)$ for all real x.

Which of these conditions is/are **necessary and sufficient** for the graph of $y = f(x)$ to have reflection symmetry in the line $x = a$?

	Condition I is necessary and sufficient	**Condition II is necessary and sufficient**	**Condition III is necessary and sufficient**
(A)	yes	yes	yes
(B)	yes	yes	no
(C)	yes	no	yes
(D)	yes	no	no
(E)	no	yes	yes
(F)	no	yes	no
(G)	no	no	yes
(H)	no	no	no

[TMUA, 2018S2Q11]

Consider the equation $2^x = mx + c$, where m and c are real constants.

Which of the following statements is/are true?

I $\quad$ The equation has a negative real solution **only if** $c > 1$.

II $\quad$ The equation has two distinct real solutions **if** $c > 1$.

III $\quad$ The equation has two distinct positive real solutions **if and only if** $c \leq 1$.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2018S2Q12]

Consider the following statement:

For any positive integer N there is a positive integer K such that $N(Km+1)-1$ is not prime for any positive integer m.

Which one of the following is the negation of this statement?

(A) For any positive integer N there is a positive integer K such that there is a positive integer m for which $N(Km+1)-1$ is prime.

(B) For any positive integer N there is a positive integer K such that there is a positive integer m for which $N(Km+1)-1$ is not prime.

(C) For any positive integer N there is a positive integer K such that for any positive integer m, $N(Km+1)-1$ is not prime.

(D) For any positive integer N, any positive integer K and any positive integer m, $N(Km+1)-1$ is not prime.

(E) There is a positive integer N such that for any positive integer K there is a positive integer m for which $N(Km+1)-1$ is not prime.

(F) There is a positive integer N such that for any positive integer K there is a positive integer m for which $N(Km+1)-1$ is prime.

(G) There is a positive integer N such that for any positive integer K and any positive integer m, $N(Km+1)-1$ is prime.

(H) There is a positive integer N and a positive integer K for which there is no positive integer m for which $N(Km+1)-1$ is prime.

[TMUA, 2018S2Q13]

The following is an attempted proof of the conjecture:

$$\textbf{if } \tan\theta > 0, \textbf{ then } \sin\theta + \cos\theta > 1.$$

Suppose $\tan\theta > 0$, so in particular $\cos\theta \neq 0$.

Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, then $\sin\theta\cos\theta = \tan\theta\cos^2\theta > 0$. (I)

It follows that $1 + 2\sin\theta\cos\theta > 1$. (II)

Therefore $\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta > 1$, (III)

which factorises to give $(\sin\theta + \cos\theta)^2 > 1$. (IV)

Therefore $\sin\theta + \cos\theta > 1$. (V)

Which one of the following is the case?

(A) The proof is correct.

(B) The proof is incorrect, and the first error occurs in line (I).

(C) The proof is incorrect, and the first error occurs in line (II).

(D) The proof is incorrect, and the first error occurs in line (III).

(E) The proof is incorrect, and the first error occurs in line (IV).

(F) The proof is incorrect, and the first error occurs in line (V).

[TMUA, 2018S2Q14]

In the triangle PQR, $PR = 2$, $QR = p$ and $\angle RPQ = 30°$.

What is the set of **all** the values of p for which this information uniquely determines the length of PQ ?

(A) $p = 1$

(B) $p = \sqrt{3}$

(C) $1 \leq p < 2$

(D) $\sqrt{3} \leq p < 2$

(E) $p = 1$ or $p \geq 2$

(F) $p = \sqrt{3}$ or $p \geq 2$

(G) $p < 2$

(H) $p \geq 2$

[TMUA, 2018S2Q15]

It is given that $f(x) = x^3 + 3qx^2 + 2$, where q is a real constant.

The equation $f(x) = 0$ has 3 distinct real roots.

Which of the following statements is/are **necessarily** true?

I The equation $f(x) + 1 = 0$ has 3 distinct real roots.

II The equation $f(x + 1) = 0$ has 3 distinct real roots.

III The equation $f(-x) - 1 = 0$ has 3 distinct real roots.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2018S2Q16]

In this question, $x_1, x_2, x_3, \ldots$ is an **arithmetic progression**, all of whose terms are integers.

Let n be a positive integer. If the median of the first n terms of the sequence is an integer, which of the following three statements **must** be true?

I The median of the first $n + 2$ terms is an integer.

II The median of the first $2n$ terms is an integer.

III The median of $x_2, x_4, x_6, \ldots, x_{2n}$ is an integer.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2018S2Q17]

A positive integer is called a *squaresum* **if and only if** it can be written as the sum of the squares of two integers. For example, 61 and 9 are both squaresums since $61 = 5^2 + 6^2$ and $9 = 3^2 + 0^2$.

A prime number is called *awkward* **if and only if** it has a remainder of 3 when divided by 4. For example, 23 is awkward since $23 = 5 \times 4 + 3$.

A (true) theorem due to Fermat states that:

A positive integer is a squaresum **if and only if** each of its awkward prime factors occurs to an even power in its prime factorisation.

It follows that 5×23^2 is a squaresum, since 23 occurs to the power 2, but 5×23^3 is not, since 23 occurs to the power 3.

Which one of the following statements is **not** true?

(A) Every square number is a squaresum.

(B) If N and M are squaresums, then so is NM.

(C) If NM is a squaresum, then N and M are squaresums.

(D) If N is not a squaresum, then kN is a squaresum for some number k which is a product of awkward primes.

[TMUA, 2018S2Q18]

$f(x)$ is a polynomial function defined for all real x.

Which of the following is a **necessary** condition for the inequality

$$\frac{f(a)+f(b)}{2} \geq f\left(\frac{a+b}{2}\right)$$

to be true for all real numbers a and b with $a < b$?

(A) $f(x) \geq 0$ for all real x

(B) $f'(x) \geq 0$ for all real x

(C) $f''(x) \geq 0$ for all real x

(D) $f(x) \leq 0$ for all real x

(E) $f'(x) \leq 0$ for all real x

(F) $f''(x) \leq 0$ for all real x

[TMUA, 2018S2Q19]

Three **real** numbers x, y and z satisfy $x > y > z > 1$.

Which one of the following statements must be true?

(A) $\frac{2^{z+1}}{2^x} > \frac{2^x+2^z}{2^y}$

(B) $2 > \frac{3^x+3^z}{3^y}$

(C) $\frac{2\times 5^x}{5^z} > \frac{5^x+5^z}{5^y}$

(D) $2 < \frac{7^x+7^z}{7^y}$

[TMUA, 2018S2Q20]

It is given that the equation $\sqrt{x+p}+\sqrt{x}=p$ has at least one real solution for x, where p is a real constant.

What is the complete set of possible values for p ?

(A) $p = 0$ or $p = 1$

(B) $p = 0$ or $p \geq 1$

(C) $p \geq -x$

(D) $p \geq \sqrt{x}$

(E) $p \geq 0$

(F) $p \geq 1$

TMUA 2019 S1

TMUA 2019 S1
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

You also require a soft pencil and an eraser.

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You can use the question paper for rough working or notes, but **no extra paper** is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

[TMUA, 2019S1Q1]

$f(x)$ is a quadratic function in x.

The graph of $y = f(x)$ passes through the point $(1, -1)$ and has a turning point at $(-1, 3)$.

Find an expression for $f(x)$.

(A) $-x^2 - 2x + 2$

(B) $-x^2 + 2x + 3$

(C) $x^2 - 2x$

(D) $x^2 + 2x - 4$

(E) $2x^2 + 4x + 1$

(F) $-2x^2 - 4x + 5$

[TMUA, 2019S1Q2]

Find the complete set of values of the real constant k for which the expression

$$x^2 + kx + 2x + 1 - 2k$$

is positive for all real values of x.

(A) $-12 < k < 0$

(B) $k < -12$ or $k > 0$

(C) $-\sqrt{6} - 3 < k < \sqrt{6} - 3$

(D) $k < -\sqrt{6} - 3$ or $k > \sqrt{6} - 3$

(E) $-2 < k < \frac{1}{2}$

(F) $k < -2$ or $k > \frac{1}{2}$

(G) $0 < k < 4$

(H) $k < 0$ or $k > 4$

[TMUA, 2019S1Q3]

Find the coefficient of x in the expression:

$$(1+x)^0 + (1+x)^1 + (1+x)^2 + (1+x)^3 + \cdots + (1+x)^{79} + (1+x)^{80}.$$

(A) 80

(B) 81

(C) 324

(D) 628

(E) 3240

(F) 3321

(G) 6480

(H) 6642

[TMUA, 2019S1Q4]

The sequence x_n is given by:

$$x_1 = 10$$
$$x_{n+1} = \sqrt{x_n} \quad \text{for } n \geq 1$$

What is the value of x_{100}?

[Note that a^{b^c} means $a^{(b^c)}$]

(A) $10^{2^{99}}$

(B) $10^{2^{100}}$

(C) $10^{2^{-99}}$

(D) $10^{2^{-100}}$

(E) $10^{-2^{99}}$

(F) $10^{-2^{100}}$

(G) $10^{-2^{-99}}$

(H) $10^{-2^{-100}}$

[TMUA, 2019S1Q5]

S is a geometric sequence.

The sum of the first 6 terms of S is equal to 9 times the sum of the first 3 terms of S.

The 7th term of S is 360.

Find the 1st term of S.

(A) $\frac{40}{27}$

(B) $\frac{40}{9}$

(C) $\frac{40}{3}$

(D) $\frac{45}{16}$

(E) $\frac{45}{8}$

(F) $\frac{45}{4}$

[TMUA, 2019S1Q6]

The circles with equations

$$(x+4)^2 + (y+1)^2 = 64 \quad \text{and}$$
$$(x-8)^2 + (y-4)^2 = r^2 \quad \text{where } r > 0$$

have exactly one point in common.

Find the difference between the two possible values of r.

(A) 4

(B) 10

(C) 16

(D) 26

(E) 50

[TMUA, 2019S1Q7]

A curve has equation

$$y = (2q - x^2)(2qx + 3)$$

The gradient of the curve at $x = -1$ is a function of q.

Find the value of q which minimises the gradient of the curve at $x = -1$.

(A) -1

(B) $-\frac{3}{4}$

(C) $-\frac{1}{2}$

(D) 0

(E) $\frac{1}{2}$

(F) $\frac{3}{4}$

(G) 1

[TMUA, 2019S1Q8]

The function f is such that $0 < f(x) < 1$ for $0 \leq x \leq 1$.

The trapezium rule with n equal intervals is used to estimate $\int_0^1 f(x)\,\mathrm{d}x$ and produces an underestimate.

Using the same number of equal intervals, for which one of the following does the trapezium rule produce an overestimate?

(A) $\int_0^1 (f(x) + 1)\,\mathrm{d}x$

(B) $\int_0^1 2f(x)\,\mathrm{d}x$

(C) $\int_{-1}^0 f(x + 1)\,\mathrm{d}x$

(D) $\int_{-1}^0 f(-x)\,\mathrm{d}x$

(E) $\int_0^1 \left(1 - f(x)\right)\,\mathrm{d}x$

[TMUA, 2019S1Q9]

p is a positive constant.

Find the area enclosed between the curves $y = p\sqrt{x}$ and $x = p\sqrt{y}$.

(A) $\frac{2}{3}p^{\frac{5}{2}} - \frac{1}{2}p^2$

(B) $\frac{4}{3}p^{\frac{2}{5}} - p^2$

(C) $\frac{p^4}{6}$

(D) $\frac{p^4}{3}$

(E) $\frac{2}{3}p^3 - \frac{1}{2}p^4$

(F) $\frac{4}{3}p^3 - p^4$

(G) $2p^4$

[TMUA, 2019S1Q10]

Evaluate

$$\int_{-1}^{3} |x|(1-x)\,\mathrm{d}x$$

(A) $\frac{17}{3}$

(B) $-\frac{17}{3}$

(C) $\frac{16}{3}$

(D) $-\frac{16}{3}$

(E) $\frac{11}{3}$

(F) $-\frac{11}{3}$

[TMUA, 2019S1Q11]

Find the sum of the real values of x that satisfy the simultaneous equations:

$$\log_3(xy^2) = 1$$
$$(\log_3 x)(\log_3 y) = -3$$

(A) $\frac{1}{3}$

(B) 1

(C) 3

(D) $3\frac{1}{9}$

(E) $9\frac{1}{27}$

(F) $9\frac{1}{3}$

(G) 27

(H) $27\frac{1}{9}$

[TMUA, 2019S1Q12]

It is given that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{24\pi(t-1)}{(1+\sqrt{t})} \quad \text{for} \quad t \geq 1$$

and $V = 7$ when $t = 1$.

Find the value of V when $t = 9$.

(A) $208\pi + 7$

(B) $216\pi + 7$

(C) $224\pi + 7$

(D) $416\pi + 7$

(E) $608\pi + 7$

(F) $744\pi + 7$

[TMUA, 2019S1Q13]

Find the maximum value of

$$4^{\sin x} - 4 \times 2^{\sin x} + \frac{17}{4}$$

for real x.

(A) $\frac{1}{4}$

(B) $\frac{5}{2}$

(C) $\frac{13}{2}$

(D) $\frac{21}{2}$

(E) $\frac{65}{4}$

(F) There is no maximum value.

[TMUA, 2019S1Q14]

x satisfies the simultaneous equations

$$\sin 2x + \sqrt{3}\cos 2x = -1$$

and

$$\sqrt{3}\sin 2x - \cos 2x = \sqrt{3}$$

where $0° \leq x \leq 360°$.

Find the sum of the possible values of x.

(A) $210°$

(B) $330°$

(C) $390°$

(D) $660°$

(E) $780°$

(F) $930°$

[TMUA, 2019S1Q15]

Find the real non-zero solution to the equation

$$\frac{2^{(9^x)}}{8^{(3^x)}} = \frac{1}{4}$$

(A) $\log_3 2$

(B) $2\log_3 2$

(C) 1

(D) 2

(E) $\log_2 3$

(F) $2\log_2 3$

[TMUA, 2019S1Q16]

Given that

$$2\int_0^1 f(x)\,\mathrm{d}x + 5\int_1^2 f(x)\,\mathrm{d}x = 14$$

and

$$\int_0^1 f(x+1)\,\mathrm{d}x = 6$$

find the value of

$$\int_0^2 f(x)\,\mathrm{d}x$$

(A) -8

(B) -4

(C) -2

(D) 2

(E) 4

(F) $\frac{29}{5}$

(G) $\frac{32}{5}$

(H) 14

[TMUA, 2019S1Q17]

Find the fraction of the interval $0 \leq \theta \leq \pi$ for which the inequality

$$\left(\sin(2\theta) - \frac{1}{2}\right)(\sin\theta - \cos\theta) \geq 0$$

is satisfied.

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{5}{12}$

(E) $\frac{7}{12}$

(F) $\frac{3}{4}$

(G) $\frac{5}{6}$

(H) $\frac{11}{12}$

[TMUA, 2019S1Q18]

Find the shortest distance between the curve $y = x^2 + 4$ and the line $y = 2x - 2$.

(A) 2

(B) $\sqrt{5}$

(C) $\frac{6\sqrt{5}}{5}$

(D) 3

(E) $\frac{5\sqrt{5}}{3}$

(F) 5

(G) 6

[TMUA, 2019S1Q19]

Find the value of

$$\sum_{k=0}^{90} \sin(10 + 90k)°$$

(A) 0

(B) $\sin 10°$

(C) $\sin 100°$

(D) $\sin 190°$

(E) $\sin 280°$

(F) 1

[TMUA, 2019S1Q20]

What is the complete range of values of k for which the curves with equations

$$y = x^3 - 12x$$

and

$$y = k - (x - 2)^2$$

intersect at **three** distinct points, of which exactly **two** have positive x-coordinates?

(A) $-4 < k < 0$

(B) $-4 < k < 4$

(C) $-4 < k < 16$

(D) $-16 < k < 0$

(E) $-16 < k < 4$

(F) $-16 < k < 16$

TMUA 2019 S2

TMUA 2019 S2
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

You also require a soft pencil and an eraser.

This paper is the second of two papers.

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You can use the question paper for rough working or notes, but **no extra paper** is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

[TMUA, 2019S2Q1]

Find the coefficient of the x^4 term in the expansion of

$$x^2\left(2x+\frac{1}{x}\right)^6$$

(A) 15

(B) 30

(C) 60

(D) 120

(E) 240

[TMUA, 2019S2Q2]

$(2x+1)$ and $(x-2)$ are factors of $2x^3+px^2+q$.

What is the value of $2p+q$?

(A) -10

(B) $-\frac{38}{5}$

(C) $-\frac{22}{3}$

(D) $\frac{22}{3}$

(E) $\frac{38}{5}$

(F) 10

[TMUA, 2019S2Q3]

a, b and c are real numbers.

Given that $ab = ac$, which of the following statements **must** be true?

I $\quad a = 0$

II $\quad b = 0$ **or** $c = 0$

III $\quad b = c$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2019S2Q4]

Consider the following conjecture:

If N is a positive integer that consists of the digit 1 followed by an odd number of 0 digits and then a final digit 1, **then** N is a prime number.

Here are three numbers:

I $\quad N = 101$ (which is a prime number)

II $\quad N = 1001$ (which equals $7 \times 11 \times 13$)

III $\quad N = 10001$ (which equals 73×137)

Which of these provide(s) a counterexample to the conjecture?

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2019S2Q5]

Considering the following statement about the positive integers a, b and n:

$(*)$: ab is divisible by n

The condition 'either a or b is divisible by n' is:

(A) necessary but not sufficient for $(*)$

(B) sufficient but not necessary for $(*)$

(C) necessary and sufficient for $(*)$

(D) not necessary and not sufficient for $(*)$

[TMUA, 2019S2Q6]

A student attempts to solve the equation

$$\cos x + \sin x \tan x = 2\sin x - 1$$

in the range $0 \leq x \leq 2\pi$.

The student's attempt is as follows:

$$\cos x + \sin x \tan x = 2\sin x - 1$$

So $\cos x - \sin x + \sin x \tan x - \sin x = -1$ (I)

So $(\sin x - \cos x)(\tan x - 1) = -1$ (II)

So $\sin x - \cos x = -1$ or $\tan x - 1 = -1$ (III)

So $(\sin x - \cos x)^2 = 1$ or $\tan x = 0$ (IV)

So $2\sin x \cos x = 0$ or $\tan x = 0$ (V)

So $x = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi, 2\pi$ (VI)

Which of the following best describes this attempt?

(A) It is completely correct

(B) It is incorrect, and the first error occurs on line (I)

(C) It is incorrect, and the first error occurs on line (II)

(D) It is incorrect, and the first error occurs on line (III)

(E) It is incorrect, and the first error is that extra solutions were introduced on line (IV)

(F) It is incorrect, and the first error is that extra solutions were introduced on line (V)

(G) It is incorrect, and the first error is not eliminating the values where $\tan x$ is undefined on line (VI)

[TMUA, 2019S2Q7]

For which one of the following statements can the fact that $12^2 + 16^2 = 20^2$ be used to produce a **counterexample**?

(A) If a, b and c are positive integers which satisfy the equation $a^2 + b^2 = c^2$, and the three numbers have no common divisor, then two of them are odd and the other is even.

(B) The equation $a^4 + b^2 = c^2$ has no solutions for which a, b and c are positive integers.

(C) The equation $a^4 + b^4 = c^4$ has no solutions for which a, b and c are positive integers.

(D) If a, b and c are positive integers which satisfy the equation $a^2 + b^2 = c^2$, then one is the arithmetic mean of the other two.

[TMUA, 2019S2Q8]

a, b and c are real numbers with $a < b < c < 0$.

Which of the following statements must be true?

I $\quad ac < ab < a^2$

II $\quad b(c + a) > 0$

III $\quad \frac{c}{b} > \frac{a}{b}$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2019S2Q9]

A large circular table has 40 chairs round it.

What is the smallest number of people who can be sitting at the table already such that the next person to sit down **must** sit next to someone?

(A) 9

(B) 10

(C) 13

(D) 14

(E) 19

(F) 20

[TMUA, 2019S2Q10]

$PQRS$ is a quadrilateral, labelled anticlockwise.

Which one of the following is a **necessary** but **not sufficient** condition for $PQRS$ to be a parallelogram?

(A) $PQ = SR$ and PS is parallel to QR

(B) $PQ = SR$ and PQ is parallel to SR

(C) $PQ = QR = SR = PS$

(D) $PR = QS$

[TMUA, 2019S2Q11]

An **arithmetic** series has n terms, all of which are **integers**.

The sum of the series is 20.

Which of the following statements **must** be true?

I The first term of the series is even.

II n is even.

III The common difference is even.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2019S2Q12]

Most students in a large college study Mathematics. A teacher chooses three different students at random, one after the other.

Consider these three probabilities:

R = P(At least one of the students chosen studies Mathematics)

S = P(The second student chosen studies Mathematics)

T = P(All three of the students chosen study Mathematics)

Which of the following is true?

(A) $R < S < T$

(B) $R < T < S$

(C) $S < R < T$

(D) $S < T < R$

(E) $T < R < S$

(F) $T < S < R$

[TMUA, 2019S2Q13]

A student approximates the integral $\int_a^b \sin^2 x \, dx$ using the trapezium rule with 4 strips. The resulting approximation is an overestimate.

Which of the following is/are **necessarily** true?

I If the student approximates $\int_{-b}^{-a} \sin^2 x \, dx$ in the same way, the result will be an overestimate.

II If the student approximates $\int_a^b \cos^2 x \, dx$ in the same way, the result will be an underestimate.

(A) neither of them

(B) I only

(C) II only

(D) I and II

[TMUA, 2019S2Q14]

Consider the following statements about the polynomial $p(x)$, where $a < b$:

I $p(a) \leq p(b)$

II $p'(a) \leq p'(b)$

III $p''(a) \leq p''(b)$

Which of these statements is a **necessary** condition for $p(x)$ to be increasing for $a \leq x \leq b$?

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2019S2Q15]

The numbers a, b and c are each greater than 1.

The following logarithms are all to the same base:

$$\log(ab^2c) = 7$$
$$\log(a^2bc^2) = 11$$
$$\log(a^2b^2c^3) = 15$$

What is this base?

(A) a

(B) b

(C) c

(D) It is possible to determine the base, but the base is not a, b or c.

(E) There is insufficient information given to determine the base.

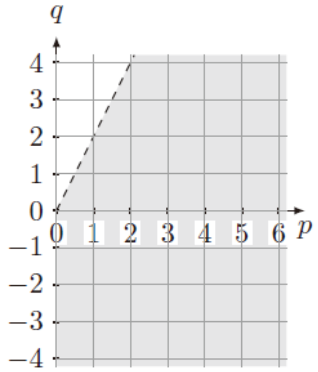
[TMUA, 2019S2Q16]

The graph of the quadratic

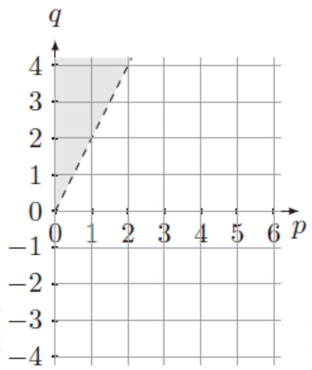
$$y = px^2 + qx + p$$

where $p > 0$, intersects the x-axis at two distinct points.

In which one of the following graphs does the shaded region show the complete set of possible values that p and q could take?



(A)



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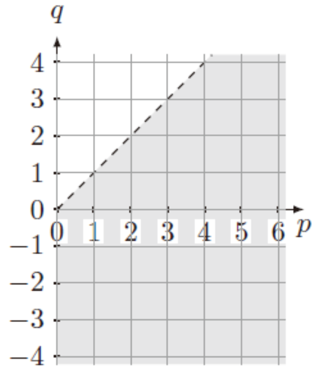

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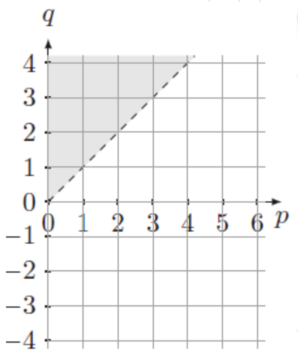

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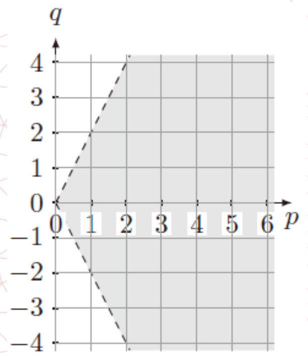
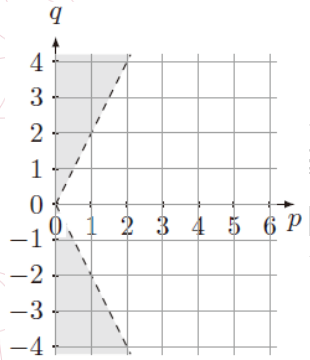
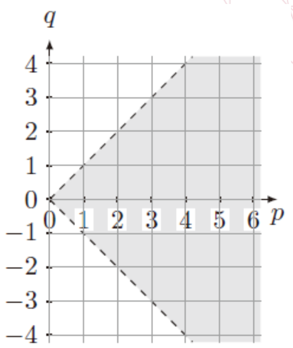


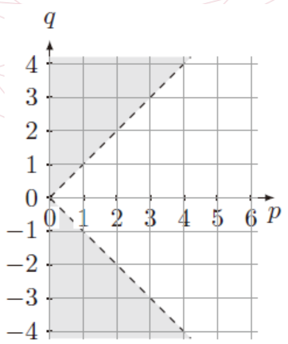
(E)



(F)



(G)



(H)

[TMUA, 2019S2Q17]

A multiple-choice test question offered the following four options relating to a certain statement:

Given that exactly one of these options was correct, which one was it?

(A) The statement is true **if and only if** $x > 1$

(B) The statement is true **if** $x > 1$

(C) The statement is true **if and only if** $x > 2$

(D) The statement is true **if** $x > 2$

[TMUA, 2019S2Q18]

Consider the following inequality:

$$(*):\ a|x| + 1 \leq |x - 2|$$

where a is a real constant.

Which one of the following describes the complete set of values of a such that $(*)$ is **true for all real** x?

(A) $a \leq \frac{3}{2}$

(B) $a \leq 1$

(C) $a \leq \frac{1}{2}$

(D) $a \leq 0$

(E) $a \leq -\frac{1}{2}$

(F) $a \leq -1$

(G) $a \leq -\frac{3}{2}$

(H) There are no such values of a.

[TMUA, 2019S2Q19]

Find the value of the expression

$$\sqrt{8-4\sqrt{2}+1}+\sqrt{9-12\sqrt{2}+8}$$

(A) $\sqrt{26-16\sqrt{2}}$

(B) $4\sqrt{2}-4$

(C) -2

(D) $4-4\sqrt{2}$

(E) 2

(F) $\sqrt{26}-4\sqrt{2}$

(G) 1

[TMUA, 2019S2Q20]

When the graph of the function $y = f(x)$, defined on the real numbers, is reflected in the y-axis and then translated by 2 units in the negative x-direction, the result is the graph of the function $y = g(x)$.

When the graph of the same function $y = f(x)$ is translated by 2 units in the negative x-direction and then reflected in the y-axis, the result is the graph of the function $y = h(x)$.

Which one of the following conditions on $y = f(x)$ is **necessary and sufficient** for the functions $g(x)$ and $h(x)$ to be identical?

(A) $f(x) = f(x+2)$ for all x

(B) $f(x) = f(x+4)$ for all x

(C) $f(x) = f(x+8)$ for all x

(D) $f(x) = f(-x)$ for all x

(E) $f(x) = f(2-x)$ for all x

(F) $f(x) = f(4-x)$ for all x

(G) $f(x) = f(8-x)$ for all x

TMUA 2020 S1

TMUA 2020 S1
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

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Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

[TMUA, 2020S1Q1]

Which of the following is an expression for the first derivative with respect to x of

$$\frac{x^3 - 5x^2}{2x\sqrt{x}}$$

(A) $-\frac{\sqrt{x}}{2}$

(B) $\frac{\sqrt{x}}{4}$

(C) $\frac{3x-5}{4\sqrt{x}}$

(D) $\frac{3\sqrt{x}-5}{4\sqrt{x}}$

(E) $\frac{3\sqrt{x}-10}{3\sqrt{x}}$

(F) $\frac{3x^2-10x}{3\sqrt{x}}$

[TMUA, 2020S1Q2]

$(2x + 1)$ and $(x - 2)$ are factors of $2x^3 + px^2 + q$

What is the value of $2p + q$?

(A) -10

(B) $-\frac{38}{5}$

(C) $-\frac{22}{3}$

(D) $\frac{22}{3}$

(E) $\frac{38}{5}$

(F) 10

[TMUA, 2020S1Q3]

Find the complete set of values of x for which

$$(x+4)(x+3)(1-x)>0 \text{ and } (x+2)(x-2)<0$$

(A) $1<x<2$

(B) $-2<x<1$

(C) $-2<x<2$

(D) $x<-2$ or $x>1$

(E) $x<-4$ or $x>2$

(F) $x<-4$ or $-3<x<1$

(G) $-4<x<-2$ or $x>1$

[TMUA, 2020S1Q4]

The 1st, 2nd and 3rd terms of a geometric progression are also the 1st, 4th and 6th terms, respectively, of an arithmetic progression.

The sum to infinity of the geometric progression is 12.

Find the 1st term of the geometric progression.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

[TMUA, 2020S1Q5]

The curve S has equation

$$y=px^2+6x-q$$

where p and q are constants.

S has a line of symmetry at $x=-\frac{1}{4}$ and touches the x-axis at exactly one point.

What is the value of $p+8q$?

(A) 6

(B) 18

(C) 21

(D) 25

(E) 38

[TMUA, 2020S1Q6]

Find the maximum value of the function

$$f(x) = \frac{1}{5^{2x} - 4(5^x) + 7}$$

(A) $\frac{1}{7}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) 3

(E) 4

(F) 7

[TMUA, 2020S1Q7]

Given that

$$2^{3x} = 8^{(y+3)}$$

and

$$4^{(x+1)} = \frac{16^{(y+1)}}{8^{(y+3)}}$$

What is the value of $x + y$?

(A) -23

(B) -22

(C) -15

(D) -14

(E) -11

(F) -10

[TMUA, 2020S1Q8]

The function f is defined for all real x as

$$f(x) = (p - x)(x + 2)$$

Find the complete set of values of p for which the maximum value of $f(x)$ is less than 4.

(A) $-2 - 4\sqrt{2} < p < -2 + 4\sqrt{2}$

(B) $-2 - 2\sqrt{2} < p < -2 + 2\sqrt{2}$

(C) $-2\sqrt{5} < p < 2\sqrt{5}$

(D) $-6 < p < 2$

(E) $-4 < p < 0$

(F) $-2 < p < 2$

[TMUA, 2020S1Q9]

The quadratic expression $x^2 - 14x + 9$ factorises as $(x - \alpha)(x - \beta)$, where α and β are positive real numbers.

Which quadratic expression can be factorised as $(x - \sqrt{\alpha})(x - \sqrt{\beta})$?

(A) $x^2 - \sqrt{10}x + 3$

(B) $x^2 - \sqrt{14}x + 3$

(C) $x^2 - \sqrt{20}x + 3$

(D) $x^2 - 178x + 81$

(E) $x^2 - 176x + 81$

(F) $x^2 + 196x + 81$

[TMUA, 2020S1Q10]

The following sequence of transformations is applied to the curve $y = 4x^2$.

1. Translation by $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$
2. Reflection in the x-axis
3. Stretch parallel to the x-axis with scale factor 2

What is the equation of the resulting curve?

(A) $y = -x^2 + 12x - 31$

(B) $y = -x^2 + 12x - 41$

(C) $y = x^2 + 12x + 31$

(D) $y = x^2 + 12x + 41$

(E) $y = -16x^2 + 48x - 31$

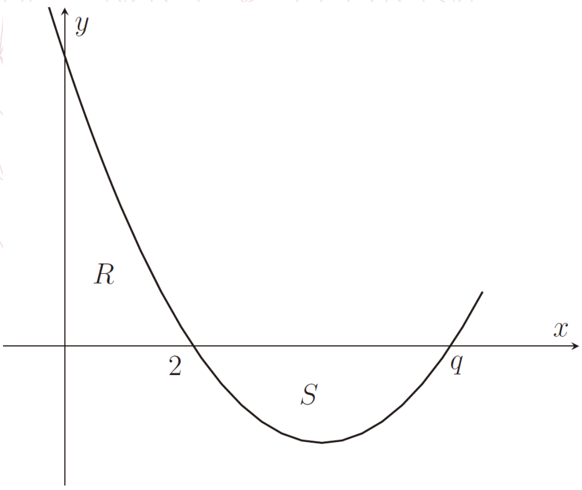
(F) $y = -16x^2 + 48x - 41$

(G) $y = 16x^2 - 48x + 31$

(H) $y = 16x^2 - 48x + 41$

[TMUA, 2020S1Q11]

The quadratic function shown passes through $(2, 0)$ and $(q, 0)$, where $q > 2$.



What is the value of q such that the area of region R equals the area of region S?

(A) $\sqrt{6}$

(B) 3

(C) $\frac{18}{5}$

(D) 4

(E) 6

(F) $\frac{33}{5}$

[TMUA, 2020S1Q12]

How many real solutions are there to the equation

$$3\cos x = \sqrt{x}$$

Where x is in radians?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) infinitely many

[TMUA, 2020S1Q13]

Find the coefficient of x^2y^4 in the expansion of $(1 + x + y^2)^7$

(A) 6

(B) 10

(C) 21

(D) 35

(E) 105

(F) 210

[TMUA, 2020S1Q14]

The area enclosed between the line $y = mx$ and the curve $y = x^3$ is 6.

What is the value of m?

(A) 2

(B) 4

(C) $\sqrt{3}$

(D) $\sqrt{6}$

(E) $2\sqrt{3}$

(F) $2\sqrt{6}$

[TMUA, 2020S1Q15]

Find the positive difference between the two real values of x for which

$$(\log_2 x)^4 + 12\left(\log_2\left(\frac{1}{x}\right)\right)^2 - 2^6 = 0$$

(A) 4

(B) 16

(C) $\frac{15}{4}$

(D) $\frac{17}{4}$

(E) $\frac{255}{16}$

(F) $\frac{257}{16}$

[TMUA, 2020S1Q16]

The circle C_1 has equation $(x+2)^2 + (y-1)^2 = 3$.

The circle C_2 has equation $(x-4)^2 + (y-1)^2 = 3$.

The straight line l is a tangent to both C_1 and C_2 and has positive gradient.

The acute angle between l and the x-axis is θ.

Find the value of $\tan\theta$.

(A) $\frac{1}{2}$

(B) 2

(C) $\frac{\sqrt{2}}{2}$

(D) $\sqrt{2}$

(E) $\frac{\sqrt{6}}{2}$

(F) $\frac{\sqrt{6}}{3}$

(G) $\frac{\sqrt{3}}{3}$

(H) $\sqrt{3}$

[TMUA, 2020S1Q17]

Find the complete set of values of m in terms of c such that the graphs of $y = mx + c$ and $y = \sqrt{x}$ have two points of intersection.

(A) $0 < m < \frac{1}{4c}$

(B) $0 < m < 4c^2$

(C) $m > \frac{1}{4c}$

(D) $m < \frac{1}{4c}$

(E) $m > 4c^2$

(F) $m < 4c^2$

[TMUA, 2020S1Q18]

Find the number of solutions and the sum of the solutions of the equation

$$1 - 2\cos^2 x = |\cos x|$$

where $0 \leq x \leq 180°$

(A) Number of solutions $= 2$ Sum of solutions $= 180°$

(B) Number of solutions $= 2$ Sum of solutions $= 240°$

(C) Number of solutions $= 3$ Sum of solutions $= 180°$

(D) Number of solutions $= 3$ Sum of solutions $= 360°$

(E) Number of solutions $= 4$ Sum of solutions $= 240°$

(F) Number of solutions $= 4$ Sum of solutions $= 360°$

[TMUA, 2020S1Q19]

Find the lowest positive integer for which $x^2 - 52x - 52$ is positive.

(A) 26

(B) 27

(C) 51

(D) 52

(E) 53

(F) 54

[TMUA, 2020S1Q20]

For how many values of a is the equation

$$(x-a)(x^2-x+a)=0$$

satisfied by exactly two distinct values of x?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) more than 4

TMUA 2020 S2

TMUA 2020 S2
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

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[TMUA, 2020S2Q1]

Find the complete set of values of k for which the line $y = x - 2$ crosses or touches the curve $y = x^2 + kx + 2$

(A) $-1 \leq k \leq 3$

(B) $-3 \leq k \leq 5$

(C) $-4 \leq k \leq 4$

(D) $k \leq -1$ or $k \geq 3$

(E) $k \leq -3$ or $k \geq 5$

(F) $k \leq -4$ or $k \geq 4$

[TMUA, 2020S2Q2]

Given that $\tan\theta = 2$ and $180° < \theta < 360°$, find the value of $\cos\theta$

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) $-\frac{\sqrt{3}}{2}$

(E) $\frac{\sqrt{5}}{5}$

(F) $-\frac{\sqrt{5}}{5}$

(G) $\frac{2\sqrt{5}}{5}$

(H) $-\frac{2\sqrt{5}}{5}$

[TMUA, 2020S2Q3]

A student makes the following claim:

For all integers n, the expression $4\left(\frac{9n+1}{2} - \frac{3n-1}{2}\right)$ is divisible by 3.

Here is the student's argument:

$$\begin{aligned} 4\left(\frac{9n+1}{2} - \frac{3n-1}{2}\right) &= 2\left(2\left(\frac{9n+1}{2} - \frac{3n-1}{2}\right)\right) && (1)\\ &= 2(9n+1-3n-1) && (2)\\ &= 2(6n) && (3)\\ &= 12n && (4)\\ &= 3(4n) && (5) \end{aligned}$$

which is always a multiple of 3. (6)

So the expansion $4\left(\frac{9n+1}{2} - \frac{3n-1}{2}\right)$ is always divisible by 3.

Which one of the following is true?

(A) The argument is correct.

(B) The argument is incorrect, and the first error occurs on line (1).

(C) The argument is incorrect, and the first error occurs on line (2).

(D) The argument is incorrect, and the first error occurs on line (3).

(E) The argument is incorrect, and the first error occurs on line (4).

(F) The argument is incorrect, and the first error occurs on line (5).

(G) The argument is incorrect, and the first error occurs on line (6).

[TMUA, 2020S2Q4]

Consider the following statement:

> Every positive integer N that is greater than 6 can be written as the sum of two non-prime integers that are greater than 1.

Which of the following is/are **counterexample**(s) to this statement?

I $\quad N = 5$

II $\quad N = 7$

III $\quad N = 9$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

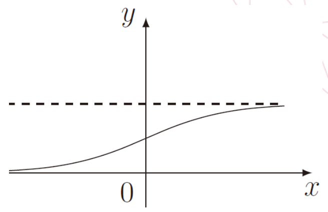
(H) I, II and III

[TMUA, 2020S2Q5]

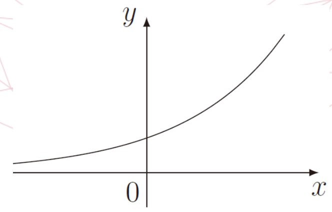
Which one of the following shows the graph of

$$y = \frac{2^x}{1 + 2^x}$$

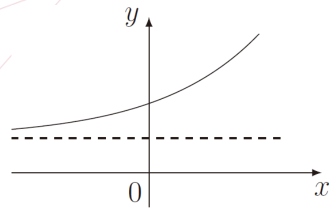
(Dotted lines indicate asymptotes.)



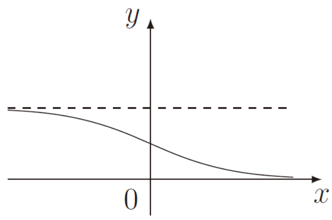
(A)



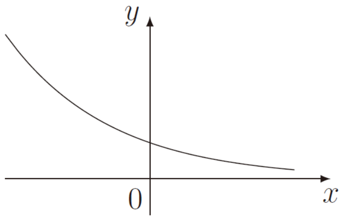
(B)



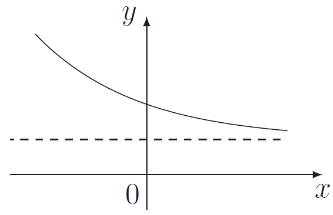
(C)



(D)



(E)



(F)

[TMUA, 2020S2Q6]

The function $f(x)$ is defined for all real values of x.

Which of the following conditions on $f(x)$ is/are **necessary** to ensure that

$$\int_{-5}^{0} f(x)\,\mathrm{d}x = \int_{0}^{5} f(x)\,\mathrm{d}x$$

Condition I: $f(x) = f(-x)$ for $-5 \leq x \leq 5$

Condition II: $f(x) = c$ for $-5 \leq x \leq 5$, where c is a constant

Condition III: $f(x) = -f(-x)$ for $-5 \leq x \leq 5$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2020S2Q7]

Consider the following conditions on a **parallelogram** $PQRS$, labelled anticlockwise:

I length of PQ = length of QR

II The diagonal PR intersects the diagonal QS at right angles

III $\angle PQR = \angle QRS$

Which of these conditions is/are individually **sufficient** for the parallelogram $PQRS$ to be a square?

(A) **Condition I is sufficient**: yes **Condition II is sufficient**: yes **Condition III is sufficient**: yes

(B) **Condition I is sufficient**: yes **Condition II is sufficient**: yes **Condition III is sufficient**: no

(C) **Condition I is sufficient**: yes **Condition II is sufficient**: no **Condition III is sufficient**: yes

(D) **Condition I is sufficient**: yes **Condition II is sufficient**: no **Condition III is sufficient**: no

(E) **Condition I is sufficient**: no **Condition II is sufficient**: yes **Condition III is sufficient**: yes

(F) **Condition I is sufficient**: no **Condition II is sufficient**: yes **Condition III is sufficient**: no

(G) **Condition I is sufficient**: no **Condition II is sufficient**: no **Condition III is sufficient**: yes

(H) **Condition I is sufficient**: no **Condition II is sufficient**: no **Condition III is sufficient**: no

[TMUA, 2020S2Q8]

A student is asked to prove whether the following statement $(*)$ is true or false:

$(*)$ For all real numbers a and b, $|a+b| < |a| + |b|$

The student's proof is as follows:

Statement $(*)$ is **false**. A counterexample is $a = 3$, $b = 4$, as $|3+4| = 7$ and $|3| + |4| = 7$, but $7 < 7$ is false.

Which of the following best describes the student's proof?

(A) The statement $(*)$ is true, and the student's proof is not correct.

(B) The statement $(*)$ is false, but the student's proof is not correct: the counterexample is not valid.

(C) The statement $(*)$ is false, but the student's proof is not correct: the student needs to give all the values of a and b where $|a+b| < |a| + |b|$ is false.

(D) The statement $(*)$ is false, but the student's proof is not correct: the student should have instead stated that for all real numbers a and b, $|a+b| \leq |a| + |b|$.

(E) The statement $(*)$ is false, and the student's proof is fully correct.

[TMUA, 2020S2Q9]

A student wishes to evaluate the function $f(x) = x \sin x$, where x is in radians, but has a calculator that only works in degrees.

What could the student type into their calculator to correctly evaluate $f(4)$?

(A) $(\pi \times 4 \div 180) \times \sin(4)$

(B) $(\pi \times 4 \div 180) \times \sin(\pi \times 4 \div 180)$

(C) $4 \times \sin(\pi \times 4 \div 180)$

(D) $(180 \times 4 \div \pi) \times \sin(4)$

(E) $(180 \times 4 \div \pi) \times \sin(180 \times 4 \div \pi)$

(F) $4 \times \sin(180 \times 4 \div \pi)$

[TMUA, 2020S2Q10]

The real numbers a, b, c and d satisfy both

$$0 < a + b < c + d$$

and

$$0 < a + c < b + d$$

Which of the following inequalities **must** be true?

I $\quad a < d$

II $\quad b < c$

III $\quad a + b + c + d > 0$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

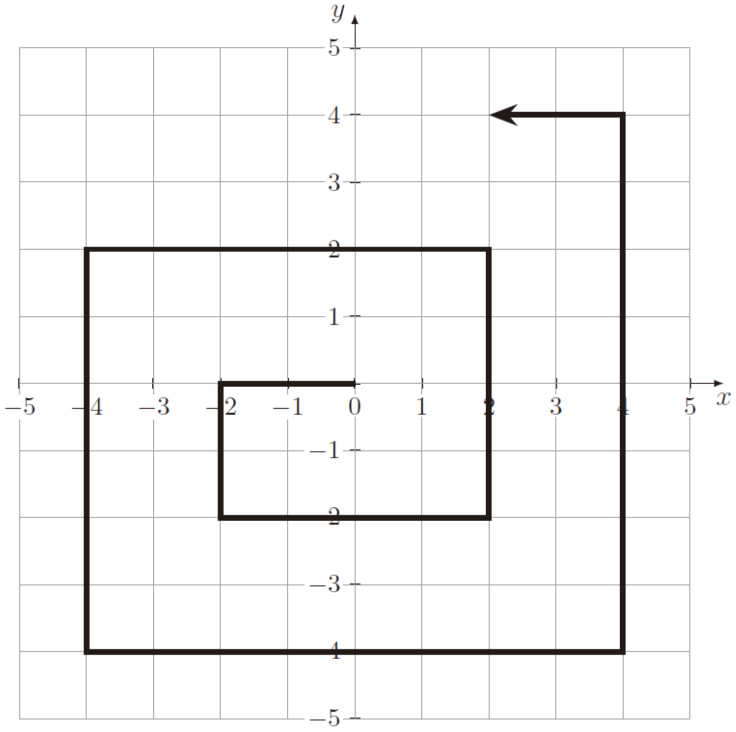
(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2020S2Q11]

A spiral line is drawn as shown.



This spiral pattern continues indefinitely.

Which one of the following points is **not** on the spiral line?

(A) $(99, 100)$

(B) $(99, -100)$

(C) $(-99, 100)$

(D) $(-99, -100)$

(E) $(100, 99)$

(F) $(100, -99)$

(G) $(-100, 99)$

(H) $(-100, -99)$

[TMUA, 2020S2Q12]

Which one of (A)–(F) correctly completes the following statement?

Given that $a < b$, and $f(x) > 0$ for all x with $a < x < b$, the trapezium rule produces an overestimate for $\int_a^b f(x)\,dx$ ...

(A) ... **if** $f'(x) > 0$ **and** $f''(x) < 0$ for all x with $a < x < b$

(B) ... **only if** $f'(x) > 0$ **and** $f''(x) < 0$ for all x with $a < x < b$

(C) ... **if and only if** $f'(x) > 0$ **and** $f''(x) < 0$ for all x with $a < x < b$

(D) ... **if** $f'(x) < 0$ **and** $f''(x) > 0$ for all x with $a < x < b$

(E) ... **only if** $f'(x) < 0$ **and** $f''(x) > 0$ for all x with $a < x < b$

(F) ... **if and only if** $f'(x) < 0$ **and** $f''(x) > 0$ for all x with $a < x < b$

[TMUA, 2020S2Q13]

$f(x)$ is a function for which

$$\int_0^3 (f(x))^2\,dx + \int_0^3 f(x)\,dx = \int_0^1 f(x)\,dx$$

Which of the following claims about $f(x)$ is/are **necessarily** true?

I $f(x) \le 0$ for some x with $1 \le x \le 3$

II $\int_0^3 f(x)\,dx \le \int_0^1 f(x)\,dx$

(A) neither of them

(B) I only

(C) II only

(D) I and II

[TMUA, 2020S2Q14]

An arithmetic sequence T has first term a and common difference d, where a and d are non-zero integers.

Property P is:

> **For some** positive integer m, the sum of the first m terms of the sequence is equal to the sum of the first $2m$ terms of the sequence.

For example, when $a = 11$ and $d = -2$, the sequence T has property P, because

$$11 + 9 + 7 + 5 = 11 + 9 + 7 + 5 + 3 + 1 + (-1) + (-3)$$

i.e. the sum of the first 4 terms equals the sum of the first 8 terms.

Which of the following statements is/are **true**?

I For T to have property P, it is **sufficient** that $ad < 0$.

II For T to have property P, it is **necessary** that d is even.

(A) neither of them

(B) I only

(C) II only

(D) I and II

[TMUA, 2020S2Q15]

Which one of the following is a **necessary and sufficient** condition for

$$\sum_{k=1}^{n} \sin\left(\frac{k\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

to be true?

(A) $n = 1$

(B) n is a multiple of 3

(C) n is a multiple of 6

(D) n is 1 more than a multiple of 3

(E) n is 1 more than a multiple of 6

(F) n is 1 more than a multiple of 6 or n is 2 more than a multiple of 6

[TMUA, 2020S2Q16]

The Fundamental Theorem of Calculus (FTC) tells us that for any polynomial f:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x f(t)\,\mathrm{d}t\right) = f(x)$$

A student calculates $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_x^{2x} t^2\,\mathrm{d}t\right)$ as follows:

(I) $\int_x^{2x} t^2\,\mathrm{d}t = \int_0^{2x} t^2\,\mathrm{d}t - \int_0^x t^2\,\mathrm{d}t$

(II) By FTC, $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x t^2\,\mathrm{d}t\right) = x^2$

(III) By FTC, $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^{2x} t^2\,\mathrm{d}t\right) = (2x)^2 = 4x^2$

(IV) So $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_x^{2x} t^2\,\mathrm{d}t\right) = 4x^2 - x^2$

(V) giving $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_x^{2x} t^2\,\mathrm{d}t\right) = 3x^2$

Which of the following best describes the student's calculation?

(A) The calculation is completely correct.

(B) The calculation is incorrect, and the first error occurs on line (I).

(C) The calculation is incorrect, and the first error occurs on line (II).

(D) The calculation is incorrect, and the first error occurs on line (III).

(E) The calculation is incorrect, and the first error occurs on line (IV).

(F) The calculation is incorrect, and the first error occurs on line (V).

[TMUA, 2020S2Q17]

A set of six **distinct** integers is split into two sets of three.

The first set of three integers has a mean of 10 and a median of 8.

The second set of three integers has a mean of 12 and a median of 9.

What is the smallest possible range of the set of all six integers?

(A) 8

(B) 10

(C) 11

(D) 12

(E) 14

(F) 15

[TMUA, 2020S2Q18]

In this question, $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = px^3 + qx^2 + rx + s$ are cubic polynomials.

If $f(x) - g(x) > 0$ for every real x, which of the following is/are **necessarily** true?

I $a > p$

II **if** $b = q$ **then** $c = r$

III $d > s$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

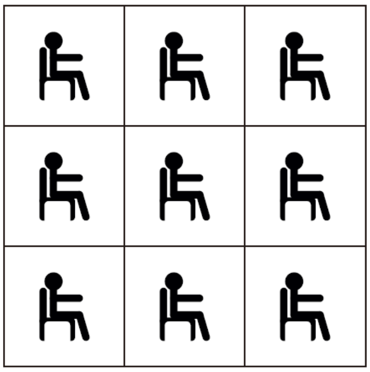
(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2020S2Q19]

Nine people are sitting in the squares of a 3 by 3 grid, one in each square, as shown. Two people are called *neighbours* if they are sitting in squares that share a side. (People in diagonally adjacent squares, which only have a point in common, are not called neighbours.)



Each of the nine people in the grid is either a truth-teller who **always** tells the truth, or a liar who **always** lies.

Every person in the grid says: 'My neighbours are all liars'.

Given only this information, what are the **smallest** number and the **largest** number of people who could be telling the truth?

(A) **smallest**: 1 **largest**: 4

(B) **smallest**: 2 **largest**: 4

(C) **smallest**: 2 **largest**: 5

(D) **smallest**: 3 **largest**: 4

(E) **smallest**: 3 **largest**: 5

(F) **smallest**: 4 **largest**: 4

(G) **smallest**: 4 **largest**: 5

(H) **smallest**: 5 **largest**: 5

[TMUA, 2020S2Q20]

x is a real number and f is a function.

Given that **exactly one** of the following statements is true, which one is it?

(A) $x \geq 0$ **only if** $f(x) < 0$

(B) $x < 0$ **if** $f(x) \geq 0$

(C) $x \geq 0$ **only if** $f(x) \geq 0$

(D) $f(x) < 0$ **if** $x < 0$

(E) $f(x) \geq 0$ **only if** $x \geq 0$

(F) $f(x) \geq 0$ **if and only if** $x < 0$

TMUA 2021 S1

TMUA 2021 S1

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[TMUA, 2021S1Q1]

Two circles have the same radius.

The centre of one circle is $(-2, 1)$.

The centre of the other circle is $(3, -2)$.

The circles intersect at two distinct points.

What is the equation of the straight line through the two points at which the circles intersect?

(A) $3x - 5y = 4$

(B) $3x + 5y = -1$

(C) $5x - 3y = -4$

(D) $5x - 3y = -1$

(E) $5x - 3y = 1$

(F) $5x - 3y = 4$

(G) $5x + 3y = 1$

[TMUA, 2021S1Q2]

The curve $y = x^3 - 6x + 3$ has turning points at $x = \alpha$ and $x = \beta$, where $\beta > \alpha$.

Find

$$\int_{\alpha}^{\beta} x^3 - 6x + 3\,\mathrm{d}x$$

(A) $-8\sqrt{2}$

(B) -10

(C) $-10 + 6\sqrt{2}$

(D) 0

(E) $12 - 8\sqrt{2}$

(F) $6\sqrt{2}$

(G) 12

[TMUA, 2021S1Q3]

An arithmetic progression and a convergent geometric progression each have first term $\frac{1}{2}$.

The sum of the second terms of the two progressions is $\frac{1}{2}$.

The sum of the third terms of the two progressions is $\frac{1}{8}$.

What is the sum to infinity of the geometric progression?

(A) -2

(B) -1

(C) $-\frac{1}{2}$

(D) $-\frac{1}{3}$

(E) $\frac{1}{3}$

(F) $\frac{1}{2}$

(G) 1

(H) 2

[TMUA, 2021S1Q4]

Find the minimum value of the function

$$2^{2x} - 2^{x+3} + 4$$

(A) -16

(B) -12

(C) -8

(D) 0

(E) 4

(F) 20

[TMUA, 2021S1Q5]

The function f is such that

$$f(mn) = \begin{cases} f(m)f(n) & \text{if } mn \text{ is a multiple of } 3 \\ mn & \text{if } mn \text{ is not a multiple of } 3 \end{cases}$$

for all positive integers m and n.

Given that $f(9) + f(16) - f(24) = 0$, what is the value of $f(3)$?

(A) $\frac{8}{3}$

(B) $2\sqrt{2}$

(C) 3

(D) $\frac{16}{5}$

(E) $3\sqrt{2}$

(F) 4

[TMUA, 2021S1Q6]

The function f is given by

$$f(x) = \frac{\cos x + 3}{7 + 5\cos x - \sin^2 x}$$

Find the positive difference between the maximum and the minimum values of $f(x)$.

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) 1

(F) 2

[TMUA, 2021S1Q7]

The function f is such that $f(0) = 0$, and $xf(x) > 0$ for all non-zero values of x.

It is given that

$$\int_{-2}^{2} f(x)\,\mathrm{d}x = 4$$

and

$$\int_{-2}^{2} |f(x)|\,\mathrm{d}x = 8$$

Evaluate

$$\int_{-2}^{0} f(|x|)\,\mathrm{d}x$$

(A) -8

(B) -6

(C) -4

(D) -2

(E) 2

(F) 4

(G) 6

(H) 8

[TMUA, 2021S1Q8]

The line $y = 2x + 3$ meets the curve $y = x^2 + bx + c$ at exactly one point.

The line $y = 4x - 2$ also meets the curve $y = x^2 + bx + c$ at exactly one point.

What is the value of $b - c$?

(A) -9

(B) -5.5

(C) -1

(D) 5

(E) 6

(F) 14

[TMUA, 2021S1Q9]

Find the area enclosed by the graph of

$$|x| + |y| = 1$$

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) 4

(E) $\frac{1}{2}\sqrt{2}$

(F) $\sqrt{2}$

(G) $2\sqrt{2}$

[TMUA, 2021S1Q10]

Use the trapezium rule with 3 strips to estimate

$$\int_{\frac{1}{2}}^{2} 2\log_{10} x \, dx$$

(A) $\log_{10}\frac{\sqrt{6}}{2}$

(B) $\log_{10}\frac{3}{2}$

(C) $\log_{10}\frac{9}{4}$

(D) $\log_{10} 3$

(E) $\log_{10}\frac{81}{16}$

(F) $\log_{10}\frac{\sqrt{23}}{2}$

[TMUA, 2021S1Q11]

The function f is given by

$$f(x) = x^{\frac{1}{7}}(x^2 - x + 1)$$

Find the fraction of the interval $0 < x < 1$ for which $f(x)$ is decreasing.

(A) $\frac{2}{15}$

(B) $\frac{1}{5}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) $\frac{2}{3}$

(F) $\frac{4}{5}$

(G) $\frac{13}{15}$

[TMUA, 2021S1Q12]

The minimum value of the function $x^4 - p^2x^2$ is -9.

p is a real number.

Find the minimum value of the function $x^2 - px + 6$.

(A) -3

(B) $6 - \frac{3\sqrt{2}}{2}$

(C) $\frac{3}{2}$

(D) 3

(E) $\frac{9}{2}$

(F) $6 + \frac{3\sqrt{2}}{2}$

[TMUA, 2021S1Q13]

The function f is such that, for every integer n

$$\int_n^{n+1} f(x)\,\mathrm{d}x = n + 1.$$

Evaluate

$$\sum_{r=1}^{8}\left(\int_0^r f(x)\,\mathrm{d}x\right).$$

(A) 36

(B) 84

(C) 120

(D) 165

(E) 204

(F) 288

[TMUA, 2021S1Q14]

This question uses radians.

Find the number of distinct values of x that satisfy the equation

$$(x+1)(3-x) = 2(1-\cos(\pi x))$$

(A) 2

(B) 3

(C) 4

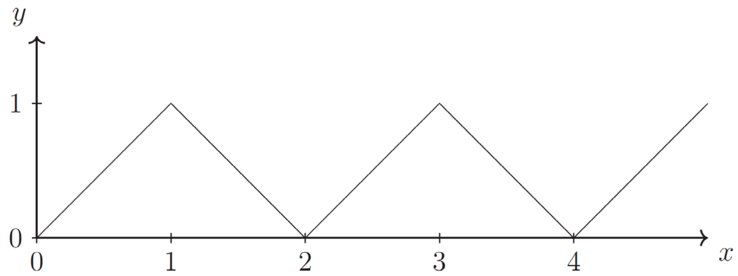
(D) 5

(E) 6

(F) 7

[TMUA, 2021S1Q15]

The diagram shows the graph of $y = f(x)$.



The graph consists of alternating straight-line segments of gradient 1 and -1 and continues in this way for all values of x.

The function g is defined as

$$g(x) = \sum_{r=1}^{10} f(2^{r-1}x).$$

Find the value of

$$\int_0^1 g(x)\,\mathrm{d}x.$$

(A) $\frac{1023}{1024}$

(B) $\frac{1023}{512}$

(C) 5

(D) 10

(E) $\frac{55}{2}$

(F) 55

[TMUA, 2021S1Q16]

Consider the expansion of

$$(a+bx)^n$$

The third term, in **ascending** powers of x, is $105x^2$.

The fourth term, in **ascending** powers of x, is $210x^3$.

The fourth term, in **descending** powers of x, is $210x^3$.

Find the value of $\left(\frac{a}{b}\right)^2$.

(A) $\frac{1}{4}$

(B) $\frac{4}{9}$

(C) $\frac{25}{36}$

(D) $\frac{5}{6}$

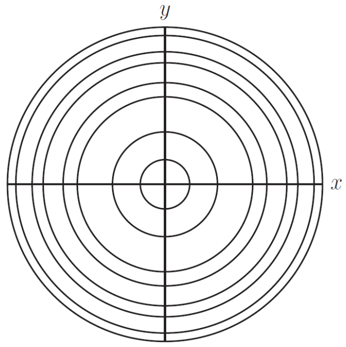
(E) 1

[TMUA, 2021S1Q17]

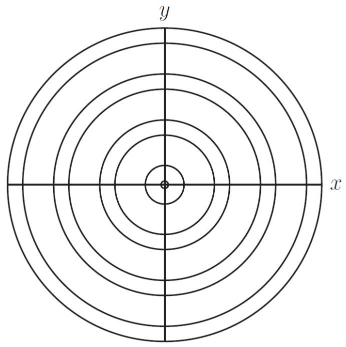
Which of the following sketches shows the graph of

$$\sin(x^2 + y^2) = \frac{1}{2}$$

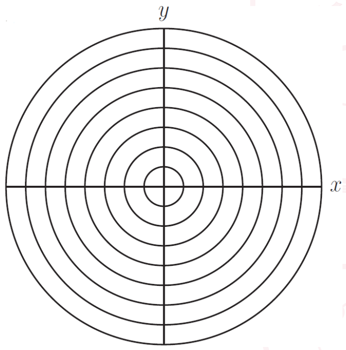
where $x^2 + y^2 \leq 8\pi$?



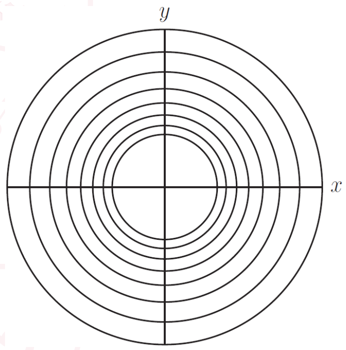
(A)



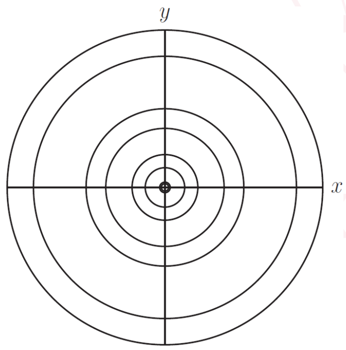
(B)



(C)



(D)



(E)

[TMUA, 2021S1Q18]

The curve with equation

$$x = y^2 - 6y + 11$$

is rotated 90° clockwise about the point P to give the curve C.

P has x-coordinate -2 and y-coordinate 3.

What is the equation of C?

(A) $y = -x^2 - 4x - 3$

(B) $y = -x^2 - 4x - 5$

(C) $y = -x^2 - 6x - 7$

(D) $y = -x^2 - 6x - 11$

(E) $y = x^2 - 4x + 5$

(F) $y = x^2 + 4x + 3$

(G) $y = x^2 - 6x + 11$

(H) $y = x^2 + 6x + 7$

[TMUA, 2021S1Q19]

The equation

$$\sin^2\left(4^{\cos\theta} \times 60°\right) = \frac{3}{4}$$

has exactly three solutions in the range $0° \leq \theta \leq x°$

What is the range of all possible values of x?

(A) $90 \leq x < 120$

(B) $90 \leq x < 270$

(C) $120 \leq x < 240$

(D) $270 \leq x < 300$

(E) $300 \leq x < 360$

(F) $450 \leq x < 630$

[TMUA, 2021S1Q20]

Find the length of the curve with equation

$$2\log_{10}(x-y) = \log_{10}(2-2x) + \log_{10}(y+5)$$

(A) 5

(B) 10

(C) 15

(D) 3π

(E) 9π

(F) 12π

TMUA 2021 S2

TMUA 2021 S2

On-line Exam

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TIME ALLOWED: 75 MINUTES

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There is no formulae booklet for this test.

[TMUA, 2021S2Q1]

Find the value of

$$\int_1^4 \left(3\sqrt{x} + \frac{4}{x^2}\right) \mathrm{d}x$$

(A) -0.75

(B) 7.125

(C) 11

(D) 17

(E) 18

(F) 21.875

(G) 34.5

[TMUA, 2021S2Q2]

$A(0, 2)$ and $C(4, 0)$ are opposite vertices of the square $ABCD$.

What is the equation of the straight line through B and D?

(A) $y = -2x + 5$

(B) $y = -\frac{1}{2}x - 3$

(C) $y = -\frac{1}{2}x + 2$

(D) $y = x$

(E) $y = 2x - 3$

(F) $y = 2x + 2$

[TMUA, 2021S2Q3]

A student is chosen at random from a class. Each student is equally likely to be chosen.

Which of the following conditions is/are **necessary** for the probability that the student wears glasses to equal $\frac{4}{15}$?

I Exactly 11 students in the class do not wear glasses.

II The number of students in the class is divisible by 3.

III The class contains 30 students, and 8 of them wear glasses.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2021S2Q4]

Consider the following claim about positive integers a, b and c:

if a is a factor of bc, **then** a is a factor of b **or** a is a factor of c

Which of the following provide(s) a **counterexample** to this claim?

I $a = 5, b = 10, c = 20$

II $a = 8, b = 4, c = 4$

III $a = 6, b = 7, c = 12$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2021S2Q5]

On which line is the first error in the following argument?

(A) $\sin^2 x + \cos^2 x = 1$ for all values of x.

(B) Therefore $\cos x = \sqrt{1 - \sin^2 x}$ for all values of x.

(C) Hence $1 + \cos x = 1 + \sqrt{1 - \sin^2 x}$ for all values of x.

(D) Thus $(1 + \cos x)^2 = \left(1 + \sqrt{1 - \sin^2 x}\right)^2$ for all values of x.

(E) Substituting $x = \pi$ gives $0 = 4$.

[TMUA, 2021S2Q6]

Consider the following two statements about the polynomial $f(x)$:

P: $f(x) = 0$ for exactly three real values of x

Q: $f(x) = 0$ for exactly two real values of x

Which one of the following is correct?

(A) P is **necessary** but **not sufficient** for Q.

(B) P is **sufficient** but **not necessary** for Q.

(C) P is **necessary and sufficient** for Q.

(D) P is **not necessary** and **not sufficient** for Q.

[TMUA, 2021S2Q7]

A circle has equation $(x - 9)^2 + (y + 2)^2 = 4$

A square has vertices at $(1, 0)$, $(1, 2)$, $(-1, 2)$ and $(-1, 0)$.

A straight line bisects both the area of the circle and the area of the square.

What is the x-coordinate of the point where this straight line meets the x-axis?

(A) 2

(B) 3

(C) 4

(D) 4.5

(E) 5

(F) 6

(G) The straight line is not uniquely determined by the information given, so there is more than one possible point of intersection.

(H) There is no straight line that bisects both the area of the circle and the area of the square.

[TMUA, 2021S2Q8]

Consider the following statement about the polynomial $p(x)$, where a and b are real numbers with $a < b$:

(∗) There exists a number c with $a < c < b$ such that $p(c) = 0$.

Which one of the following is true?

(A) The condition $p(a) = p(b)$ is **necessary and sufficient** for (∗)

(B) The condition $p(a) = p(b)$ is **necessary** but **not sufficient** for (∗)

(C) The condition $p(a) = p(b)$ is **sufficient** but **not necessary** for (∗)

(D) The condition $p(a) = p(b)$ is **not necessary** and **not sufficient** for (∗)

[TMUA, 2021S2Q9]

Consider the following statements about a polynomial $f(x)$:

I $f(x) = px^3 + qx^2 + rx + s$, where $p = 0$.

II There is a real number t for which $f(t) = 0$.

III There are real numbers u and v for which $f(u)f(v) < 0$.

Which of these statements is/are **sufficient** for the equation $f(x) = 0$ to have a real solution?

	Statement I is sufficient	Statement II is sufficient	Statement III is sufficient
(A)	Yes	Yes	Yes
(B)	Yes	Yes	No
(C)	Yes	No	Yes
(D)	Yes	No	No
(E)	No	Yes	Yes
(F)	No	Yes	No
(G)	No	No	Yes
(H)	No	No	No

[TMUA, 2021S2Q10]

The first seven terms of a sequence of positive integers are:

$$u_1 = 15$$
$$u_2 = 21$$
$$u_3 = 30$$
$$u_4 = 37$$
$$u_5 = 44$$
$$u_6 = 51$$
$$u_7 = 59$$

Consider the following statement about this sequence:

($*$) **If** n is a prime number, **then** u_n is a multiple of 3 **or** u_n is a multiple of 5.

What is the smallest value of n that provides a **counterexample** to ($*$)?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

(G) 7

[TMUA, 2021S2Q11]

A student attempts to solve the following problem, where a and b are non-zero real numbers:

Show that **if** $a^2 - 4b^3 \geq 0$ then there exist real numbers x and y such that $a = xy(x+y)$ and $b = xy$.

Consider the following attempt:

$$\begin{array}{ll} (x-y)^2 \geq 0 & \text{(I)} \\ \text{so } x^2 + y^2 - 2xy \geq 0 & \text{(II)} \\ \text{so } (x+y)^2 - 4xy \geq 0 & \text{(III)} \\ \text{so } x^2y^2(x+y)^2 - 4x^3y^3 \geq 0 & \text{(IV)} \\ \text{so } a^2 - 4b^3 \geq 0 & \text{(V)} \end{array}$$

Which of the following best describes this attempt?

(A) It is completely correct.

(B) It is incorrect, but it would be correct if written in the reverse order.

(C) It is incorrect, but the student has correctly proved the converse.

(D) It is incorrect because there is an error in line (II).

(E) It is incorrect because there is an error in line (III).

(F) It is incorrect because there is an error in line (IV).

[TMUA, 2021S2Q12]

Which of the following statements about polynomials f and g is/are true?

I **If** $f(x) \geq g(x)$ for all $x \geq 0$, **then** $\int_0^x f(t)\,\mathrm{d}t \geq \int_0^x g(t)\,\mathrm{d}t$ for all $x \geq 0$.

II **If** $f(x) \geq g(x)$ for all $x \geq 0$, **then** $f'(x) \geq g(x)$ for all $x \geq 0$.

III **If** $f'(x) \geq g(x)$ for all $x \geq 0$, **then** $f(x) \geq g(x)$ for all $x \geq 0$.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2021S2Q13]

A region R in the (x, y)-plane is defined by the simultaneous inequalities

$$y - x < 3$$
$$y - x^2 < 1$$

Which of the following statements is/are true for **every** point in R?

I $\quad -1 < x < 2$

II $\quad (y - x)(y - x^2) < 3$

III $\quad y < 5$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2021S2Q14]

Consider the following simultaneous equations, where p is a real number:

$$p2^x + \log_2 y = 2$$
$$2^x + \log_2 y = 1$$

What is the complete range of p for which these simultaneous equations have a real solution (x, y)?

(A) $p < 1$

(B) $p = 1$

(C) $p > 1$

(D) $p < 1$ or $p > 2$

(E) $p = 1$ and $p < 2$

(F) $p > 1$ and $p < 2$

(G) $p > 2$

(H) All real values of p

[TMUA, 2021S2Q15]

A circle has equation

$$x^2 + ax + y^2 + by + c = 0$$

where a, b and c are non-zero real constants.

Which one of the following is a **necessary and sufficient** condition for the circle to be tangent to the y-axis?

(A) $a^2 = 4c$

(B) $b^2 = 4c$

(C) $\frac{a}{2} = \sqrt{\frac{a^2+b^2}{4} - c}$

(D) $\frac{b}{2} = \sqrt{\frac{a^2+b^2}{4} - c}$

(E) $-\frac{a}{2} = \sqrt{\frac{a^2+b^2}{4} - c}$

(F) $-\frac{b}{2} = \sqrt{\frac{a^2+b^2}{4} - c}$

[TMUA, 2021S2Q16]

p and q are real numbers, and the equation

$$x|x| = px + q$$

has exactly k distinct real solutions for x.

Which one of the following is the complete list of possible values for k?

(A) 0, 1, 2

(B) 0, 1, 2, 3

(C) 0, 1, 2, 3, 4

(D) 0, 2, 4

(E) 1, 2, 3

(F) 1, 2, 3, 4

[TMUA, 2021S2Q17]

Consider the following functions defined for $x > 1$:

$$f(x) = \log_2(\log_2 \sqrt{x})$$

$$g(x) = \log_2(\sqrt{\log_2 x})$$

Which one of the following is true for **all** values of $x > 1$?

(A) $0 \le f(x) \le g(x)$ **or** $g(x) \le f(x) \le 0$

(B) $0 \le g(x) \le f(x)$ **or** $f(x) \le g(x) \le 0$

(C) $\frac{1}{2} \le f(x) \le g(x)$ **or** $g(x) \le f(x) \le \frac{1}{2}$

(D) $\frac{1}{2} \le g(x) \le f(x)$ **or** $f(x) \le g(x) \le \frac{1}{2}$

(E) $1 \le f(x) \le g(x)$ **or** $g(x) \le f(x) \le 1$

(F) $1 \le g(x) \le f(x)$ **or** $f(x) \le g(x) \le 1$

[TMUA, 2021S2Q18]

A student chooses two distinct real numbers x and y with $0 < x < y < 1$.

The student then attempts to draw a triangle ABC with:

$$AB = 1$$
$$\sin A = x$$
$$\sin B = y$$

Which of the following statements is/are correct?

I For some choice of x and y, there is exactly **one** triangle the student could draw.

II For some choice of x and y, there are exactly **two** different triangles the student could draw.

III For some choice of x and y, there are exactly **three** different triangles the student could draw.

(Note that congruent triangles are considered to be the same.)

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2021S2Q19]

The angle θ can take any of the values $1°, 2°, 3°, \ldots, 359°, 360°$.

For how many of these values of θ is it true that

$$\sin\theta\sqrt{1+\sin\theta}\sqrt{1-\sin\theta} + \cos\theta\sqrt{1+\cos\theta}\sqrt{1-\cos\theta} = 0$$

(A) 0

(B) 1

(C) 2

(D) 4

(E) 93

(F) 182

(G) 271

(H) 360

[TMUA, 2021S2Q20]

A sequence of functions $f_1, f_2, f_3, \ldots$ is defined by

$$f_1(x) = |x|$$
$$f_{n+1}(x) = |f_n(x) + x| \quad \text{for } n \geq 1$$

Find the value of

$$\int_{-1}^{1} f_{99}(x)\,dx$$

(A) 0

(B) 0.5

(C) 1

(D) 49.5

(E) 50

(F) 99

(G) 99.5

(H) 100

TMUA 2022 S1

TMUA 2022 S1

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[TMUA, 2022S1Q1]

How many real solutions are there to the equation

$$2\cos^4\theta - 5\cos^2\theta + 3 = 0$$

in the interval $0 \leq \theta \leq 2\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

(G) 7

(H) 8

[TMUA, 2022S1Q2]

Find the complete set of values of p for which the equation

$$x^2 - 2px + y^2 - 6y - p^2 + 8p + 9 = 0$$

describes a circle in the xy-plane.

(A) $p < -\frac{9}{4}$

(B) $0 < p < 4$

(C) $-1 < p < 9$

(D) $p < 0$ or $p > 4$

(E) $p < -1$ or $p > 9$

(F) all real values of p

[TMUA, 2022S1Q3]

Given the following statements about a function f

- $f''(x) = a$ for all x
- $f(0) = 1, f(1) = 2$
- $\int_0^1 f(x)\,\mathrm{d}x = 1$

find the value of a.

(A) -6

(B) -3

(C) -2

(D) 2

(E) 3

(F) 6

[TMUA, 2022S1Q4]

These sectors of circles are similar.

The arc length of the smaller sector is 6.

The difference between the areas of the sectors is 21.

Find the positive difference between the perimeters of the sectors.

(A) 4.5

(B) 7

(C) 8

(D) 9

(E) 10.5

(F) 14

(G) 15

[TMUA, 2022S1Q5]

The terms x_n of a sequence follow the rule

$$x_{n+1} = \frac{x_n + p}{x_n + q}$$

where p and q are real numbers.

Given that $x_1 = 3$, $x_2 = 5$, and $x_3 = 7$, find the value of x_4.

(A) -5

(B) 5

(C) $\frac{51}{7}$

(D) $\frac{15}{2}$

(E) $\frac{23}{3}$

(F) 9

(G) 11

(H) 13

[TMUA, 2022S1Q6]

Given that

$$\int_{\log_2 5}^{\log_2 20} x \, \mathrm{d}x = \log_2 M$$

what is the value of M?

(A) 4

(B) 15

(C) 16

(D) 20

(E) 25

(F) 100

(G) $10,000$

[TMUA, 2022S1Q7]

Find the finite area enclosed between the line $y = 0$ and the curve $y = x^2 - 4|x| - 12$.

(A) $\frac{128}{3}$

(B) $\frac{176}{3}$

(C) $\frac{256}{3}$

(D) 108

(E) 144

(F) 288

[TMUA, 2022S1Q8]

A geometric sequence has first term a and common ratio r, where a and r are positive integers and r is greater than 1.

The sum of the first n terms of this sequence is denoted by S_n.

It is given that the terms of the sequence satisfy

$$S_{30} - S_{20} = kS_{10}$$

for some positive integer k.

What is the smallest possible value of k?

(A) 2^{10}

(B) 2^{20}

(C) 2^{30}

(D) $\frac{2^{10}}{2^{10}-1}$

(E) $2^{10}(2^{10} - 1)$

[TMUA, 2022S1Q9]

This question is about pairs of functions f and g that satisfy

$$f(x) - g(x) = 2\sin x$$
$$f(x)g(x) = \cos^2 x$$

for all real numbers x.

Across all solutions for $f(x)$, what is the minimum value that $f(x)$ attains for any x?

(A) $1 - \sqrt{2}$

(B) $-1 - \sqrt{2}$

(C) 0

(D) -1

(E) -2

(F) -3

(G) -4

[TMUA, 2022S1Q10]

A sequence of translations is applied to the graph of $y = x^3$.

Which of the following graphs could be the result of this sequence of translations?

I $y = x^3 - 3x^2 + 9x - 27$

II $y = x^3 - 9x^2 + 27x - 3$

III $y = 27x^3 - 9x^2 + x - 3$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2022S1Q11]

Evaluate

$$\sum_{n=1}^{100} \log_{10}(3^{1-n})$$

(A) $-4950\log_{10} 3$

(B) $4950\log_{10} 3$

(C) $-5050\log_{10} 3$

(D) $5050\log_{10} 3$

(E) $1-4950\log_{10} 3$

(F) $1+4950\log_{10} 3$

(G) $1–5050\log_{10} 3$

(H) $1+5050\log_{10} 3$

[TMUA, 2022S1Q12]

A family of quadratic curves is given by

$$y_k = 2\left(x-\frac{k}{2}\right)^2 + \frac{k^2}{2} + 4k + 3$$

where k is any real number and y_k is a function of x.

All these curves are sketched, and the point with the lowest y-coordinate among all the curves y_k is (a, b).

Find the value of $a+b$.

(A) -1

(B) -3

(C) -5

(D) -7

(E) -9

[TMUA, 2022S1Q13]

Given that

$$\left(a^3 + \frac{2}{b^3}\right)\left(\frac{2}{a^3} - b^3\right) = \sqrt{2}$$

where a and b are real numbers, what is the least value of ab?

(A) $-\sqrt{2}$

(B) $\sqrt{2}$

(C) $-2\sqrt{2}$

(D) $2\sqrt{2}$

(E) $-\frac{\sqrt{2}}{2}$

(F) $\frac{\sqrt{2}}{2}$

(G) $-2^{\frac{1}{6}}$

(H) $2^{\frac{1}{6}}$

[TMUA, 2022S1Q14]

A circle has centre O and radius 6.

P, Q and R are points on the circumference with angle $POQ \geq \frac{\pi}{2}$.

The area of the triangle POQ is $9\sqrt{3}$.

What is the greatest possible area of triangle PRQ?

(A) $18 + 9\sqrt{3}$

(B) $18\sqrt{3}$

(C) $27 + 9\sqrt{3}$

(D) $27\sqrt{3}$

(E) $36 + 9\sqrt{3}$

(F) $36\sqrt{3}$

[TMUA, 2022S1Q15]

A rectangle is drawn in the region enclosed by the curves p and q, where

$$p(x) = 8 - 2x^2$$
$$q(x) = x^2 - 2$$

such that the sides of the rectangle are parallel to the x- and y-axes.

What is the maximum possible area of the rectangle?

(A) $\frac{26}{9}$

(B) $\frac{52}{9}$

(C) $\frac{4\sqrt{6}}{3}$

(D) $\frac{8\sqrt{6}}{3}$

(E) $4\sqrt{2}$

(F) $8\sqrt{2}$

(G) $\frac{20\sqrt{10}}{9}$

(H) $\frac{40\sqrt{10}}{9}$

[TMUA, 2022S1Q16]

The solutions to $7x^4 - 6x^2 + 1 = 0$ are $\pm\cos\theta$ and $\pm\cos\beta$.

Which one of the following equations has solutions $\pm\sin\theta$ and $\pm\sin\beta$?

(A) $7x^4 - 8x^2 - 5 = 0$

(B) $7x^4 - 8x^2 + 2 = 0$

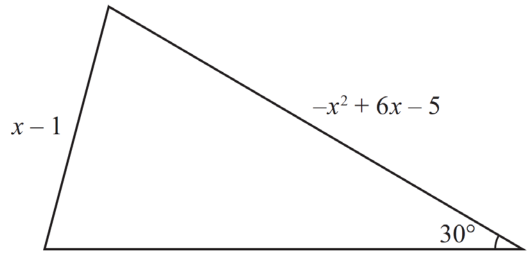
(C) $7x^4 - 6x^2 - 2 = 0$

(D) $7x^4 - 6x^2 + 1 = 0$

(E) $7x^4 + 6x^2 - 1 = 0$

(F) $7x^4 + 6x^2 + 5 = 0$

[TMUA, 2022S1Q17]



Find the complete set of values of x for which there are two non-congruent triangles with the side lengths and angle as shown in the diagram.

(A) $1 < x < 3$

(B) $1 < x < 4$

(C) $1 < x < 5$

(D) $3 < x < 4$

(E) $3 < x < 5$

(F) $4 < x < 5$

[TMUA, 2022S1Q18]

It is given that

$$f(x) = x^2(x-1)^2(x-2)$$
$$g(x) = -p(x-q)^2(x-r)^2$$

where p, q and r are positive and $q < r$.

Find the set of values of q and r that guarantees the greatest number of distinct real solutions of the equation $f(x) = g(x)$ for all p.

(A) $q < 1$ and $r < 1$

(B) $q < 1$ and $1 < r < 2$

(C) $q < 1$ and $r > 2$

(D) $1 < q < 2$ and $1 < r < 2$

(E) $1 < q < 2$ and $r > 2$

(F) $q > 2$ and $r > 2$

[TMUA, 2022S1Q19]

Circle C_1 is defined as $x^2 + y^2 = 25$.

A second circle C_2 has radius 4 and centre (a, b) where

$$-2 \leq a \leq 2 \quad \text{and} \quad -3 \leq b \leq 3$$

If the centre of C_2 is equally likely to be located anywhere within the given range, what is the probability that C_2 intersects C_1?

(A) $\frac{1}{25}$

(B) $\frac{9}{25}$

(C) $\frac{16}{25}$

(D) $\frac{6-\pi}{6}$

(E) $\frac{16-\pi}{24}$

(F) $\frac{24-\pi}{24}$

[TMUA, 2022S1Q20]

n is the number of points of intersection of the graphs

$$y = |x^2 - a^2| \quad \text{and} \quad y = a^2|x - 1|$$

where a is a real number.

What is the smallest value of n that is **not** possible?

(A) $n = 1$

(B) $n = 2$

(C) $n = 3$

(D) $n = 4$

(E) $n = 5$

TMUA 2022 S2

TMUA 2022 S2
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

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Calculators and dictionaries are NOT permitted.

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[TMUA, 2022S2Q1]

Determine the number of stationary points on the curve with equation

$$y = 3x^4 + 4x^3 + 6x^2 - 5$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

[TMUA, 2022S2Q2]

Find the coefficient of the x^5 term in the expansion of

$$(1 + x)^5 \times \sum_{i=0}^{5} x^i$$

(A) 1

(B) 5

(C) 16

(D) 25

(E) 32

[TMUA, 2022S2Q3]

Consider the following statement about the positive integer n

if n is prime, **then** $n^2 + 2$ is **not** prime

Which of the following is a **counterexample** to this statement?

I $\quad n = 2$

II $\quad n = 3$

III $\quad n = 4$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2022S2Q4]

The point P has coordinates (p, q), and the equation of a circle is

$$x^2 + 2fx + y^2 + 2gy + h = 0$$

where f, g, h, p and q are all real constants.

Let L be the distance between the centre of the circle and the point P.

Which one of the following is **sufficient** on its own to be able to calculate L?

(A) the values of f, g and h

(B) the values of f, g, p and q

(C) the values of f, h, p and q

(D) the values of g, h, p and q

(E) none of the options **A-D** is sufficient on its own

[TMUA, 2022S2Q5]

A straight line L passes through $(1, 2)$.

Let P be the statement

if the y-intercept of L is negative, **then** the x-intercept of L is positive.

Which of the following statements **must** be true?

I P

II the converse of P

III the contrapositive of P

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2022S2Q6]

A list consists of n integers.

Consider the following statements:

P: n is odd.

Q: The median of the list is one of the numbers in the list.

Which one of the following is true?

(A) P is **necessary and sufficient** for Q.

(B) P is **necessary** but **not sufficient** for Q.

(C) P is **sufficient** but **not necessary** for Q.

(D) P is **not necessary** and **not sufficient** for Q.

[TMUA, 2022S2Q7]

Consider the following claim:

The difference between two consecutive positive cube numbers is always prime.

Here is an attempted proof of this claim:

I $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

II Taking x to be a positive integer, the difference between two consecutive cube numbers can be expressed as $(x+1)^3 - x^3 = 3x^2 + 3x + 1$

III It is impossible to factorise $3x^2 + 3x + 1$ into two linear factors with integer coefficients because its discriminant is negative.

IV Therefore for every positive integer value of x the integer $3x^2 + 3x + 1$ cannot be factorised.

V Hence, the difference between two consecutive cube numbers will always be prime.

Which of the following best describes this proof?

(A) The proof is completely correct, and the claim is true.

(B) The proof is completely correct, but there are counterexamples to the claim.

(C) The proof is wrong, and the first error occurs on line I.

(D) The proof is wrong, and the first error occurs on line II.

(E) The proof is wrong, and the first error occurs on line III.

(F) The proof is wrong, and the first error occurs on line IV.

(G) The proof is wrong, and the first error occurs on line V.

[TMUA, 2022S2Q8]

A selection, S, of n terms is taken from the arithmetic sequence $1, 4, 7, 10, \ldots, 70$.

Consider the following statement:

$$(*) \text{ There are two distinct terms in } S \text{ whose sum is } 74.$$

What is the smallest value of n for which $(*)$ is **necessarily** true?

(A) 12

(B) 13

(C) 14

(D) 21

(E) 22

(F) 23

[TMUA, 2022S2Q9]

Consider the following statement:

$$(*) \textbf{ For all } \text{real numbers } x, \textbf{ if } x < k \textbf{ then } x^2 < k$$

What is the complete set of values of k for which $(*)$ is true?

(A) no real numbers

(B) $k > 0$

(C) $k < 1$

(D) $k \leq 1$

(E) $0 < k < 1$

(F) $0 < k \leq 1$

(G) all real numbers

[TMUA, 2022S2Q10]

Which of the following statements is/are true?

I **For all** real numbers x and **for all** positive integers n, $x < n$

II **For all** real numbers x, **there exists** a positive integer n such that $x < n$

III **There exists** a real number x such that **for all** positive integers n, $x < n$

(A) none of them

(B) I only

(C) II only

(D) III only

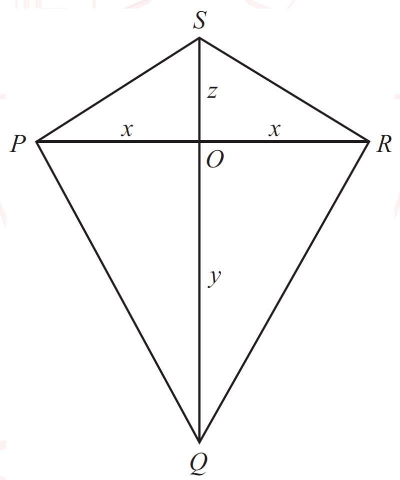
(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2022S2Q11]



The diagram shows a kite $PQRS$ whose diagonals meet at O.

$$OP = x$$
$$OQ = y$$
$$OR = x$$
$$OS = z$$

Which of the following is **necessary and sufficient** for angle SPQ to be a right angle?

(A) $x = y = z$

(B) $2x = y + z$

(C) $x^2 = yz$

(D) $y = z$

(E) $y^2 = x^2 + z^2$

[TMUA, 2022S2Q12]

Place the following integrals in order of size, starting with the smallest.

$$P = \int_0^1 2^{\sqrt{x}}\,dx$$
$$Q = \int_0^1 2^x\,dx$$
$$R = \int_0^1 \left(\sqrt{2}\right)^x dx$$

(A) $P < Q < R$

(B) $P < R < Q$

(C) $Q < P < R$

(D) $Q < R < P$

(E) $R < P < Q$

(F) $R < Q < P$

[TMUA, 2022S2Q13]

Consider the statement $(*)$ about a real number x:

$(*)$ **There exists** a real number y such that $x - xy + y$ is negative.

For how many real values of x is $(*)$ true?

(A) no values of x

(B) exactly one value of x

(C) exactly two values of x

(D) all except exactly two values of x

(E) all except exactly one value of x

(F) all values of x

[TMUA, 2022S2Q14]

Consider the two inequalities:

$$|x+5| < |x+11|$$
$$|x+11| < |x+1|$$

Which one of the following is correct?

(A) There is no real number for which both inequalities are true.

(B) There is exactly one real number for which both inequalities are true.

(C) The real numbers for which both inequalities are true form an interval of length 1.

(D) The real numbers for which both inequalities are true form an interval of length 2.

(E) The real numbers for which both inequalities are true form an interval of length 3.

(F) The real numbers for which both inequalities are true form an interval of length 4.

(G) The real numbers for which both inequalities are true form an interval of length 5.

[TMUA, 2022S2Q15]

The real numbers x, y and z are all greater than 1, and satisfy the equations

$$\log_x y = z \quad \text{and} \quad \log_y z = x$$

Which one of the following equations for $\log_z x$ **must** be true?

(A) $\log_z x = y$

(B) $\log_z x = \frac{1}{y}$

(C) $\log_z x = xy$

(D) $\log_z x = \frac{1}{xy}$

(E) $\log_z x = xz$

(F) $\log_z x = \frac{1}{xz}$

(G) $\log_z x = yz$

(H) $\log_z x = \frac{1}{yz}$

[TMUA, 2022S2Q16]

In this question, $a_1, \dots, a_{100}$ and $b_1, \dots, b_{100}$ and $c_1, \dots, c_{100}$ are three sequences of integers such that

$$a_n \leq b_n + c_n$$

for each n.

Which of the following statements **must** be true?

I (minimum of $a_1, \dots, a_{100}$) $\leq$ (minimum of $b_1, \dots, b_{100}$) + (minimum of $c_1, \dots, c_{100}$)

II (minimum of $a_1, \dots, a_{100}$) $\geq$ (minimum of $b_1, \dots, b_{100}$) + (minimum of $c_1, \dots, c_{100}$)

III (maximum of $a_1, \dots, a_{100}$) $\leq$ (maximum of $b_1, \dots, b_{100}$) + (maximum of $c_1, \dots, c_{100}$)

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2022S2Q17]

A student answered the following question:

a and b are non-zero real numbers.

Prove that the equation $x^3 + ax^2 + b = 0$ has three distinct real roots if $27b\left(b + \frac{4a^3}{27}\right) < 0$.

Here is the student's solution:

I We differentiate $y = x^3 + ax^2 + b$ to get $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2ax = x(3x + 2a)$.

Solving $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ shows that the stationary points are at $(0, b)$ and $\left(-\frac{2a}{3}, b + \frac{4a^3}{27}\right)$.

II If $27b\left(b + \frac{4a^3}{27}\right) < 0$, then b and $b + \frac{4a^3}{27}$ must have opposite signs, and so one of the stationary points is above the x-axis and one is below.

III If the cubic has three distinct real roots, then one of the stationary points is above the x-axis and one is below.

IV Hence if $27b\left(b + \frac{4a^3}{27}\right) < 0$, then the equation has three distinct real roots.

Which one of the following options best describes the student's solution?

(A) It is a completely correct solution.

(B) The student has instead proved the converse of the statement in the question.

(C) The solution is wrong, because the student should have stated step II after step III.

(D) The solution is wrong, because the student should have shown the converse of the result in step II.

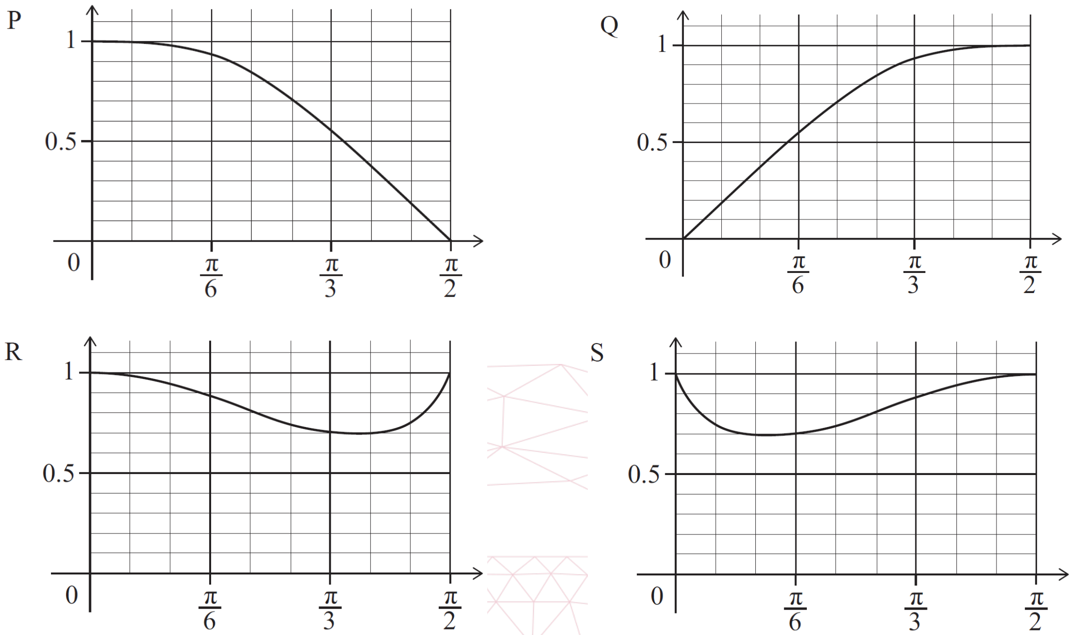
(E) The solution is wrong, because the student should have shown the converse of the result in step III.

[TMUA, 2022S2Q18]

P, Q, R and S show the graphs of

$$y = (\cos x)^{\cos x}, y = (\sin x)^{\sin x}, y = (\cos x)^{\sin x} \text{ and } y = (\sin x)^{\cos x}$$

for $0 < x < \frac{\pi}{2}$ in some order.



Which row in the following table correctly identifies the graphs?

	$y = (\cos x)^{\cos x}$	$y = (\sin x)^{\sin x}$	$y = (\cos x)^{\sin x}$	$y = (\sin x)^{\cos x}$
(A)	P	Q	R	S
(B)	P	Q	S	R
(C)	Q	P	R	S
(D)	Q	P	S	R
(E)	R	S	P	Q
(F)	R	S	Q	P
(G)	S	R	P	Q
(H)	S	R	Q	P

[TMUA, 2022S2Q19]

A polygon has n vertices, where $n \geq 3$. It has the following properties:

- Every vertex of the polygon lies on the circumference of a circle C.
- The centre of the circle C is inside the polygon.
- The radii from the centre of the circle C to the vertices of the polygon cut the polygon into n triangles of equal area.

For which values of n are these properties **sufficient** to deduce that the polygon is regular?

(A) no values of n

(B) $n = 3$ only

(C) $n = 3$ and $n = 4$ only

(D) $n = 3$ and $n \geq 5$ only

(E) all values of n

[TMUA, 2022S2Q20]

The functions f_1 to f_5 are defined on the real numbers by

$$\begin{aligned} f_1(x) &= \cos x \\ f_2(x) &= \sin(\cos x) \\ f_3(x) &= \cos(\sin(\cos x)) \\ f_4(x) &= \sin(\cos(\sin(\cos x))) \\ f_5(x) &= \cos(\sin(\cos(\sin(\cos x)))) \end{aligned}$$

where all numbers are taken to be in radians.

These functions have maximum values m_1, m_2, m_3, m_4 and m_5, respectively.

Which one of the following statements is true?

(A) m_1, m_2, m_3, m_4 and m_5 are all equal to 1

(B) $0 < m_5 < m_4 < m_3 < m_2 < m_1 = 1$

(C) $m_1 = m_3 = m_5 = 1$ and $0 < m_2 = m_4 < 1$

(D) $m_1 = m_3 = m_5 = 1$ and $0 < m_4 < m_2 < 1$

(E) $m_1 = m_3 = 1$ and $0 < m_2 = m_4 < 1$ and $0 < m_5 < 1$

(F) $m_1 = m_3 = 1$ and $0 < m_4 < m_2 < 1$ and $0 < m_5 < 1$

TMUA 2023 S1

TMUA 2023 S1

On-line Exam

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[TMUA, 2023S1Q1]

Given that

$$\int_0^1 (ax+b)\,\mathrm{d}x = 1$$

and

$$\int_0^1 x(ax+b)\,\mathrm{d}x = 1$$

find the value of $a + b$.

(A) -1

(B) 0

(C) 1

(D) 2

(E) 3

(F) 4

(G) 5

[TMUA, 2023S1Q2]

The graphs of $y = x^2 + 5x + 6$ and $y = mx - 3$, where m is a constant, are plotted on the same set of axes.

Given that the graphs do not meet, what is the complete range of possible values of m?

(A) $-1 < m < 11$

(B) $m < -1,\ m > 11$

(C) $-\sqrt{11} < m < \sqrt{11}$

(D) $m < -\sqrt{11}, m > \sqrt{11}$

(E) $-11 < m < 1$

(F) $m < -11,\ m > 1$

[TMUA, 2023S1Q3]

For any integer $n \geq 0$,

$$\int_n^{n+1} f(x)\,\mathrm{d}x = n + 1.$$

Evaluate

$$\int_0^3 f(x)\,\mathrm{d}x + \int_1^3 f(x)\,\mathrm{d}x + \int_2^3 f(x)\,\mathrm{d}x + \int_4^3 f(x)\,\mathrm{d}x + \int_5^3 f(x)\,\mathrm{d}x.$$

(A) -2

(B) 0

(C) 1

(D) 4

(E) 18

(F) 27

[TMUA, 2023S1Q4]

Evaluate

$$\sum_{n=0}^{\infty} \frac{\sin\left(n\pi + \frac{\pi}{3}\right)}{2^n}.$$

(A) 0

(B) $\frac{1}{3}$

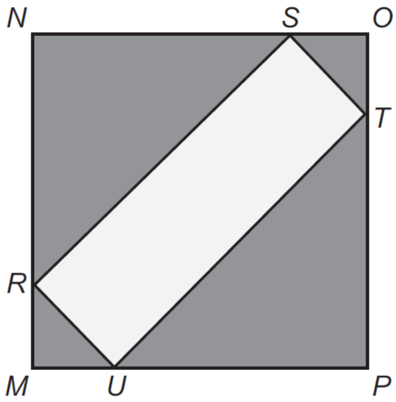
(C) $\frac{\sqrt{3}}{3}$

(D) $\sqrt{3}$

(E) 3

[TMUA, 2023S1Q5]

The following shape has two lines of reflectional symmetry.



[diagram not to scale]

$MNOP$ is a square of perimeter 40 cm.

The vertices of rectangle $RSTU$ lie on the edge of square $MNOP$.

MR has length x cm.

What is the largest possible value of x such that $RSTU$ has area 20 cm²?

(A) $\sqrt{2}$

(B) $\sqrt{10}$

(C) $2\sqrt{15}$

(D) $10\sqrt{2}$

(E) $5+\sqrt{5}$

(F) $5+\sqrt{15}$

[TMUA, 2023S1Q6]

In the simplified expansion of $(2+3x)^{12}$, how many of the terms have a coefficient that is divisible by 12?

(A) 0

(B) 2

(C) 5

(D) 10

(E) 11

(F) 12

(G) 13

[TMUA, 2023S1Q7]

$P(x)$ and $Q(x)$ are defined as follows:

$$P(x) = 2^x + 4$$
$$Q(x) = 2^{(2x-2)} - 2^{(x+2)} + 16$$

Find the largest value of x such that $P(x)$ and $Q(x)$ are in the ratio $4 : 1$, respectively.

(A) 5

(B) 12

(C) 32

(D) $\log_2 3$

(E) $\log_2 5$

(F) $\log_2 12$

(G) $\log_2 20$

[TMUA, 2023S1Q8]

A triangle XYZ is called fun if it has the following properties:

$$\text{angle } YXZ = 30°$$
$$XY = \sqrt{3}a$$
$$YZ = a$$

where a is a constant.

For a given value of a, there are two distinct fun triangles S and T, where the area of S is greater than the area of T.

Find the ratio

$$\text{area of } S : \text{area of } T$$

(A) $1 : 1$

(B) $2 : 1$

(C) $2 : \sqrt{3}$

(D) $\sqrt{3} : 1$

(E) $3 : 1$

[TMUA, 2023S1Q9]

How many solutions are there to

$$(1 + 3\cos 3\theta)^2 = 4$$

in the interval $0° \leq \theta \leq 180°$?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

[TMUA, 2023S1Q10]

The trapezium rule with 4 strips is used to estimate the integral:

$$\int_{-2}^{2} \sqrt{4 - x^2}\, \mathrm{d}x.$$

What is the positive difference between the estimate and the exact value of the integral?

(A) $2(\pi - 2 - 2\sqrt{3})$

(B) $2(\pi - 1 - \sqrt{3})$

(C) $2(2\pi - 1 - \sqrt{3})$

(D) $4(\pi - 1 - \sqrt{3})$

(E) $2\pi - 3\sqrt{3}$

(F) $4\pi - 6\sqrt{3}$

[TMUA, 2023S1Q11]

It is given that $f(x) = x^2 - 6x$.

The curves $y = f(kx)$ and $y = f(x - c)$ have the same minimum point, where $k > 0$ and $c > 0$.

Which of the following is a correct expression for k in terms of c?

(A) $k = \frac{3-c}{3}$

(B) $k = \frac{3}{c+3}$

(C) $k = \frac{c-6}{6}$

(D) $k = \frac{6}{6-c}$

(E) $k = \frac{c+9}{9}$

(F) $k = \frac{9}{c-9}$

[TMUA, 2023S1Q12]

How many solutions are there to the equation

$$\frac{2^{\tan^2 x}}{4^{\sin^2 x}} = 1$$

in the range $0 \leq x \leq 2\pi$?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

(F) 7

(G) 8

[TMUA, 2023S1Q13]

Point P lies on the circle with equation $(x-2)^2+(y-1)^2=16$.

Point Q lies on the circle with equation $(x-4)^2+(y+5)^2=16$.

What is the maximum possible length of PQ?

(A) 10

(B) 14

(C) 16

(D) $2\sqrt{34}$

(E) $10\sqrt{2}$

(F) $8+2\sqrt{10}$

(G) $16+2\sqrt{6}$

[TMUA, 2023S1Q14]

The function

$$f(x)=\frac{2}{3}x^3+2mx^2+n,\ m>0$$

has three distinct real roots.

What is the complete range of possible values of n, in terms of m?

(A) $-\frac{8}{3}m^3<n<0$

(B) $-\frac{4}{3}m^3<n<0$

(C) $0<n<\frac{3}{2}m^2$

(D) $0<n<\frac{40}{3}m^3$

(E) $n<-\frac{8}{3}m^3$

(F) $n<\frac{3}{2}m^2$

(G) $n>-\frac{4}{3}m^3$

(H) $n>\frac{40}{3}m^3$

[TMUA, 2023S1Q15]

The difference between the maximum and minimum values of the function $f(x) = a^{\cos x}$, where $a > 0$ and x is real, is 3.

Find the sum of the possible values of a.

(A) 0

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) 3

(E) $\sqrt{10}$

(F) $\sqrt{13}$

[TMUA, 2023S1Q16]

A right-angled triangle has vertices at $(2,3)$, $(9,-1)$ and $(5,k)$.

Find the sum of all the possible values of k.

(A) -8

(B) -6

(C) 0.25

(D) 2

(E) 2.25

(F) 8.25

(G) 10.25

[TMUA, 2023S1Q17]

A circle C_n is defined by

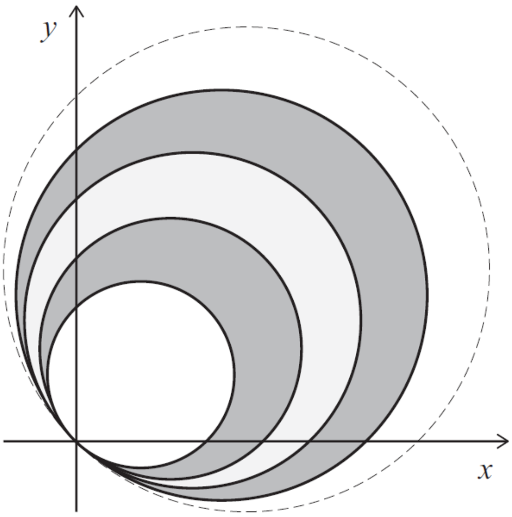
$$x^2 + y^2 = 2n(x + y)$$

where n is a positive integer.

C_1 and C_2 are drawn and the area between them is shaded.

Next, C_3 and C_4 are drawn and the area between them is shaded.

This is shown in the diagram.



[diagram not to scale]

This process continues until 100 circles have been drawn.

What is the total shaded area?

(A) 100π

(B) 500π

(C) 2500π

(D) 5050π

(E) 10100π

(F) 40400π

[TMUA, 2023S1Q18]

You are given that

$$S = 4 + \frac{8k}{7} + \frac{16k^2}{49} + \frac{32k^3}{343} + \cdots + 4\left(\frac{2k}{7}\right)^n + \cdots$$

The value for k is chosen as an integer in the range $-5 \le k \le 5$.

All possible values for k are equally likely to be chosen.

What is the probability that the value of S is a finite number greater than 3?

(A) $\frac{1}{11}$

(B) $\frac{1}{10}$

(C) $\frac{3}{11}$

(D) $\frac{3}{10}$

(E) $\frac{5}{11}$

(F) $\frac{1}{2}$

(G) $\frac{7}{11}$

(H) $\frac{7}{10}$

[TMUA, 2023S1Q19]

The solution to the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = |-6x| \text{ for all } x$$

is $y = f(x) + c$, where c is a constant.

Which one of the following is a correct expression for $f(x)$?

(A) $-\frac{6x}{|x|}$

(B) $\frac{6x}{|x|}$

(C) $-3x|x|$

(D) $3x|x|$

(E) $-3x^2$

(F) $3x^2$

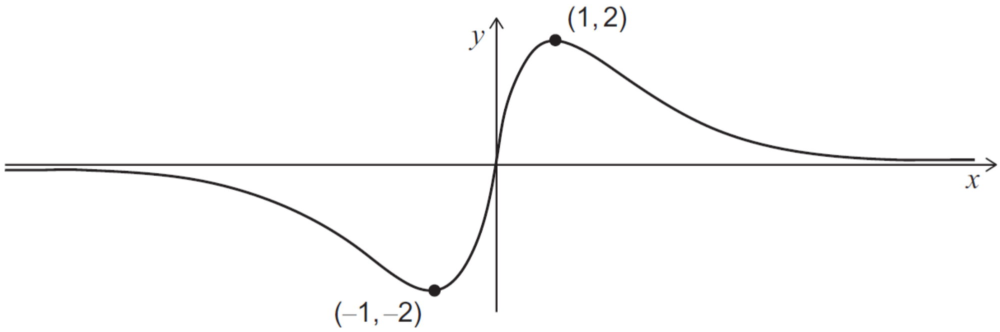
(G) $-x^3$

(H) x^3

[TMUA, 2023S1Q20]

The diagram shows the graph of $y = f(x)$.

The function f attains its maximum value of 2 at $x = 1$, and its minimum value of -2 at $x = -1$.



Find the difference between the maximum and minimum values of $\left(f(x)\right)^2 - f(x)$.

(A) 2

(B) $\frac{9}{4}$

(C) 4

(D) $\frac{17}{4}$

(E) 6

(F) $\frac{25}{4}$

(G) 8

(H) $\frac{33}{4}$

TMUA 2023 S2

TMUA 2023 S2

On-line Exam

Scan the QR code or click on the link to take an on-line exam.

Full solutions can be accessed after submission.

TIME ALLOWED: 75 MINUTES

A separate answer sheet is provided for this paper. Please check you have one.

You also require a soft pencil and an eraser.

This paper is the second of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must **NOT** be used.

There is no formulae booklet for this test.

[TMUA, 2023S2Q1]

Given that

$$\frac{1}{\sqrt{x}-6}-\frac{1}{\sqrt{x}+6}=\frac{3}{11}$$

what is the value of x?

(A) $2\sqrt{15}$

(B) $4\sqrt{5}$

(C) $5\sqrt{2}$

(D) $\sqrt{58}$

(E) 50

(F) 58

(G) 60

(H) 80

[TMUA, 2023S2Q2]

Evaluate

$$\int_9^{16}\left(\frac{1}{\sqrt{x}}+\sqrt{x}\right)^2 \mathrm{d}x-\int_9^{16}\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right)^2 \mathrm{d}x.$$

(A) 0

(B) 2

(C) 4

(D) 7

(E) 14

(F) 28

(G) 75

(H) 175

[TMUA, 2023S2Q3]

Consider the claim:

For all positive real numbers x and y,

$$\sqrt{x^y} = x^{\sqrt{y}}$$

Which of the following is/are a **counterexample** to the claim?

I $x = 1, y = 16$

II $x = 2, y = 8$

III $x = 3, y = 4$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2023S2Q4]

A student attempts to answer the following question.

What is the largest number of consecutive odd integers that are all prime?

The student's attempt is as follows:

I There are two consecutive odd integers that are prime (for example: 17, 19).

II Any three consecutive odd integers can be written in the form $n-2, n, n+2$ for some n.

III If n is one more than a multiple of 3, then $n+2$ is a multiple of 3.

IV If n is two more than a multiple of 3, then $n-2$ is a multiple of 3.

V The only other possibility is that n is a multiple of 3.

VI In each case, one of the integers is a multiple of 3, so not prime.

VII Therefore the largest number of consecutive odd integers that are all prime is two.

Which of the following best describes this attempt?

(A) It is completely correct.

(B) It is incorrect, and the first error is on line I.

(C) It is incorrect, and the first error is on line II.

(D) It is incorrect, and the first error is on line III.

(E) It is incorrect, and the first error is on line IV.

(F) It is incorrect, and the first error is on line V.

(G) It is incorrect, and the first error is on line VI.

(H) It is incorrect, and the first error is on line VII.

[TMUA, 2023S2Q5]

Consider the two statements:

R: k is an integer multiple of π.

S: $\int_0^k \sin 2x \, dx = 0$.

Which of the following statements is true?

(A) R is **necessary and sufficient** for S.

(B) R is **necessary** but **not sufficient** for S.

(C) R is **sufficient** but **not necessary** for S.

(D) R is **not necessary** and **not sufficient** for S.

[TMUA, 2023S2Q6]

Consider the following equation where a is a real number and $a > 1$:

$$(*) \quad a^x = x$$

Which of the following equations must have the same number of real solutions as $(*)$?

I $\quad \log_a x = x$

II $\quad a^{2x} = x^2$

III $\quad a^{2x} = 2x$

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2023S2Q7]

The graph of the line $ax + by = c$ is drawn, where a, b and c are real non-zero constants.

Which one of the following is a **necessary** but **not sufficient** condition for the line to have a positive gradient **and** a positive y-intercept?

(A) $\frac{c}{b} > 0$ and $\frac{a}{b} < 0$

(B) $\frac{c}{b} < 0$ and $\frac{a}{b} > 0$

(C) $a > b > c$

(D) $a < b < c$

(E) a and c have opposite signs

(F) a and c have the same sign

[TMUA, 2023S2Q8]

A student draws a triangle that is acute-angled or obtuse-angled but **not** right-angled.

The student counts the number of straight lines that divide the triangle into two triangles, **at least one** of which is right-angled.

Which of the following statements is/are true?

I The student can draw a triangle for which there is exactly 1 such straight line.

II The student can draw a triangle for which there are exactly 2 such straight lines.

III The student can draw a triangle for which there are exactly 3 such straight lines.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2023S2Q9]

Consider the following statement about a pentagon P:

($*$) **If** at least one of the interior angles in P is $108°$, then all the interior angles in P form an arithmetic sequence.

Which of the following is/are true?

I The statement ($*$)

II The contrapositive of ($*$)

III The converse of ($*$)

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2023S2Q10]

Here is an attempt to solve the inequality $x^4 - 2x^2 - 3 < 0$ by completing the square:

$$x^4 - 2x^2 - 3 < 0$$

I if and only if $x^4 - 2x^2 + 1 < 4$

II if and only if $(x^2 - 1)^2 < 4$

III if and only if $-2 < x^2 - 1 < 2$

IV if and only if $x^2 - 1 < 2$

V if and only if $x^2 < 3$

VI if and only if $-\sqrt{3} < x < \sqrt{3}$

Which of the following statements is true?

(A) The argument is completely correct.

(B) The first error occurs in line I.

(C) The first error occurs in line II.

(D) The first error occurs in line III.

(E) The first error occurs in line IV.

(F) The first error occurs in line V.

(G) The first error occurs in line VI.

[TMUA, 2023S2Q11]

In this question, k is a positive integer.

Consider the following theorem:

$$\textbf{If } 2^k + 1 \text{ is a prime, } \textbf{then } k \text{ is a power of 2. } (*)$$

Which of the following statements, taken individually, is/are equivalent to $(*)$?

I **If** k is a power of 2, **then** $2^k + 1$ is prime.

II $2^k + 1$ is **not** prime **only if** k is **not** a power of 2.

III A **sufficient** condition for k to be a power of 2 is that $2^k + 1$ is prime.

	Statement I is equivalent to $(*)$	Statement II is equivalent to $(*)$	Statement III is equivalent to $(*)$
(A)	Yes	Yes	Yes
(B)	Yes	Yes	No
(C)	Yes	No	Yes
(D)	Yes	No	No
(E)	No	Yes	Yes
(F)	No	Yes	No
(G)	No	No	Yes
(H)	No	No	No

[TMUA, 2023S2Q12]

In this question, p is a real constant.

The equation $\sin x \cos^2 x = p^2 \sin x$ has n distinct solutions in the range $0 \leq x \leq 2\pi$. Which of the following statements is/are true?

I $n = 3$ is **sufficient** for $p > 1$

II $n = 7$ **only if** $-1 < p < 1$

(A) none of them

(B) I only

(C) II only

(D) I and II

[TMUA, 2023S2Q13]

Let x be a real number.

Which **one** of the following statements is a **sufficient** condition for **exactly** three of the other four statements?

(A) $x \geq 0$

(B) $x = 1$

(C) $x = 0$ **or** $x = 1$

(D) $x \geq 0$ **or** $x \leq 1$

(E) $x \geq 0$ **and** $x \leq 1$

[TMUA, 2023S2Q14]

Three lines are given by the equations:

$$\begin{aligned} ax + by + c &= 0 \\ bx + cy + a &= 0 \\ cx + ay + b &= 0 \end{aligned}$$

where a, b and c are non-zero real numbers.

Which one of the following is correct?

(A) **If** two of the lines are parallel, **then** all three are parallel.

(B) **If** two of the lines are parallel, **then** the third is perpendicular to the other two.

(C) **If** two of the lines are parallel, **then** the third is parallel to $y = x$.

(D) **If** two of the lines are parallel, **then** the third is perpendicular to $y = x$.

(E) **If** two of the lines are perpendicular, **then** all three meet at a point.

(F) **If** two of the lines are perpendicular, **then** the third is parallel to $y = x$.

(G) **If** two of the lines are perpendicular, **then** the third is perpendicular to $y = x$.

[TMUA, 2023S2Q15]

The base 10 number 0.03841 has the value

$$0 \times 10^{-1} + 3 \times 10^{-2} + 8 \times 10^{-3} + 4 \times 10^{-4} + 1 \times 10^{-5} = 0.03841.$$

Similarly, the base 2 number 0.01101 has the value

$$0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5} = \frac{13}{32}.$$

What is the value of the recurring base 2 number $0.\dot{0}01\dot{1} = 0.001100110011\ldots$?

(A) $\frac{1}{3}$

(B) $\frac{1}{5}$

(C) $\frac{1}{15}$

(D) $\frac{2}{15}$

(E) $\frac{4}{15}$

(F) $\frac{3}{16}$

(G) $\frac{5}{16}$

(H) $\frac{6}{31}$

[TMUA, 2023S2Q16]

A sequence is defined by:

$$u_1 = a$$
$$u_2 = b$$
$$u_{n+2} = u_n + u_{n+1} \text{ for } n \geq 1$$

where a and b are positive integers. The highest common factor of a and b is 7.

Which of the following statements **must** be true?

I u_{2023} is a multiple of 7.

II If u_1 is not a factor of u_2, then u_1 is not a factor of u_n for any $n > 1$.

III The highest common factor of u_1 and u_5 is 7.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2023S2Q17]

The ceiling of x, written $\lceil x \rceil$, is defined to be the value of x rounded up to the nearest integer.

For example: $\lceil \pi \rceil = 4$, $\lceil 2.1 \rceil = 3$, $\lceil 8 \rceil = 8$.

What is the value of the following integral?

$$\int_0^{99} 2^{\lceil x \rceil}\,dx$$

(A) 2^{99}

(B) $2^{99} - 1$

(C) $2^{99} - 2$

(D) 2^{100}

(E) $2^{100} - 1$

(F) $2^{100} - 2$

[TMUA, 2023S2Q18]

The equation $x^4 + bx^2 + c = 0$ has four distinct real roots **if and only if** which of the following conditions is satisfied?

(A) $b^2 > 4c$

(B) $b^2 < 4c$

(C) $c > 0$ **and** $b > 2\sqrt{c}$

(D) $c > 0$ **and** $b < -2\sqrt{c}$

(E) $c < 0$ **and** $b < 0$

(F) $c < 0$ **and** $b > 0$

[TMUA, 2023S2Q19]

In this question, $f(x)$ is a non-constant polynomial, and $g(x) = xf'(x)$.

$f(x) = 0$ for exactly M real values of x.

$g(x) = 0$ for exactly N real values of x.

Which of the following statements is/are true?

I It is possible that $M < N$.

II It is possible that $M = N$.

III It is possible that $M > N$.

(A) none of them

(B) I only

(C) II only

(D) III only

(E) I and II only

(F) I and III only

(G) II and III only

(H) I, II and III

[TMUA, 2023S2Q20]

Let f be a polynomial with real coefficients.

The integral $I_{p,q}$ where $p < q$ is defined by

$$I_{p,q} = \int_p^q \big(f(x)\big)^2 - \big(f(|x|)\big)^2 \, \mathrm{d}x.$$

Which of the following statements must be true?

1 $\quad I_{p,q} = 0$ **only if** $0 < p$

2 $\quad f'(x) < 0$ for all x **only if** $I_{p,q} < 0$ for all $p < q < 0$

3 $\quad I_{p,q} > 0$ **only if** $p < 0$

(A) none of them

(B) 1 only

(C) 2 only

(D) 3 only

(E) 1 and 2 only

(F) 1 and 3 only

(G) 2 and 3 only

(H) 1, 2 and 3

UEIE TMUA Mock 2022

TMUA Mock 2022
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TIME ALLOWED: 75 MINUTES

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