UE OXBRIDGE-PREP

OXFORD PHYSICS ADMISSIONS TEST

PAT

PAST PAPERS

2006-2023

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Introduction

"PAT Past Papers" is presented by UE International Education (ueie.com), which is designed as a companion to the PAT Standard Course and the PAT Question Practice. It aims to help students to prepare the Oxford Physics Admissions (Aptitude) Test. It is also a useful reference for teachers who are teaching PAT.

All questions in this collection are reproduced from the official past papers released by the University of Oxford, with a few typos from the source files corrected. The 2024 Edition collects a total of **430** PAT questions from 2006 to 2023.

In addition, subscribed users can access two more on-line PAT mock papers, which are made up by our professional teachers based on the latest research on PAT questions.

How to Access Full Solutions

Although this document is free for everyone to use, the detailed solutions to all questions are **only available for subscribed users** who have purchased one of the following products of the UE Oxbridge-Prep series (click on the link to learn more):

PAT Standard Course

PAT Question Practice

At least one of the official solution, hand-written solution or video solution is provided for each question. Hand-written solutions are provided if official solutions are unavailable. There are video solutions for some questions.

All solutions can be accessed **ON-LINE ONLY**.

PAT Past Scores

You may look up PAT past scores through the following page:

Oxford PAT Scores

Statistics of Solutions

Year	Number of Questions	Official Solutions	Handwritten Solutions	Video Solutions
2006	**26**	0	26	2
2007	**26**	0	26	2
2008	**27**	0	27	1
2009	**27**	0	27	1
2010	**25**	0	25	2
2011	**26**	0	26	2
2012	**22**	0	22	2
2013	**21**	0	21	3
2014	**19**	0	19	3
2015	**21**	0	21	2
2016	**21**	0	21	2
2017	**23**	0	23	3
2018	**23**	0	23	2
2019	**24**	0	24	1
2020	**26**	0	26	3
2021	**24**	0	24	0
2022	**23**	0	23	0
2023	**26**	0	26	0
Total	**430**	0 (0%)	430 (100%)	31 (7.2%)

简介

《PAT 历年真题集》由优易国际教育（ueie.com）出品，是 PAT 标准课程和 PAT 刷题训练的配套资料之一。其主要用途是帮助学生提高备考牛津 PAT 考试的效率，以及为教授 PAT 考试的同行老师提供参考。

真题集中的所有真题均由牛津大学官方发布的真题重新排版制作而成，并修订了源文件中的若干印刷错误。2024 版收录了 2006 年至 2023 年共 **430** 道 PAT 真题。

此外，我们还为付费订阅用户提供两套线上 PAT 模考题。这些模考题是由我们的专业教师团队依据近几年 PAT 考试命题趋势而命制的。

真题解析在哪里可以看到

所有用户均可免费使用真题集，但所有题目的解析仅向购买以下任意优易牛剑备考系列产品之一的**付费用户开放**：

PAT 标准课

PAT 刷题训练

所有真题都有详细解析，解析形式为官方解析、手写解析或视频讲解中的一种或多种。如果没有官方解析，则提供手写解析。部分题目提供视频讲解。

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PAT 历年分数线

你可以通过下方页面查询 PAT 历年分数线：

牛津 PAT 分数

解析数量统计

年份	真题数量	官方解析题量	手写解析题量	视频講解题量
2006	**26**	0	26	2
2007	**26**	0	26	2
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簡介

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此外，我們還為付費訂閱用戶提供兩套在線 PAT 模擬題。這些模擬題系我們的專業教師團隊依據近幾年 PAT 考試命題趨勢而命製的。

真題解析在哪裡可以看到

所有用戶均可免費使用真題集，但所有題目的解析僅向購買以下任意優易牛劍備考系列產品之一的**付費用戶開放**：

PAT 標準課

PAT 刷題訓練

所有真題都有詳細解析，解析形式為官方解析、手寫解析或視訊講解中的一種或多種。如果沒有官方解析，則提供手寫解析。部分題目提供影片講解。

所有解析均**只能線上查看**。

PAT 歷年分數線

你可以透過下方頁面查詢 PAT 歷年分數線：

牛津 PAT 分數

解析數量統計

年份	真題數量	官方解析題量	手寫解析題量	影片講解題量
2006	**26**	0	26	2
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2023	**26**	0	26	0
總計	**430**	0 (0%)	430 (100%)	31 (7.2%)

PAT Syllabus

6 June 2018

Please note that any formulae included in this syllabus do not represent an exhaustive list of formulae which might be used within the exam.

Syllabus for the Mathematics content of Physics Aptitude Test

Elementary mathematics:

- Knowledge of elementary mathematics, in particular topics in arithmetic, geometry including coordinate geometry, and probability, will be assumed. Questions may require the manipulation of mathematical expressions in a physical context.

Algebra:

- Knowledge of the properties of polynomials, including the solution of quadratics either using a formula or by factorising.
- Graph sketching including the use of differentiation to find stationary points.
- Transformations of variables.
- Solutions to inequalities.
- Elementary trigonometry including relationships between sine, cosine and tangent (sum and difference formulae will be stated if required).
- Properties of logarithms and exponentials and how to combine logarithms, e.g. $\log(a) + \log(b) = \log(ab)$.
- Knowledge of the formulae for the sum of arithmetic and geometric progressions to n (or infinite) terms.
- Use of the binomial expansion for expressions such as $(a + bx)^n$, using only positive integer values of n.

Calculus:

- Differentiation and integration of polynomials including fractional and negative powers.
- Differentiation to find the slope of a curve, and the location of maxima and minima.
- Integration as the reverse of differentiation and as finding the area under a curve.
- Simplifying integrals by symmetry arguments including use of the properties of even and odd functions (where an even function has $f(x) = f(-x)$, an odd function has $f(-x) = -f(x)$).

Syllabus for the Physics content of Physics Aptitude Test

Mechanics:

- Distance, velocity, speed, acceleration, and the relationships between them, e.g. velocity as the rate of change of distance with time, acceleration as rate of change of velocity with time. Understand the difference between vector quantities (e.g. velocity) and scalar quantities (e.g. speed). Knowledge and use of equations such as speed = distance / time, acceleration = change in velocity / time or the SUVAT equations.
- Interpretation of graphs, e.g. force-distance, distance-time, velocity-time graphs and what the gradient of a curve or area underneath a curve represents.
- Response of a system to multiple forces; Newton's laws of motion; know the difference between weight (= mg) and mass; vector addition of forces.
- Circular motion including equations for centripetal force ($F = m\omega^2 r$ or $F = mv^2/r$) and acceleration ($a = v^2/r$ or $a = \omega^2 r$).
- The meaning of the terms friction, air resistance and terminal velocity and how they can be calculated.
- Levers (including taking moments about a point on an object), pulleys (including calculating the tension in a rope or the overall motion in a system of ropes and pulleys) and other simple machines combining levers, springs and pulleys.
- Springs, including knowledge of Hooke's law (Force $= -kx$) and stored potential energy ($= 1/2kx^2$).
- Kinetic energy (= $1/2mv^2$) and gravitational potential energy (= mgh in a constant gravitational field) and their inter-conversion; what other forms of energy exist (e.g. thermal, sound).
- Conservation of energy and momentum (=mass × velocity); power (= energy transfer/time) and work (= force × distance moved in direction of force).

Waves and optics:

- An understanding of the terms longitudinal and transverse waves; and that waves transfer energy without net movement of matter.
- Be able to define the amplitude, frequency, period, wavelength and speed of a wave. Knowledge and use of formulae for the wave speed = wavelength × frequency and frequency = 1 / period (with units of hertz, Hz).
- Basic properties of the electromagnetic spectrum, e.g. identify and correctly order parts of the spectrum by wavelength or frequency (radio waves, microwaves, IR, visible light, UV, X rays and gamma rays) and the nature and properties of electromagnetic waves (transverse, travel at the speed of light in a vacuum).
- Description of reflection at plane mirrors, where the angle of incidence (the angle between the incident ray and the normal) = angle of reflection (angle between the reflected ray and the normal).
- Refraction, including the definition of refractive index(n) as the ratio of the speed of light in a vacuum to the speed of light in a material and Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ Elementary properties of prisms and optical fibres including total internal reflection, where total internal reflection occurs at an angle θ_c when $\sin\theta_c = n_2/n_1$.
- Qualitative understanding of how interference, diffraction and standing waves can occur.

Electricity and magnetism:

- Understanding of the terms current (= charge / time), voltage (potential difference = energy / charge), charge, resistance (= voltage / current) and links to energy and power (power = voltage × current, power = energy / time). Knowledge of transformers, including how the number of turns on the primary and secondary coils affect the voltage and current.
- Understanding circuit diagrams including batteries, wires, resistors, filament lamps, diodes, capacitors, light dependent resistors and thermistors. Knowledge of current, voltage and resistance rules for series and parallel circuits.
- Knowledge of the force between two point charges (Force = kQ_1Q_2/r^2 (where k is a constant)) and on a point charge in a constant electric field (Force = charge × electric field).
- Understanding that current is a flow of electrons; the photoelectric effect, where photoelectrons are emitted if they are given sufficient energy to overcome the work function of the material, and how to find the energy of accelerated electron beams (energy = charge × potential difference).

Natural world:

- Atomic structure; that atoms consist of protons, neutrons and electrons, definition of the atomic number, Bohr model of the atom.
- Basic knowledge of bodies in our Solar System, including planets, moons, comets and asteroids. (Name and relative positions of the planets should be known but detailed knowledge of their physical parameters is not required).
- Know what is meant by the phrases 'phases of the moon' and 'eclipses' and how the position of the observer on the Earth affects their view of these events.
- Knowledge of circular orbits under gravity including orbital speed, radius, period, centripetal acceleration, and gravitational centripetal force. This may include equating the force between two masses due to gravity ($F = GM_1M_2/r^2$) to centripetal force of a smaller body orbiting a larger body ($F = m\omega^2 r$ or $F = mv^2/r$) and use of centripetal acceleration ($a = v^2/r$ or $a = \omega^2 r$).
- Understanding of the terms satellites; geostationary and polar orbits.

Problem solving:

- Problems may be set which require problem solving based on information provided rather than knowledge about a topic.

If there are parts of the syllabus which you think won't be covered at school by the time of the PAT, we expect you to work on them by yourself. Your teachers might be able to advise you.

Calculators and tables

- Non-graphical calculators may be used but no tables or lists of formulae are allowed. Candidates may be expected to perform standard arithmetical operations by hand, including simple powers and roots, and the manipulation of fractions. Numeric answers should be calculated to 2 significant figures unless indicated otherwise. Specifications for Calculators used in the PAT.

PAT 2006

PAT 2006
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 26 questions [100 marks].

Calculator

You may use any calculator.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics and Physics and Philosophy.

[PAT, 2006Q1][M1]

In the equation $x = ut + \frac{1}{2}at^2$ the term ut represents:

(A) a speed

(B) an acceleration

(C) a displacement

(D) an impulse

[PAT, 2006Q2][M1]

A block of Niobium, a metal with density 8570 kg/m³, has sides of length 3 cm, 4 cm and 5 cm. What is the maximum pressure that can be exerted by this block when it is stood upright on one of its faces?

(A) 4.3 kPa

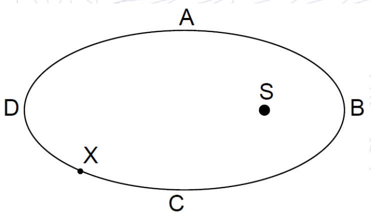
(B) 430 Pa

(C) 2.6 kPa

(D) 510 kPa

[PAT, 2006Q3][M1]

The diagram below shows the approximate orbit of the dwarf planet Eris (X) around the sun (S).



Which of the following statements is **false**?

(A) Eris moves fastest at point D.

(B) Eris moves at the same speed at points A and C.

(C) Eris moves in an ellipse with the sun at one focus.

(D) The potential energy of Eris changes during the orbit.

[PAT, 2006Q4][M1]

A hollow toy boat is floating in a bath. If you take a teaspoon full of water out of the bath and put it in the boat, what happens to the water level in the bath?

(A) The level goes down.

(B) The level goes up.

(C) The level stays the same.

(D) There isn't enough information to say.

[PAT, 2006Q5][M1]

In quantum mechanics the de Broglie wavelength of an object depends on its momentum according to $\lambda = h/p$ where h is Planck's constant. Protons of charge e and mass m are accelerated from rest through a potential V. What is their de Broglie wavelength?

(A) $2h/\sqrt{meV}$

(B) $h/\sqrt{2meV}$

(C) $h\sqrt{meV}$

(D) h/eV

[PAT, 2006Q6][M1]

A car accelerates steadily from 0 m/s to 20 m/s in a distance d and a time t. Another car takes a time $2t$ to accelerate steadily from stationary to the same final velocity. What distance does the second car cover during the new acceleration?

(A) $d/4$

(B) $d/2$

(C) d

(D) $2d$

[PAT, 2006Q7][M1]

An alien civilization is in the business of building custom solar systems. Their basic model has five planets in circular orbits at distances D that are perfect square multiples of a basic length, so that they are in the ratio 1 : 4 : 9 : 16 : 25. For this model the year lengths Y of the planets are in ratios 1 : 8 : 27 : 64 : 125. How are D and Y related?

(A) $Y/D = k$

(B) $D = k\sqrt{Y}$

(C) $Y = kD\sqrt{D}$

(D) $Y^3/D^2 = k$

[PAT, 2006Q8][M1]

A Martian attempts to measure his mass using a set of bathroom scales in his house on Mars, and gets a reading of 93 kg. Unfortunately his scales were designed for use on Venus. Given that the gravitational strengths at the surface of Mars and Venus are 3.8 N/kg and 8.8 N/kg respectively, what is his true mass?

(A) 40 kg

(B) 106 kg

(C) 215 kg

(D) 245 kg

[PAT, 2006Q9][M1]

When a metal bar is cooled it contracts. Which of the following is true?

(A) The density and mass increase.

(B) The density increases and the mass remains constant.

(C) The density and mass are unchanged.

(D) The mass remains constant and the density decreases.

[PAT, 2006Q10][M1]

A hot air balloon is descending at a steady speed of 11 m/s. The pilot drops a sandbag, which takes 7 s to fall to the ground. What was the height of the balloon when the sandbag was released?

(A) 168 m

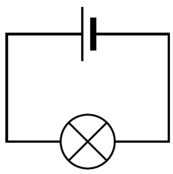
(B) 245 m

(C) 322 m

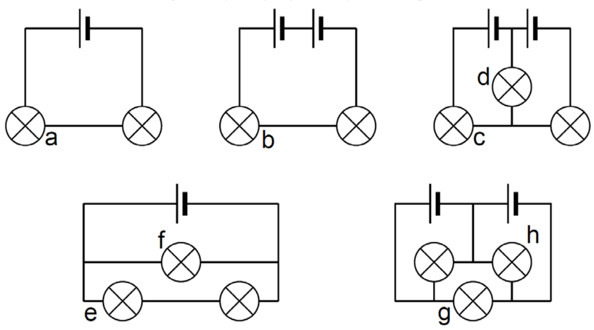
(D) 528 m

[PAT, 2006Q11][M8]

The diagram below shows a circuit in which the bulb lights up with normal brightness.



In the circuits below the bulbs and cells all have the same specifications as the bulb and cell shown above.



Determine whether the bulbs marked by letters in these circuits are brighter than normal, normal, dimmer than normal, or off. [8]

[PAT, 2006Q12][M6]

While exploring the north pole of Mars an astronaut stumbles into a cave, containing a pool of glowing liquid and a collection of coloured cubes. Lacking any proper instruments she uses her spare oxygen tank as a measure and determines that

(a) A red cube and a green cube together are as long as the tank.

(b) Two green cubes and a blue cube together are twice as long as the tank.

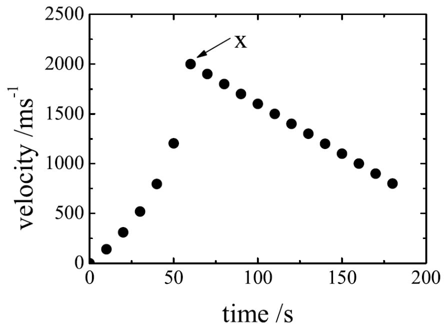
(c) A red cube and a blue cube together are as long as two green cubes.

(d) A red cube, a green cube and a blue cube together weigh the same as the tank.

She also tries dropping the cubes into the pool, and notices that they all float with half their volume exposed. On returning to her base she discovers that her tank is 35 cm long and has a mass of 20 kg. Find the density of the glowing liquid. [6]

[PAT, 2006Q13][M6]

The graph below shows the velocity of a rocket as a function of time.



(a) What has happened at point X? [1]

(b) When is the acceleration of the rocket a maximum? Suggest an explanation of this. [2]

(c) Describe the subsequent motion of the rocket. [2]

(d) How in principle would you determine from this graph the maximum height reached by the rocket? [1]

[PAT, 2006Q14][M20]

This problem concerns the mathematical treatment of a simple model of an electric toy car of mass m, which is initially stationary. The batteries in the car can be considered as an electrical power source with constant power P. You may neglect air-resistance and other experimental imperfections in the calculations.

(a) Suppose that the car is placed on a level surface and that the motor can be treated as a device which converts electrical energy directly into kinetic energy. Calculate the kinetic energy of the car as a function of time, and hence its velocity as a function of time. [3]

(b) Hence calculate the acceleration of the car and the distance traveled by the car as a function of time. [4]

(c) Find the limiting value of the velocity at very large times and comment on whether your result seems reasonable. [2]

(d) Find the limiting value of the acceleration at very large and very small times and comment on whether your results seem reasonable. [4]

(e) Instead of driving the car forward the motor could be used to lift the car up a vertical rope. Calculate the height which the car can reach as a function of time, and hence calculate the velocity at which it climbs the rope. [3]

(f) The calculation in part (e) ignored the fact that as the car is moving then some of the motor's power must be used to give kinetic energy to the car. Assuming that the approach you used in part (e) is correct, calculate the ratio between the kinetic energy and the potential energy of the car climbing the rope as a function of time. Use your result to determine under what circumstances it is reasonable to ignore kinetic energy in this way. [4]

2006

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8

[PAT, 2006Q15][M3]

Evaluate

(i) $2007^2 - 2006^2$ precisely;

(ii) $1.001^6 - 1.001^5$ to one significant figure. [3]

[PAT, 2006Q16][M2]

Find the gradient of the line joining the points $(4, 8)$ and $(5, -2)$. [2]

[PAT, 2006Q17][M3]

Find all the values of x for which

(i) $\log_e(e^{3x}) = 6$;

(ii) $\log_3 x^2 = 2$. [3]

[PAT, 2006Q18][M3]

Simplify

$$13 \sin\left[\tan^{-1}\left(\frac{12}{5}\right)\right]$$

[3]

[PAT, 2006Q19][M8]

Sketch the functions

(i) $\sin^2 x$;

(ii) $\frac{1}{x^2-1}$. [8]

[PAT, 2006Q20][M4]

Circle A has a radius which is 1 cm bigger than circle B, and its area is 2π cm^2 bigger. Find the radii of the two circles. [4]

[PAT, 2006Q21][M3]

Three dice are thrown, one after the other. Calculate the probability that

(i) all three dice give a six;

(ii) all three dice give the same number;

(iii) only the third die gives a six. [3]

[PAT, 2006Q22][M3]

The volume of a spherical balloon increases by 1 cm^3 every second. What is the rate of growth of the radius when the surface area of the balloon is 100 cm^2? [3]

[PAT, 2006Q23][M5]

Find the area between the curve $y = |x^n|$, where n is a positive constant, the line defined by $y = -2$, and the lines defined by $|x| = 2$. [5]

[PAT, 2006Q24][M4]

Sum the following series:

(i) $1 + e^y + e^{2y} + e^{3y} + \cdots$, where $e^y \ll 1$;

(ii) $\log_2 1 + \log_2 2 + \log_2 4 + \cdots + \log_2 2^n$. [4]

[PAT, 2006Q25][M6]

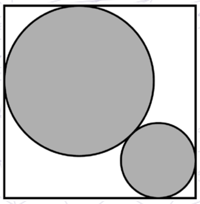
Identify and classify the stationary points of the function

$$y = 5 + 24x - 9x^2 - 2x^3.$$

[6]

[PAT, 2006Q26][M6]

In the figure below, the radius of the larger circle is twice that of the smaller circle. Find an expression for the fraction of the area of the square which is occupied by the two circles. [6]



PAT 2007

PAT 2007
On-line Exam

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TIME ALLOWED: 2 HOURS

Questions and Score

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Calculator

You may use any calculator.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10$ m s^{-2}.

Applicants

For candidates applying to Physics and Physics and Philosophy.

[PAT, 2007Q1][M1]

A cube of metal has sides of length x. The electrical resistance between opposite faces of the cube is:

(A) directly proportional to x

(B) proportional to x^2

(C) inversely proportional to x

(D) independent of x

[PAT, 2007Q2][M1]

An astronaut in the International space station experiences weightlessness because

(A) She is outside the earth's gravitational field

(B) The attractive force of the moon cancels that of the earth

(C) She is moving

(D) She is accelerating at the same rate as the space station

[PAT, 2007Q3][M1]

A 9 V battery is connected across a 100 Ω resistor. Given that the charge on an electron is 1.6×10^{-19} C. What is the number of electrons passing through the resistor every second?

(A) 5.6×10^{17}

(B) 6.9×10^{19}

(C) 5.6×10^{21}

(D) 6.9×10^{15}

[PAT, 2007Q4][M1]

A drop slide in a fairground has a very steep initial slope which gradually curves into a more gentle slope. If a child drops down the slide, what happens to his speed v and the magnitude of his acceleration a?

(A) v and a both increase

(B) v increases and a stays the same

(C) v increases and a decreases

(D) v decreases and a increases

[PAT, 2007Q5][M1]

A spring that obeys Hooke's law has a spring constant k. Two such springs are linked to form a spring of twice the length. What is the spring constant of this new longer spring?

(A) $k/2$

(B) $k/\sqrt{2}$

(C) $\sqrt{2}k$

(D) $2k$

[PAT, 2007Q6][M1]

Two resistors R_1 and R_2 are in parallel with a potential difference V across them. The total power dissipated in this circuit is

(A) $V^2 \times \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(B) $\frac{V^2}{R_1+R_2}$

(C) $V^2 \div \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(D) $V^2(R_1 + R_2)$

[PAT, 2007Q7][M1]

Positron Emission Tomography (PET) scanners frequently operate using the radioactive isotope ^{18}F, which has a half-life of about two hours. The isotope is incorporated into a drug, half of which is excreted by the body every two hours. How long will it take before the quantity of radioactive drug in the body halves?

(A) 0.5 hours

(B) 1 hour

(C) 1.5 hours

(D) 2 hours

[PAT, 2007Q8][M1]

A pressure cooker has an escape valve that is essentially a 125 g weight resting on a circular hole of radius 1 mm. What pressure will lift the weight off the hole?

(A) 400 Pa

(B) 40 kPa

(C) 400 kPa

(D) 400 MPa

[PAT, 2007Q9][M2]

A car of mass 1500 kg is towing a trailer of mass 1000 kg at a steady speed. The driver decides to overtake another car and accelerates at 4 m/s^2. If the frictional force on the trailer is 2500 N what is the force on the towbar during the manoeuvre?

(A) 6500 N

(B) 8500 N

(C) 10000 N

(D) 12500 N

[PAT, 2007Q10][M7]

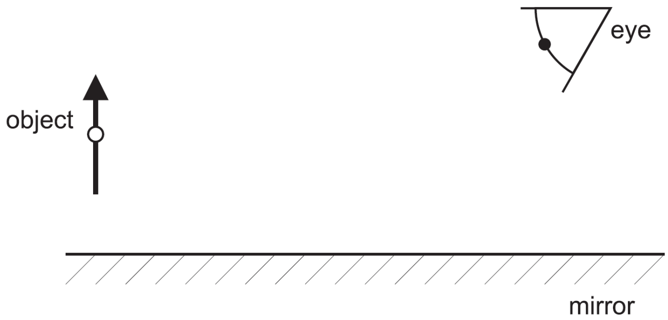
A pilot plans to fly an aeroplane from his base to another airport 300 km due north. His aeroplane can travel at a speed of 170 km/h relative to the air.

(a) He sets off at 9:00 am and aims his aeroplane due north. Calculate his estimated arrival time assuming there is no wind. [2]

(b) After flying for one hour he discovers that he is only 153 km from his starting point and has travelled along a line bearing 010°. Assuming the wind has been steady throughout the journey calculate its speed. [5]

[PAT, 2007Q11][M3]

Make a sketch copy of the diagram below and indicate clearly the position and nature of the image formed by the mirror. Draw rays corresponding to light coming from the open circle, and mark any relevant angles. [3]



[PAT, 2007Q12][M6]

The planet Pluto (radius 1180 km) is populated by three species of purple caterpillar. Studies have established the following facts:

(a) A line of 5 mauve caterpillars is as long as a line of 7 violet caterpillars

(b) A line of 3 lavender caterpillars and 1 mauve caterpillar is as long as a line of 8 violet caterpillars.

(c) A line of 5 lavender caterpillars, 5 mauve caterpillars and 2 violet caterpillars is 1m long in total.

(d) A lavender caterpillar takes 10 s to crawl the length of a violet caterpillar.

(e) Violet and mauve caterpillars both crawl twice as fast as lavender caterpillars.

How long would it take a mauve caterpillar to crawl around the equator of Pluto? [6]

[PAT, 2007Q13][M4]

The current in amperes through a certain type of non-linear resistor is given by $I = 0.05V^3$, where V is the potential difference in volts across the resistor. This resistor is connected in series to a fixed resistor and a constant voltage source of 9 V is connected across the series combination. What value of resistance should the fixed resistor have so that a current of 0.40 A flows? [4]

[PAT, 2007Q14][M20]

You should already be familiar with the mathematical treatment of an ideal pendulum, in which the pendulum bob is modelled as a point mass on the end of a rigid rod of negligible mass. In this problem you will consider the behaviour of more complex types of pendulum. You will be given all the information you need in the sections below.

For a general pendulum of any shape and size the period P is given by

$$P = 2\pi\sqrt{\frac{I}{gML_{CM}}}$$

where g is the acceleration due to gravity, M is the total mass of the pendulum, L_{CM} is the effective length of the pendulum, defined as the distance from the pivot to the centre of mass, and I is the moment of inertia around the pivot point. For a point mass m fixed at a distance r from the pivot $I = mr^2$, while for a uniform rod of mass m and length r attached to the pivot at one end $I = \frac{1}{3}mr^2$. For more complex objects the total moment of inertia can be calculated by adding together values for the component parts.

(a) Identify L_{CM} and I for an ideal pendulum of length L with a bob of mass M, and hence calculate its period. [3]

(b) Repeat this calculation for a pendulum made from a uniform rod of mass M and length L. [3]

(c) Now consider the case of a real pendulum, with a bob of mass M_b (which you may treat as a point mass) attached to the pivot using a uniform rod of mass M_r and length L, and find the period in this case. Show that the result for a real pendulum reduces to the results for an ideal pendulum and a rod pendulum by taking appropriate limits. [7]

For the remainder of this problem you can consider an ideal pendulum, for which you will calculate the effect of changing its environment.

(d) Most substances expand with increasing temperature, and so a metal rod will expand in length by a fraction $\alpha\delta T$ if the temperature is changed by δT, where α is called the coefficient of linear thermal expansion. Consider the effect of this expansion on a pendulum clock with a pendulum made from brass, with $\alpha = 19 \times 10^{-6}$ K^{-1}. What temperature change can this clock tolerate if it is to remain accurate to 1 second in 24 hours? [5]

(e) Repeat the calculation for a pendulum made from Invar alloy, with $\alpha = 1.2 \times 10^{-6}$ K^{-1}. [2]

[PAT, 2007Q15][M2]

Evaluate $6667^2 - 3333^2$. [2]

[PAT, 2007Q16][M3]

Find the equation of the line which is tangent to the curve $y = x^4$ at the point $(-2, 16)$. [3]

[PAT, 2007Q17][M3]

Evaluate

$$\frac{2\log 125}{3\log 25}$$

[3]

[PAT, 2007Q18][M4]

Two dice are thrown. What is the probability that their numbers add up to (i) six (ii) eleven? [4]

[PAT, 2007Q19][M3]

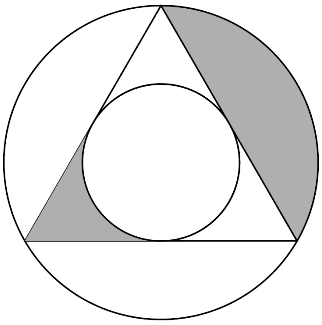
Expand $(2 + x)^5$ in ascending powers of x as far as the term in x^3. [3]

[PAT, 2007Q20][M6]

In the figure shown below, the triangle is equilateral. Find the ratio of the areas of

(i) the larger to the smaller circle;

(ii) the larger to the smaller shaded region. [6]



[PAT, 2007Q21][M5]

Sketch the curves $y = x^2$, $y = (x-2)^2$ and $y = x^2 + (x-2)^2$ on the same graph. [5]

[PAT, 2007Q22][M4]

Solve the equation $\tan\theta = 2\sin\theta$ for $0 \le \theta \le 2\pi$. [4]

[PAT, 2007Q23][M4]

Show that the points $(-5,4)$, $(-1,-2)$ and $(5,2)$ lie at three corners of a square. Find the coordinates of the fourth corner and the area of the square. [4]

[PAT, 2007Q24][M4]

Evaluate

$$\int_1^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) \mathrm{d}x.$$

[4]

[PAT, 2007Q25][M6]

The first two terms of a geometric progression (first term a and common ratio r) are the same as the first two terms of an arithmetic progression (first term a and common difference d). The third term of the geometric progression is twice as big as the third term of the arithmetic progression. Find two different expressions for d and hence or otherwise find the two possible values of r. [6]

[PAT, 2007Q26][M6]

An isosceles triangle has sides of length x, x and $p-2x$ where p is the length of the perimeter of the triangle. Find the value of x which maximises the area of the triangle for fixed p and all the angles of the triangle for this value of x. [6]

PAT 2008

PAT 2008
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 27 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$.

Applicants

For candidates applying to Physics and Physics and Philosophy.

[PAT, 2008Q1][M3]

Evaluate the sum of integers $1 + 2 + 3 + \cdots + 99 + 100$. [3]

[PAT, 2008Q2][M4]

Evaluate $(0.25)^{-\frac{1}{2}}$ and $(0.09)^{\frac{3}{2}}$. [4]

[PAT, 2008Q3][M5]

The first three terms of the series expansion of $(1 + x)^m$ are:

$$1 + mx + \frac{m(m-1)x^2}{2}.$$

Find the first three terms in the series expansion of $(1 + x)^{m+1}(1 - 2x)^m$. [5]

[PAT, 2008Q4][M3]

Find the set of values of x for which

$$\frac{x^2 + 2}{1 - x^2} < 3.$$

[3]

[PAT, 2008Q5][M4]

Given that $x = \log_9 2$, find, in terms of x, (i) $\log_2 9$; (ii) $\log_8 3$ [4]

[PAT, 2008Q6][M3]

Find the two values of x for which $1, x^2, x$ are successive terms of an arithmetic progression. [3]

[PAT, 2008Q7][M4]

Determine the value of a such that the curve $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ and the line $y = ax$ have the same gradient at $x = 0$. What value will a have if instead they have the same gradient at $x = \frac{1}{4}$? [4]

[PAT, 2008Q8][M4]

The points $(5, 2)$ and $(-3, 8)$ are at opposite ends of the diameter of a circle. Determine the equation of the circle. [4]

[PAT, 2008Q9][M5]

A die is biased so that the numbers 5 and 6 are obtained three times as often as 2, 3 and 4, and the number 1 is never obtained. Calculate the probability that (i) a two is thrown; (ii) two consecutive throws give a total ≥ 10. [5]

[PAT, 2008Q10][M3]

A cube has side a. Find the length of its body diagonal. [3]

[PAT, 2008Q11][M4]

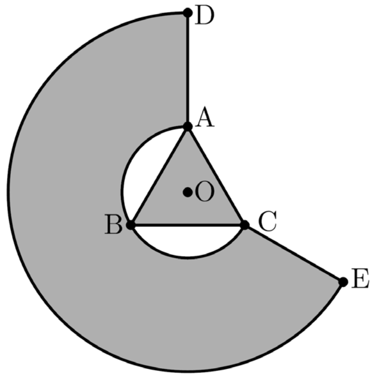
Evaluate

(i) $\int_{-1}^{1} x + x^3 + x^5 + x^7 \, \mathrm{d}x$

(ii) $\int_0^1 \frac{x^9 + x^{99}}{11} \mathrm{d}x.$ [4]

[PAT, 2008Q12][M8]

In the figure below, the shaded area $ADEC$ is defined by concentric circles which share a common centre O with the shaded triangle ABC. The straight lines AD and CE, if extended, pass through O. The lengths $AD = AB = BC = CA = CE$. Find the ratio of the two shaded areas $ADEC$ and ABC. [8]



[PAT, 2008Q13][M1]

A symmetric seesaw is 3 m long from end to end. If a boy of mass 20 kg sits on one end, how far away from him should a girl of mass 30 kg sit to balance the seesaw?

(A) 0.5 m

(B) 1.0 m

(C) 2.0 m

(D) 2.5 m

[PAT, 2008Q14][M1]

When nuclear fission occurs in a commercial nuclear reactor the mass of the products compared with the mass of the reactants is

(A) increased

(B) decreased

(C) stays the same

(D) it depends on the reaction

[PAT, 2008Q15][M1]

The visible universe contains about 400 billion galaxies (where 1 billion equals 10^9). Our galaxy contains about 250 billion stars. The mass of our sun is about 2×10^{30} kg. NASA estimates that dark matter out-masses stars by about 20 : 1. Use this data to estimate the total mass of the visible universe.

(A) 4.2×10^{36} kg

(B) 9.5×10^{51} kg

(C) 2.0×10^{53} kg

(D) 4.2×10^{54} kg

[PAT, 2008Q16][M1]

A solar eclipse can only occur when the moon's phase is

(A) new moon

(B) full moon

(C) waning

(D) waxing

[PAT, 2008Q17][M1]

When an ideal gas is heated in a container of fixed volume then

(A) the pressure and density both rise

(B) the pressure rises and the density falls

(C) the pressure rises and the density stays the same

(D) the pressure stays the same and the density falls

[PAT, 2008Q18][M1]

A physics lecture theatre is situated 3 m east and 4 m above reception. Calculate the minimum energy a 60 kg receptionist would have to expend to reach the lecture theatre.

(A) 1800 J

(B) 2400 J

(C) 3000 J

(D) 4200 J

[PAT, 2008Q19][M1]

A 3.6 V mobile phone battery can produce 0.7 A of current for 1 hour. This can be charged using a square solar panel 25 cm on each side. Assuming an efficiency of 10% and an incident solar power of 1 kW m^{-2}. What time is needed to charge the battery?

(A) 0.10 hours

(B) 0.28 hours

(C) 0.40 hours

(D) 1.5 hours

[PAT, 2008Q20][M1]

A light dependent resistor is connected across an ideal 12 V source and placed in the open in the middle of a desert. When is the power dissipated in the resistor highest?

(A) dawn

(B) mid morning

(C) noon

(D) midnight

[PAT, 2008Q21][M1]

A bullet with a mass of 10 g is fired at a velocity of 400 m s^{-1} into a cubical tank of water 2 m on each side and is brought to a halt by friction. Given that the heat capacity of water is 4.2 kJ K^{-1} kg^{-1}, and its density is 1000 kg m^{-3}, calculate the temperature rise of the water in the tank

(A) 2.4×10^{-5} K

(B) 4.8×10^{-5} K

(C) 1.9×10^{-4} K

(D) 2.4×10^{-2} K

[PAT, 2008Q22][M1]

When using Einstein's formula $E = mc^2$ a student enters the mass m in grams. If he uses a value of $c = 3.0 \times 10^8$ m s^{-1} for the speed of light, what are the units of the energy E?

(A) mJ

(B) J

(C) kJ

(D) MJ

[PAT, 2008Q23][M7]

This problem will consider the possibility of storing electrical energy in capacitors, made up of two parallel metal plates, each of area A, separated by a thickness d of a dielectric with electrical permittivity p. A capacitor will store a charge $q = CV$ where the capacitance is given by $C = pA/d$ and V is the voltage across the capacitor. Given that the work done in charging the capacitor is

$$W = \frac{q^2}{2C}$$

show that the work done to charge a parallel plate capacitor is

$$W = \frac{pAV^2}{2d}.$$

This work done is equal to the energy stored in the capacitor. In practice a capacitor is limited by the breakdown voltage of its dielectric which is proportional to the thickness, $V_{\text{max}} = Bd$, where B is a constant that depends on the material. Determine how the maximum energy stored in a capacitor depends on the mass m of the dielectric, its density D and other constants.

For a dielectric with $p = 2 \times 10^{-11}$ F m^{-1}, $B = 2 \times 10^{7}$ V m^{-1}, and $D = 1000$ kg m^{-3} (which is about right for common plastic dielectrics) calculate the maximum energy can be stored in a capacitor with a mass of 1 kg. Comment on the practicality of using a capacitor of this type to smooth the output from a 1 kW domestic wind turbine in a gusty area. [7]

[PAT, 2008Q24][M4]

A pilot takes off from an airfield 5 km west of her house and flies in a direction 60° east of north. After 5 minutes she sees that direction to her house is now at angle of 135° to her course. How far away is she from her house? (Use $\sqrt{2} \approx 1.4$) [4]

[PAT, 2008Q25][M6]

A forest is inhabited by three species of macaw which are all the same shape but are different sizes and colours. The food consumption of each type is proportional to the square of its length. Given that

(a) a crimson macaw and a ruby macaw put together are twice as long as a scarlet macaw

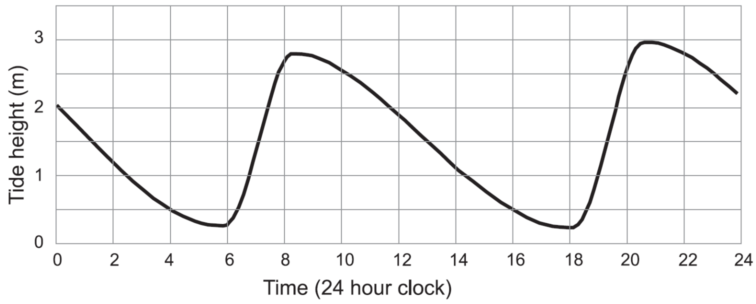
(b) a crimson macaw and a scarlet macaw put together eat as much as a ruby macaw

(c) 2 crimson macaws and a scarlet macaw put together are 1 m long

determine the lengths of the three types of macaw. [6]

[PAT, 2008Q26][M3]

The graph below shows tide heights at Wadebridge on July 7 2008.



At what time in the morning (to the nearest hour) is the tide height changing most rapidly? What is the rate of change in cm per minute at this time? [3]

[PAT, 2008Q27][M20]

A 2 m tall birdwatcher with a mass of 100 kg sees a nest in a tree. When he stands 18 m away from the tree his line of sight to the nest makes an angle of 45° to a line parallel to the ground.

(a) The birdwatcher sees an egg fall from the nest. How long does it take to reach the ground? (You may neglect the effects of air resistance.) [3]

(b) How fast is the egg travelling when it reaches the ground? [2]

(c) The egg strikes the ground and is brought to a stop in a distance of 1 mm. Assuming a mass of 20 g for the egg calculate the force required. (You may assume a constant braking force.) [3]

(d) Calculate the work done by this braking force and compare it with the gravitational potential energy of the egg. [3]

(e) Unsurprisingly the egg is smashed by the impact. To prevent this happening again the birdwatcher places a pad of foam which is 10 cm thick around the tree. This responds to impacts by compressing to half its initial thickness. Find the new braking force when a second egg falls and the time taken to bring the egg to a halt. [3]

(f) The birdwatcher considers the possibility of returning the egg to the nest by climbing the tree. Calculate the minimum energy he would have to expend to achieve this. [2]

(g) Instead he decides to use this energy to boil the egg using a small electrical heater powered from a hand generator. Calculate the minimum efficiency required for the system so that it requires no more effort to boil the egg that to return it to the nest. Assume that the egg has a specific heat capacity of 4 kJ kg^{-1}, the same boiling point as water, and starts at 20 °C. [4]

PAT 2009

PAT 2009
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 27 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$.

Applicants

For candidates applying to Physics and Physics and Philosophy.

[PAT, 2009Q1][M3]

If $x = \sin t$ and $y = \tan t$, express y in terms of x. [3]

[PAT, 2009Q2][M5]

Sketch the function $y = x + \frac{4}{x^2}$ over the range $-4 < x < +4$. Find the stationary point of this function. [5]

[PAT, 2009Q3][M5]

Find the equations of the two lines which pass through the point $(0, 4)$ and form tangents to a circle of radius 2, centred on the origin. [5]

[PAT, 2009Q4][M5]

(i) Find x where $\log_2 \sqrt[3]{x} = \frac{1}{2}$, [2]

(ii) Calculate $\sqrt{\log_8 16}$. [3]

[PAT, 2009Q5][M4]

Find all the solutions to $x^4 - 13x^2 + 36 = 0$. [4]

[PAT, 2009Q6][M5]

Find the range of values for x (where $x > 0$) for which

$$x + 2 < 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots$$

[5]

[PAT, 2009Q7][M5]

If two identical dice are thrown, what is the probability that the total of the numbers is 10 or higher? [Hint: list the combinations that can give a total of 10 or higher.] [2]

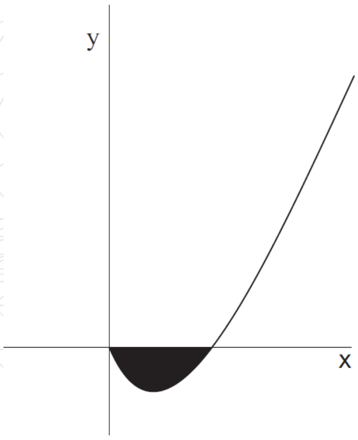
Two dice have been thrown, giving a total of at least 10. What is the probability that the throw of a third die will bring the total of the three numbers shown to 15 or higher? [3]

[PAT, 2009Q8][M3]

Sketch the function $y = 1 - 2\sin^2 x$ over the range $-\pi < x < +\pi$. [3]

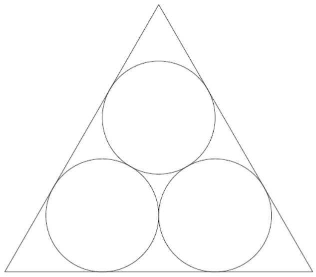
[PAT, 2009Q9][M5]

The plot below shows the function $y = \frac{(x^2-4x)}{\sqrt{x}}$. Calculate the area of the shaded regions between the x-axis and the section of the line below the x-axis. [5]



[PAT, 2009Q10][M5]

The figure below shows three circles arranged within a triangle. Find an expression for the ratio of the area of the circles to the area of the triangle. [5]



[PAT, 2009Q11][M2]

Determine the value of b for which the sequence $-a, -\frac{a}{b}, \frac{a}{b}, a$ is an arithmetic progression (where $a \neq 0$). [2]

[PAT, 2009Q12][M3]

Evaluate 2.1^5 to one decimal place. [3]

[PAT, 2009Q13][M1]

The sun produces 3.8×10^{26} W through fusion. How much mass is it losing every second? (The speed of light is $c = 3.0 \times 10^{8}$ m/s.)

(A) 4.2×10^{9} kg s^{-1}

(B) 4.2×10^{12} kg s^{-1}

(C) 3.4×10^{8} kg s^{-1}

(D) 1.3×10^{7} kg s^{-1}

[PAT, 2009Q14][M1]

A battery is replaced by two identical batteries connected in parallel. The combination can deliver

(A) the same maximum voltage and the same maximum current

(B) the same maximum voltage and a lower maximum current

(C) the same maximum voltage and a higher maximum current

(D) a higher maximum voltage and a lower maximum current

[PAT, 2009Q15][M1]

The moon Titan has an angular diameter of 4.4 mrad as seen from the surface of Saturn. The Sun has an angular diameter of 9.3 mrad as seen from the surface of the Earth. Which of the following eclipses *cannot* be seen from the surface of Saturn?

(A) A lunar eclipse of Titan by Saturn.

(B) A partial solar eclipse due to Titan.

(C) A total solar eclipse due to Titan.

(D) An annular solar eclipse due to Titan.

[PAT, 2009Q16][M1]

A yacht on a lake drops its anchor overboard. What happens to the water level in the lake?

(A) It rises very slightly.

(B) It stays exactly the same.

(C) It falls very slightly.

(D) It's impossible to say.

[PAT, 2009Q17][M1]

Estimate the change in temperature of the water in Fell Beck before and after it falls into the Gaping Gill pothole (depth 105 m). The specific heat capacity of water is 4.2 kJ kg^{-1} K^{-1}.

(A) 4 °C

(B) 0.25 °C

(C) 0.025 °C

(D) 9.2×10^{-4} °C

[PAT, 2009Q18][M1]

A time-of-flight mass spectrometer can be used to determine the mass of charged molecules through the equation $t = d\sqrt{m/2qU}$, where t is the time-of-flight, $d = 1.5$ m is the length of the tube, m is the mass of the molecule, q is its charge, and $U = 16$ kV is the accelerating voltage. Assuming that q is a single elementary charge (1.6×10^{-19} C). What is the mass that corresponds with a time-of-flight of 30 μs?

(A) 1.4×10^{-12} kg

(B) 1.0×10^{-23} kg

(C) 1.0×10^{-24} kg

(D) 2.0×10^{-24} kg

[PAT, 2009Q19][M1]

When an object moves at high velocity in a fluid the drag force on it is given by $F = Kv^2A$, where v is the object's velocity and A its area. What sort of quantity is K?

(A) A mass

(B) An acceleration

(C) A length

(D) A density

[PAT, 2009Q20][M1]

A battery is connected across two identical resistors in series. If one of the resistors is instantaneously replaced by an uncharged capacitor, what happens to the current in the circuit?

(A) It rises

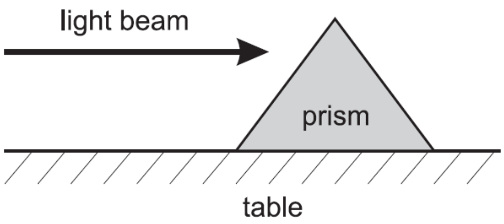
(B) It falls

(C) It initially rises, but then falls

(D) It initially falls, but then rises

[PAT, 2009Q21][M1]

A triangular glass prism sits on a table point upwards, and a beam of coloured light is directed horizontally near the top of the prism. What happens to the light beam at the prism?



(A) It is bent up

(B) It is bent down

(C) It continues horizontally

(D) It depends on the colour

[PAT, 2009Q22][M1]

The moon orbits the earth at a distance of 400,000 km with a period of 2.4×10^6 s. What is its acceleration towards the earth?

(A) 2.7×10^{-3} m s^{-2}

(B) 2.7×10^{-6} m s^{-2}

(C) 10 m s^{-2}

(D) 6.6×10^{3} m s^{-2}

[PAT, 2009Q23][M3]

A hot object will glow, emitting radiation with a total power approximately given by $P = AkT^4$ where A is the surface area, T is the temperature in Kelvin, and $k \approx 6 \times 10^{-8}$ W m^{-2} K^{-4} is a constant (the Stefan–Boltzmann constant). Estimate the surface area of the filament of a 75 W incandescent light bulb working at 5000 K. [3]

[PAT, 2009Q24][M6]

Freight transport on Titan is mostly by ship, with three types of ship called pangs, quizzers, and roodles in common use. All three ships have the same shape and design but differ in size. The cargo capacity depends on the hold volume, while the number of crew required is proportional to the surface area of the deck. A quizzer and a roodle taken together have the same length as two pangs, and the crew of a quizzer is just sufficient to provide crew for two pangs and a roodle.

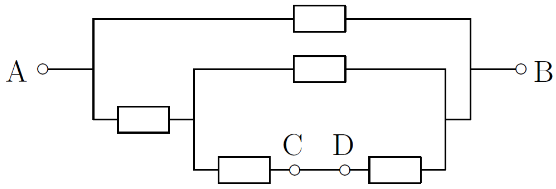
A fully loaded quizzer wishes to transfer all its cargo to smaller pangs and roodles, while minimising the number of crew required for the resultant fleet.

How many pangs and roodles are needed?

[Hint: Note that for objects of any shape the surface area is proportional to the square of the object's size, and the volume is proportional to the cube of its size.] [6]

[PAT, 2009Q25][M6]

Five identical 1 kΩ resistors are arranged in a network as shown



What resistance would be measured between terminals A and B? If a 6 V battery with an internal resistance of 125 Ω is connected across terminals A and B, what current flows in the wire connecting terminals C and D? [6]

[PAT, 2009Q26][M5]

An electron gun in a cathode ray tube accelerates an electron with mass m and charge $-e$ across a potential difference of 50 V and directs it horizontally towards a fluorescent screen 0.4 m away. How far does the electron fall during its journey to the screen? Take $m \approx 10^{-30}$ kg and $e \approx 1.6 \times 10^{-19}$ C [5]

[PAT, 2009Q27][M20]

In this question you will use a simple model to estimate how the energy used by a car depends on its design and how it is driven. Begin by neglecting air and ground resistance, and assume that the car travels at constant velocity between regular equally spaced stops.

(a) A stationary car of mass m is rapidly accelerated to a velocity v, driven for a distance s, and is rapidly brought to a halt by its brakes. Calculate the energy dispersed by the brakes. [1]

(b) Assuming the car restarts immediately, calculate the time between subsequent stops and hence the average power dissipated. [2]

(c) Hence or otherwise calculate the energy used in travelling a total distance d. [2]

(d) Taking $m = 1000$ kg, $v = 10$ m s^{-1} and $s = 100$ m calculate the energy used in travelling 1 km. What would be the effect of doubling the speed to 20 m s^{-1}? [3]

Now consider the effect of air resistance. This can be estimated by assuming that the car has to accelerate all the air it travels through to the same average velocity as itself. (You may ignore the rapid random motion of individual air molecules in this calculation.)

(e) Treating the car as a disc with cross sectional area A travelling at velocity v calculate the volume of air swept out in a time t. If the air has density D calculate the kinetic energy transferred to the air in this time, and hence the power needed to overcome the air resistance. [5]

(f) Taking $A = 1$ m^2, $v = 10$ m s^{-1} and $D = 1$ kg m^{-3} calculate the energy used in travelling 1 km. [2]

(g) Using the data above calculate the distance between stops at which the energy dissipated in the brakes is the same as that lost to air resistance. [2]

(h) Comment on the significance of these calculations for the design of cars optimised for driving in cities and cars optimised for driving on highways. [3]

PAT 2010

PAT 2010
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 25 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$.

Applicants

For candidates applying to Physics and Physics and Philosophy.

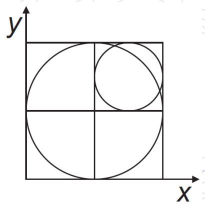
[PAT, 2010Q1][M6]

(i) Solve $\sin 3x = \sqrt{3}\cos 3x$ for x in the range $0 \leq x \leq \pi$. [3]

(ii) Solve $\cos^2 x - \sin x + 1 = 0$ for x also in the range $0 \leq x \leq \pi$. [3]

[PAT, 2010Q2][M4]

The equation of the larger circle in the figure below is $(x-1)^2 + (y-1)^2 = 1$. Find the equation of the smaller circle. [4]



[PAT, 2010Q3][M5]

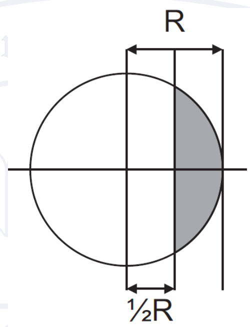
Show that $x = -1$ is a root of the polynomial equation $x^3 + 2x^2 - 5x - 6 = 0$, and find the other two roots. [5]

[PAT, 2010Q4][M4]

Find the equation of the line passing through the points $A(2,3)$ and $B(1,5)$ in the $x-y$ plane. [4]

[PAT, 2010Q5][M5]

Find the area of the shaded region of the circle in the figure below, as a function of the radius R. [5]



[PAT, 2010Q6][M4]

A rectangle is formed by bending a length of wire of length L around four pegs. Calculate the area of the largest rectangle which can be formed this way (as a function of L). [4]

[PAT, 2010Q7][M4]

(i) Calculate $\log_3 9$. [2]

(ii) Simplify $\log 4 + \log 16 - \log 2$. [2]

[PAT, 2010Q8][M4]

(i) Calculate $(16.1)^2$ [2]

(ii) Calculate 10.11×3.2 [2]

[PAT, 2010Q9][M5]

The first, fourth, and seventh terms of an arithmetic progression are given by x^3, x, and x^2 respectively (where $x \neq 0$ and $x \neq 1$). Find x, and the common difference of the progression. [5]

[PAT, 2010Q10][M4]

In a game of dice, a player initially throws a single die, and receives the number of points shown. If the die shows a 6, the player then throws the die again and adds the number shown to his/her score. The player does not throw the die more than twice. Calculate the probability that the player will gain an even number of points. [4]

[PAT, 2010Q11][M5]

Sketch the curves: $y = x^2$ and $x = y^2$, label the points of intersection and calculate the area between the two curves. [5]

[PAT, 2010Q12][M1]

A rock sample contains two radioactive elements A and B, with half lives of 8000 and 16000 years respectively. If the relative proportion of $A : B$ is initially $1 : 1$, what is their relative proportion after 16000 years?

(A) $2 : 1$

(B) $1 : 2$

(C) $3 : 1$

(D) $1 : 3$

[PAT, 2010Q13][M1]

Two resistors R_1 and R_2 are connected in series with a potential difference V across them. The power dissipated by the resistor R_1 is:

(A) $V^2 R_1/(R_1 + R_2)^2$

(B) $V^2 R_2^2/(R_1(R_1 + R_2)^2)$

(C) $V^2 R_1 \times (R_1 + R_2)^2$

(D) $V^2 R_2^2 \times (R_1(R_1 + R_2)^2)$

[PAT, 2010Q14][M1]

A block of concrete, of mass 100 kg, lies on a 2 m-long plank of wood at a distance 0.5 m from one end. If a builder lifts up the other end of the plank, how much force must he apply to lift the block?

(A) 125 N

(B) 12.5 N

(C) 250 N

(D) 25 N

[PAT, 2010Q15][M1]

A plane flies in a direction NW (according to the plane's internal compass) at an airspeed of 141 km/hr. If the wind at the plane's cruise altitude is blowing with a speed of 100 km/hr directly from the north, what is the plane's actual speed and direction relative to the ground?

(A) 141 km/hr, SW

(B) 100 km/hr, W

(C) 141 km/hr, S

(D) 223 km/hr, NNW

[PAT, 2010Q16][M1]

A teacher wants to listen to a programme on his favourite radio station, broadcasting at a frequency of 1000 kHz, but his radio only indicates the wavelength of the station. To what wavelength must the teacher tune his radio to hear the programme?

(A) 300 m

(B) 300000 m

(C) 0.0033 m

(D) 50 m

[PAT, 2010Q17][M1]

Two mirrors are set at right angles to each other. A beam of light, which is perpendicular to the line of intersection of the two mirrors is incident on the first mirror M_1 at an angle A with respect to its normal. The light reflected by M_1 is then reflected by M_2. What is the angle through which the resultant beam is turned with respect to the incident beam direction?

(A) Greater than 180°

(B) Exactly 180°

(C) Less than 180°

(D) It depends on the wavelength

[PAT, 2010Q18][M1]

A capacitor, of capacitance 3 nF, is charged to a voltage of 10 V. What is the charge held by the capacitor?

(A) 3.3×10^{9} C

(B) 3×10^{-10} C

(C) 3×10^{-8} C

(D) 3×10^{-9} C

[PAT, 2010Q19][M1]

The suspension spring of a car, which has a spring constant of $k = 80000$ N m^{-1} is sat on by a person weighing 80 kg. By how much is the spring compressed?

(A) 1 mm

(B) 10 mm

(C) 5 mm

(D) 20 mm

[PAT, 2010Q20][M1]

A comet orbits a star. At its closest approach to the star at a distance of 4×10^{10} km, the comet has a speed of 50 km/s. How fast is it travelling when it is at its maximum distance from the star of 10×10^{10} km?

(A) 50 km/s

(B) 30 km/s

(C) 20 km/s

(D) 10 km/s

[PAT, 2010Q21][M1]

A fisherman sees a fish in a river at an apparent depth below the surface of the water of 0.75 m. Given that the refractive index of water is 1.33, is the true depth of the fish below the water's surface:

(A) 0.75 m?

(B) Less than 0.75 m?

(C) 1 m?

(D) More than 1 m?

[PAT, 2010Q22][M6]

A bathroom contains three rubber ducks, red, green and blue, of identical shape and density, but different overall sizes. The following observations are made:

a) The length of a red duck is equal to the length of a blue duck added to that of a green duck.

b) The area of the base of the green duck is four times larger than the area of the base of the blue duck.

c) The blue duck has a mass of 3 g.

What are the masses of the red and green ducks?

If, when fully submersed, the green duck displaces a total mass of water of 32 g, what is the density of the ducks (the density of water is 1000 kg m^{-3})?

[Hint: Note that for objects of any shape the surface area is proportional to the square of the object's size, and the volume is proportional to the cube of its size.] [6]

[PAT, 2010Q23][M7]

Light from the Sun has an approximate flux level of 1×10^3 W m^{-2} at the distance of the Earth and is incident on a steel frying pan of area 0.07 m^2, total mass 2 kg, and initially at a temperature of 20 °C. Assuming that the frying pan is perfectly thermally insulated from its surroundings and absorbs all the sunlight incident upon it, how long does it take for the pan to reach a temperature of 70 °C and thus be hot enough to fry an egg? [3]

The frying pan, still at 70 °C, is then plunged into a bowl containing 4 kg of water at 20 °C. Assuming the bowl has negligible heat capacity and assuming that there is no heat flow to or from the surroundings, what is the final temperature of the water in the bowl to the nearest °C? [4]

(The specific heat capacity of steel is 490 J kg^{-1} K^{-1} and the specific heat capacity of water is 4200 J kg^{-1} K^{-1}).

[Hint: For the second part of the question, you may find it easier to write the problem in terms of the temperature change of the water.]

[PAT, 2010Q24][M7]

An astronaut arrives on the planet Oceania and climbs to the top of a cliff overlooking the sea. The astronaut's eye is 100 m above the sea level and he observes that the horizon in all directions appears to be at angle of 5 mrad below the local horizontal. What is the radius of the planet Oceania at sea level? [4]

How far away is the horizon from the astronaut? [3]

[Hint: the line of sight from the astronaut to the horizon is tangential to surface of the planet at sea level.]

[PAT, 2010Q25][M20]

(a) A gun is designed that can launch a projectile, of mass 10 kg, at a speed of 200 m/s. The gun is placed close to a straight, horizontal railway line and aligned such the projectile will land further down the line. A small rail car, of mass 200 kg and travelling at a speed of 100 m/s passes the gun just as it is fired. Assuming the gun and the car are at the same level, at what angle upwards must the projectile be fired in order that it lands in the rail car? [3]

(b) How long does it take for the projectile to reach its maximum altitude?

(You may use $\sqrt{3} \approx 1.732$) [3]

(c) How far is the rail car from the gun when the projectile lands in it? [3]

(d) Without considering energy, calculate the projectile's maximum altitude. [2]

(e) Now consider energy. What is the initial kinetic energy of the rail car and what is the initial kinetic energy of the projectile in both the vertical and horizontal directions? [3]

(f) Using your calculation of the projectile's initial kinetic energy, again calculate the projectile's maximum altitude. [2]

(g) When the projectile lands in the rail car, why does the velocity of the car not change? [2]

(h) Assuming that the projectile remains in the car, what is the combined kinetic energy of the car plus projectile after the projectile has landed (to three significant figures)? [2]

PAT 2011

PAT 2011
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 26 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics and Physics and Philosophy.

[PAT, 2011Q1][M4]

Find the values of θ between 0 and 2π which solve: $\sin\theta - 2\cos^2\theta = -1$. [4]

[PAT, 2011Q2][M3]

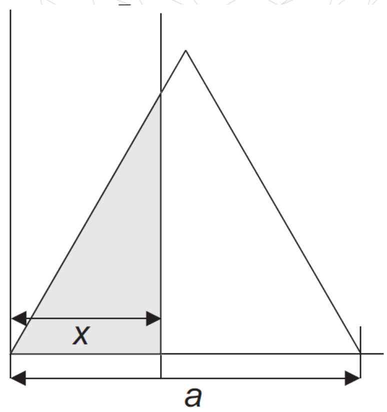
Sketch the function $y = 3 + 2\sin\left((x-3)\frac{\pi}{3}\right)$ between $x = 0$ and $x = 12$. [3]

[PAT, 2011Q3][M5]

The area A is defined as the area to the left of point x, within an equilateral triangle with sides of length a. Find an expression for A as a function of x,

(i) for $0 \le x \le \frac{a}{2}$

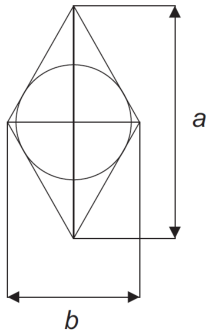
(ii) for $\frac{a}{2} \le x \le a$ [5]



[PAT, 2011Q4][M6]

The figure below shows a rhombus with diagonals a and b, which contains a circle. Find an expression for the ratio of the area of the circle to the area of the rhombus in terms of a and b.

[6]



[PAT, 2011Q5][M4]

Given that $\log 5 = 0.7$ (to one decimal place). Find the value of x such that $2^x = 10$ (again to one decimal place).

[4]

[PAT, 2011Q6][M4]

Evaluate: $\sum_{r=1}^{6}\left(2^r + \frac{2r}{3}\right)$ [4]

[PAT, 2011Q7][M4]

Given that $x^2 - x - 6$ is a factor of $x^4 + 4x^3 - 17x^2 - 24x + 36 = 0$, find all the roots of this polynomial. [4]

[PAT, 2011Q8][M6]

Evaluate the following integrals:

(i) $\int \frac{x+2}{(x+1)(x-1)} \mathrm{d}x$ [3]

(ii) $\int_0^1 \frac{1}{\sqrt{x+1}} \mathrm{d}x$ [3]

[PAT, 2011Q9][M6]

Given the functions:

$$y_1 = x^3 - 3x^2 + 2x + 3$$
$$y_2 = x^2 - 3x - 4$$

Find the values of x between 0.8 and 1.9 which give the maximum and minimum difference between y_1 and y_2. [6]

[PAT, 2011Q10][M4]

Given that $s = x^2 + y^2$ and $t = 2xy$, find expressions for x and y in terms of s and t. [4]

[PAT, 2011Q11][M4]

The numbers shown by two dice are labelled d_1 and d_2; a score is constructed from these by the expression: $S = Ad_1 + Bd_2 + C$, where A, B and C are constants. Determine the values of A, B and C such that the range of possible values for S covers all integers from 0 to 35, with an equal probability of each score. [4]

[PAT, 2011Q12][M1]

A boy sitting on a harbour wall observes waves on the water's surface. He sees that the waves have a period of 2 s and that a single wave travels the length of the harbour wall in 25 s. If the harbour wall is of length 45 m, what is the wavelength of the wave?

(A) 4.0 m

(B) 1.8 m

(C) 3.6 m

(D) 1.0 m

[PAT, 2011Q13][M1]

The contents of a refrigerator, which are kept at a temperature $T = 6\,°\mathrm{C}$, has to be cooled at a rate of $\alpha(T_S - T)$, where T_S is the temperature of the surroundings and $\alpha = 15\ \mathrm{W/K}$. If the refrigerator has an efficiency of 30%, what is its power consumption on a day when $T_S = 26\,°\mathrm{C}$?

(A) 1 kW

(B) 2 kW

(C) 3 kW

(D) 4 kW

[PAT, 2011Q14][M1]

A lunar eclipse can only occur when the moon's phase is

(A) New moon.

(B) Full moon.

(C) First quarter.

(D) Last quarter.

[PAT, 2011Q15][M1]

Two stars in the night sky are observed to have the same apparent brightness, but one is known to be at a distance of 10 light years and the other at a distance of 20 light years. What is the ratio of the total power radiated by the more distant star to that radiated by the nearer star?

(A) 1.0

(B) 2.0

(C) 3.0

(D) 4.0

[PAT, 2011Q16][M1]

An exoplanet is observed to orbit a nearby star at a distance of 0.4 A.U. with a period of 3 days. If a second exoplanet is observed to orbit the same star with a period of 24 days, what must be its orbital radius?

(N.B. 1 A.U. is an Astronomical Unit and is the distance the Earth orbits the Sun (1 A.U. = 1.49×10^8 km)).

(A) 1.6 A.U.

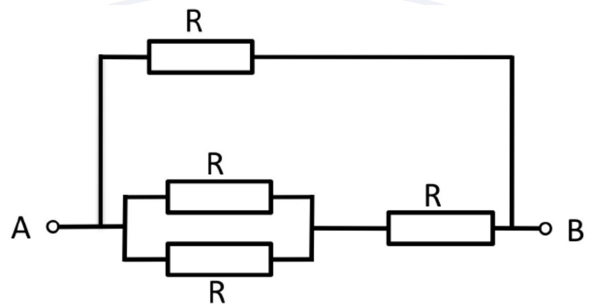
(B) 3.2 A.U.

(C) 1 A.U.

(D) 16 A.U.

[PAT, 2011Q17][M1]

What is the resistance of the following network of resistors between points A and B?



(A) $R/2$

(B) $5R/3$

(C) R

(D) $3R/5$

[PAT, 2011Q18][M1]

The primary coil of a transformer is connected to an alternating voltage supply of 240 V and draws a current of 1 A. The secondary coil is connected to a resistor and delivers a voltage of 120 V. If there are 50 turns in the primary coil, how many turns are there in the secondary coil?

(A) 240

(B) 100

(C) 25

(D) 50

[PAT, 2011Q19][M1]

An electric motor is driven by a battery of voltage 6 V and draws a current of 1 A. If the motor is used to lift vertically a block of mass 100 g, what is the vertical velocity of the mass?

(A) 12 m/s

(B) 6 m/s

(C) 10 m/s

(D) 0.6 m/s

[PAT, 2011Q20][M1]

A toy car of mass 10 g rests on a slope of inclination 30°. Neglecting friction, what is its acceleration down the slope?

(A) 10 m/s^2

(B) 2.5 m/s^2

(C) 8.7 m/s^2

(D) 5.0 m/s^2

[PAT, 2011Q21][M1]

A boat crosses a river of width 100 m and flowing in the east-west direction. The water in the river flows from east to west at a speed of 5 m/s. The boat can travel at a speed of 10 m/s. The boat leaves one bank and the skipper wants to reach the point directly on the opposite bank. What course must she steer?

(A) 30 °W

(B) 20 °W

(C) 30 °E

(D) 60 °E

[PAT, 2011Q22][M6]

(a) A catapult consists of a massless cup attached to a massless spring of length l and of spring constant k. If a ball of mass m is loaded into the cup and the catapult pulled back to extend the spring to a total length x, what velocity does the ball reach when launched horizontally? [2]

(b) The catapult is then used to launch the ball vertically. If the spring is extended to the same total length of x before release, to what velocity does the catapult now accelerate the ball? [2]

(c) What height above its position at launch will the ball reach if launched vertically? [2]

[PAT, 2011Q23][M4]

An electron, initially at rest, is accelerated by a potential V in a vacuum and then travels horizontally in a region of space where there is an electric field, E, and a magnetic field, B. The fields are aligned such that the electron is subjected to a force eE upwards and a force evB downwards, where e is the charge of the electron and v is its velocity. If $E = 1000$ V/m and $B = 1 \times 10^{-5}$ T, what is the value of the initial accelerating voltage V for the electron to continue flying undeflected?

(Take $m \approx 10^{-30}$ kg and $e \approx 1.6 \times 10^{-19}$ C) [4]

[PAT, 2011Q24][M4]

A radioactive source emits a parallel beam of alpha, beta and gamma radiation and is placed 10 cm from a detector, which is sensitive to all forms of radiation and receives 100 counts/sec.

When a sheet of aluminium of thickness 1 cm is placed in front of the detector, the radiation level is seen to fall to 50 counts/sec.

When the source is taken away completely, the radiation levels are seen to fall to 10 counts/sec.

When the source is placed 1 cm from the detector, the radiation levels are seen to increase to 400 counts/sec.

In what proportion is the source emitting alpha : beta : gamma particles? [4]

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[PAT, 2011Q25][M6]

A packing company supplies storage boxes in three different sizes: small, medium, and large. All three types of box have the same ratio of width : length and height : length. It is noted that:

A. Eight small boxes fit neatly inside one medium box.

B. The length of the small box is the same as the height of the medium box.

C. The base area (i.e. width times length) of a large box is 9 times larger than the base area of the small box.

D. The lengths of all three boxes added together is 2.4 m.

E. The width of the medium box is twice the height of the small box.

What are the lengths of the three different boxes?

What are the ratios of the width : height and width : length of the boxes? [6]

[PAT, 2011Q26][M20]

(a) An archer draws the string of her bow back a distance of 0.6 m and holds it there with a force of 120 N before releasing an arrow of mass 20 g. What is the speed of the arrow when it leaves the bowstring, assuming that all the energy in the bow is imparted to the arrow? [6]

(b) In fact, the stored energy of the bow not only accelerates the arrow, but also the arms of the bow and only a fraction h of the original stored energy is imparted to the arrow. If $h = 25/36$, what is the actual speed of the arrow leaving the bow? [2]

(c) The archer aims at a target, which is a distance 50 m away. How long will it take for the arrow to reach the target, assuming the arrow does not slow through air friction? [2]

(d) To account for the effects of gravity, estimate how far above the centre of the target the archer must aim to ensure that the arrow strikes the middle. [2]

(e) If the arrow is brought to rest in a distance of 5 mm, what is the average force of the arrow strike? [4]

(f) If the target has a mass of 5 kg, at what velocity is it thrown back by the arrow strike? [4]

PAT 2012

PAT 2012
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 22 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2012Q1][M4]

Find the area between $y = x^2$ and $y = |x|$. [4]

[PAT, 2012Q2][M4]

(i) Write down the binomial expansion of $(4 + x)^4$.

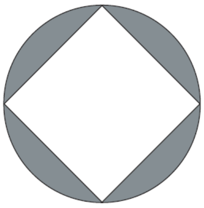
(ii) Hence or otherwise evaluate $(4.2)^4$ to 2 d.p (decimal places). [4]

[PAT, 2012Q3][M3]

Evaluate $\sum_{r=1}^{8}(2 + 4^r)$. [3]

PAT, 2012Q4][M3]

Consider a square inside a circle of radius r as shown. What is the shaded area in terms of r?
[3]



[PAT, 2012Q5][M4]

Show $x = 1$ is a solution to $x^3 - 6x^2 - 9x + 14 = 0$ and find the other solutions. [4]

[PAT, 2012Q6][M4]

Find the equation of the line passing through $(0, 2)$ and just touching $y = (x - 2)^2$ at $x > 0$.[4]

[PAT, 2012Q7][M4]

If $5 = \log_2 16 + \log_{10} \sqrt{0.01} + \log_3 x$, determine x. [4]

[PAT, 2012Q8][M4]

Consider two dice – one contains the numbers 1-6, the other contains only 1, 2, 3 each shown twice (i.e. 1, 2, 3, 1, 2, 3). What is the probability that when we roll the two dice we will obtain a score of 7? [4]

[PAT, 2012Q9][M4]

Solve $\cos^2 \theta + \sin \theta = 0$ for θ. Leave your answer in terms of $\sin \theta$. [4]

[PAT, 2012Q10][M4]

Sketch an example of a real function $f(x)$ defined for all real arguments x, which has all of the following properties:

(a) $f(x) > 0$ for all x,

(b) $f(x)$ is a continuous function,

(c) $\frac{\mathrm{d}f}{\mathrm{d}x} = 0$ only for $x = 4$,

(d) $\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0$ only for $x = 2$ and $x = 6$. [4]

[PAT, 2012Q11][M6]

Solve $-1 < -\frac{1}{x} + 2x < 1$. [6]

[PAT, 2012Q12][M6]

Sketch $y = \frac{1-x-x^2}{x^2}$. [6]

[PAT, 2012Q13][M2]

A vintage steam locomotive made of iron has a mass of 6.5×10^4 kg and is 10 m long. How long is its scale model which is also made out of iron and has a mass of 1 kg?

(A) ≈ 4 cm

(B) ≈ 20 cm

(C) ≈ 25 cm

(D) ≈ 30 cm

[PAT, 2012Q14][M2]

A gas cylinder has a volume of $0.02\ \text{m}^3$ and contains 88 g of carbon dioxide at a temperature of 27 °C. The molar gas constant $R \approx 8.3\ \text{J mol}^{-1}\ \text{K}^{-1}$. What is the gas pressure?

(A) ≈ 101 kPa

(B) ≈ 149 kPa

(C) ≈ 201 kPa

(D) ≈ 249 kPa

[PAT, 2012Q15][M2]

An electric car has a battery pack delivering 160 V and 100 A of steady current when moving at 36 km/h. What is the air resistance, assuming 100% efficiency?

(A) ≈ 440 N

(B) ≈ 1600 N

(C) ≈ 2000 N

(D) ≈ 3200 N

[PAT, 2012Q16][M2]

A cube painted black is cut into 125 identical cubes. How many of them are not painted at all?

(A) 21

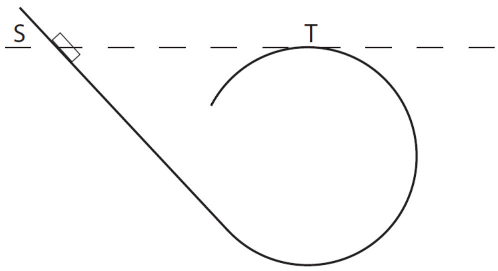
(B) 25

(C) 27

(D) 30

[PAT, 2012Q17][M2]

A massive slider starts from rest from a point S (which is at the same height as a point T at the top of the track) and slides along a frictionless circular track as sketched in figure below. The slider



(A) does not get to T.

(B) gets to T and falls straight down.

(C) gets to T but then, leaves the track and falls down following a parabola trajectory to the left.

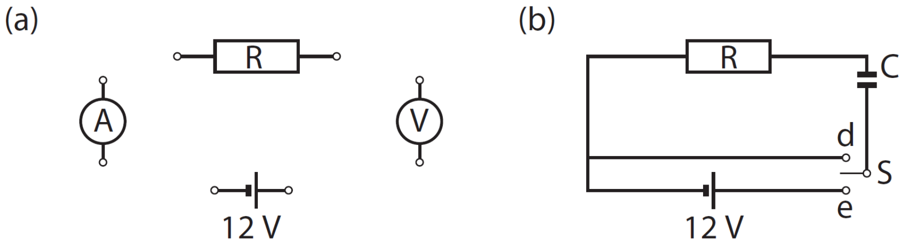
(D) passes T staying on the track all the way through.

[PAT, 2012Q18][M4]

A 12 V battery, a voltmeter, an ammeter and a resistor $R = 2\ \text{k}\Omega$ are sketched in figure (a) below.

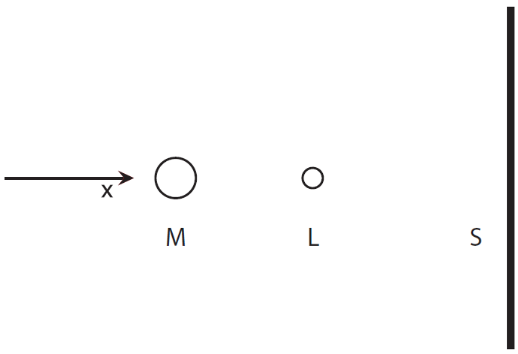
Sketch connections to create a circuit to measure a potential difference across the resistor and an electric current. How big is the current? [1]

A capacitor $C = 4\ \mu\text{F}$ and a switch S, as sketched in figure (b), are inserted into the circuit. Sketch how the current depends on time from the moment t_e when the switch is moved to e closing the circuit. Estimate the time T after which the current is not changing significantly. After a time t_d much longer than T, the switch is moved to d. Sketch the current from that moment until the moment when the current is not changing significantly, indicating on your sketch the time interval T. [3]



[PAT, 2012Q19][M4]

A loudspeaker L is placed between a microphone M and a screen S reflecting sound waves as sketched below. The loudspeaker emits sound of fixed wavelength λ in all directions. When the screen is moving slowly, to the right along the x direction (slowly in comparison with the speed of sound), the microphone records minima and maxima of the sound intensity. What is the distance between two screen positions giving two successive maxima? Would the microphone record minima and maxima if (a) the loudspeaker or (b) the microphone is moving along x direction instead of the screen? [4]



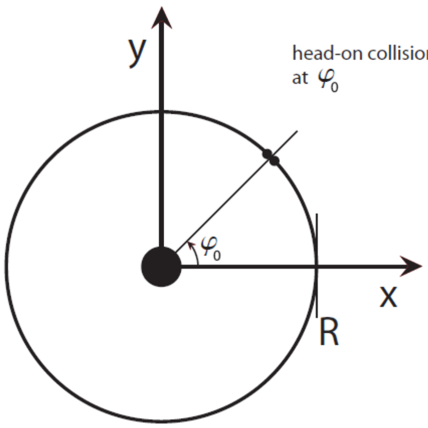
[PAT, 2012Q20][M4]

The ^{238}U isotope has a half-life $T_{238} = 4.5 \times 10^9$ years and the ^{235}U has $T_{235} = 7.0 \times 10^8$ years. $N_{238}(t)$ is the number of ^{238}U nuclei at time t and $N_{235}(t)$ is the corresponding number for ^{235}U. The relative abundance $r(t)$ is defined as $r(t) = \frac{N_{235}(t)}{N_{238}(t)}$. At present, $r = 0.0072$. Estimate the relative abundance of these two isotopes 10^9 years ago. You might use the following approximations: $e^x \approx 1 + x$ for small x, $e \approx 2.7$ and $\ln 2 \approx 0.7$. [4]

[PAT, 2012Q21][M8]

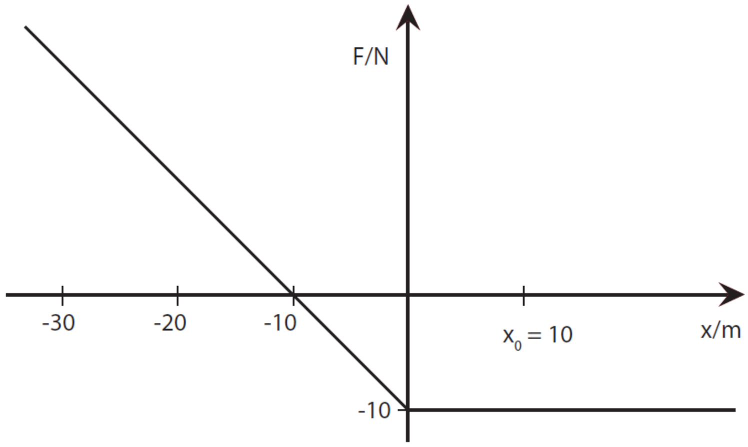
(a) A meteoroid of mass m is on a circular Earth orbit of radius R which is a few (> 2) times larger than the radius of the Earth R_E. Derive an expression for the meteoroid's speed. State the meanings of all symbols used. [2]

(b) Another meteoroid of the same mass is on the same orbit, in the same plane but rotating in the opposite direction. At an azimuthal angle φ_0, see figure below, the two meteoroids collide head-on and coalesce (combine). Sketch the complete trajectory of the newly formed double mass meteoroid showing how the azimuthal angle φ depends on the distance r. Sketch also its kinetic energy and expected meteoroid surface temperature as a function of r. Give a very brief explanation of why you expect the temperature to depend on r that way. For $r < 2R_E$ effects due to the Earth's atmosphere can not be neglected; r is the distance from the Earth's centre. [6]



[PAT, 2012Q22][M20]

A point like object with mass $m = 1$ kg starts from rest at point $x_0 = 10$ m and moves without any friction under a force F which depends on the coordinate x as illustrated in figure below. The motion is confined to one dimension along x.



(a) What is its speed at $x = 0$? [2]

(b) Sketch its kinetic energy as a function of x. [4]

(c) Sketch its velocity as well as its acceleration as a function of time t. [6]

Now consider a case when, in addition, a friction force of a magnitude of 1 N is present for $x \geq 0$.

(d) Sketch how the velocity depends on x in that case. [6]

(e) How many meters this point like object travelled during the time when its position coordinate x was ≥ 0? [2]

PAT 2013

PAT 2013
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 21 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2013Q1][M4]

What is the sum of the series $\frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \cdots$? [4]

[PAT, 2013Q2][M3]

If $y = \sqrt{xv}$ and $\left(y - \sqrt{x}\right)^2 = u$, find an expression for x in terms of u and v. [3]

[PAT, 2013Q3][M3]

50 people are in a room. 8 people in the room have red hair, 3 people have black hair and 20 people are male. You may assume that hair colour and gender are independent.

(a) If a person is selected at random from the room, what is the probability that they will be a female with red hair?

(b) If a person is selected at random from the room, what is the probability that they will be a male who does not have red or black hair? [3]

[PAT, 2013Q4][M5]

Consider the function $f(x) = x^3 - x^2 - 4x + 4$

(a) Show that $x = 1$ is a root of $f(x) = 0$ and hence factorise $f(x)$ to find the remaining roots.

(b) Having found the roots of $f(x) = 0$, find the area bounded by the curve $f(x)$ and the x-axis between the two smallest roots. [5]

[PAT, 2013Q5][M4]

If $x = \log_{10} 100 + \log_5 \sqrt{25} - \log_3 y^2$ and $\frac{x}{2} = \log_2 8 - 9\log_{10}\sqrt{10} + 2\log_3 y$, find x and y. [4]

[PAT, 2013Q6][M5]

Find the equation of the straight line that passes through the centres of the two circles: $x^2 + 4x + y^2 - 2y = -1$ and $x^2 - 4x + y^2 - 6y = 3$. [5]

[PAT, 2013Q7][M4]

How many terms in the binomial expansion would be needed to determine $(3.12)^5$ to one decimal place? [4]

[PAT, 2013Q8][M7]

Sketch the regions in the xy plane defined by the inequalities: $1 < xy < 2$ and $\frac{1}{2} < \frac{y}{x} < 2$. [7]

[PAT, 2013Q9][M8]

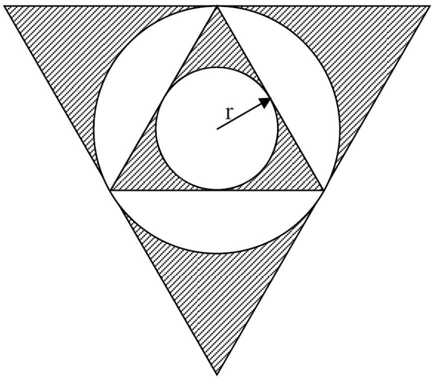
(a) Sketch $y = \exp(-x)$

(b) Sketch $y = 3\{\exp[-2(x-1)] - 2\exp[-(x-1)]\}$ for $x > 0$

[$\exp(x)$ is defined as $\exp(x) \equiv e^x$] [8]

[PAT, 2013Q10][M7]

In the figure below, all triangles are equilateral. Find the shaded area in terms of r. [7]



[PAT, 2013Q11][M2]

An ideal transformer has 100 turns on the primary coil. It is connected to an alternating supply of 100 V, 2.4 A. How many turns are required on the secondary coil to supply 4.8 A?

(A) 25 turns

(B) 50 turns

(C) 75 turns

(D) 200 turns

[PAT, 2013Q12][M2]

A radioactive sample contains two different isotopes, A and B. A has a half-life of 3 days, B has a half-life of 6 days. Initially in the sample there are twice as many atoms of A as of B. At what time will the ratio of the number of atoms of A to B be **reversed**?

(A) 3 days

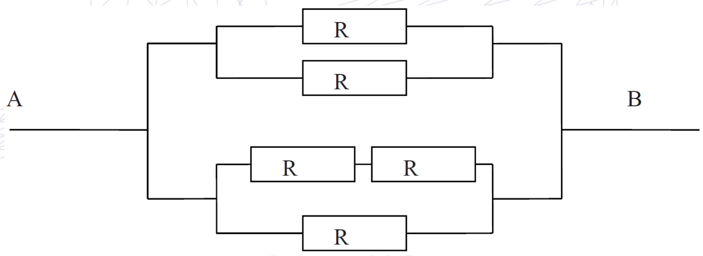
(B) 6 days

(C) 12 days

(D) ratio will never be reversed

[PAT, 2013Q13][M2]

Consider the resistor network shown below. What is the overall resistance between A and B?



(A) $2R/7$

(B) $R/2$

(C) $3R/2$

(D) $7R/2$

[PAT, 2013Q14][M2]

Two satellites are in orbit around the Earth. The first is in a geostationary orbit, the second satellite orbits at a radius half that of the first. What is the period of the second satellite?

(A) approx. 4.3 hours

(B) approx. 8.5 hours

(C) approx. 17.0 hours

(D) approx. 72.0 hours

[PAT, 2013Q15][M2]

You are in a desert and discover a radio mast. 100 m from the mast you measure 20 W of power from the transmitter. If you require a minimum power level of 1 mW, how far can you go away from the mast and still obtain the minimum power? You may assume the transmitter acts like a point source.

(A) $\frac{1}{10\sqrt{2}}$ km

(B) $\sqrt{20}$ km

(C) $10\sqrt{2}$ km

(D) 20 km

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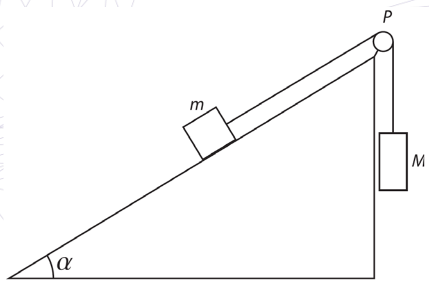
[PAT, 2013Q16][M4]

A four wheeled car, of mass 1000 kg, rests on the ground. If each tyre is inflated to 2 bar (where 1 bar = 100 kPa), what area of **each** tyre is in contact with the ground? (Assume a uniform distribution of mass across the car). [4]

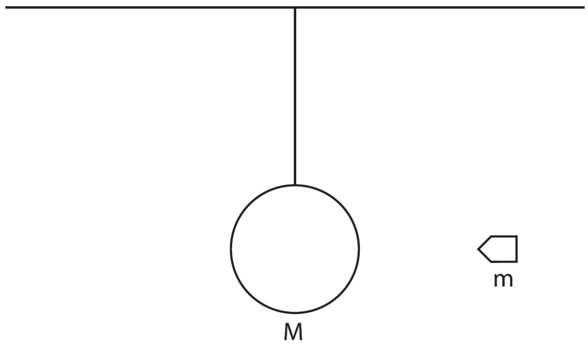
[PAT, 2013Q17][M5]

Two masses, m and M, are connected by a massless string of fixed length on a slope inclined at an angle α as sketched in the figure below. The pulley P is massless. Ignoring friction, calculate the acceleration of mass m and the tension of the string. What is the condition for the masses to be stationary? [5]



[PAT, 2013Q18][M5]

A projectile of mass 0.2 kg and speed 122 m s^{-1} hits a ball of mass 12 kg hanging on a massless string of fixed length, as sketched in the figure below. The projectile was moving at the height of the centre of the ball and after hitting the ball it stops inside the ball, i.e. it becomes stationary with respect to the ball. What is the maximum height that the ball (with the projectile inside) will reach above its original position? [5]



[PAT, 2013Q19][M6]

A monochromatic point light source of wavelength λ is shining through two narrow slits separated by a distance d (d is of the order of λ) on a screen which is a distance D away ($D \gg d$) from the slits. Sketch the pattern of light intensity observed on the screen. Explain why there are minima and maxima. If λ corresponds to red light, what would the pattern look like for green light; make a sketch on the same scale. [6]

[PAT, 2013Q20][M10]

(a) An explorer tests her gas fuelled cooking stove before setting off on an expedition. She has a pot which has a square base of side 10 cm and height 15 cm. Starting from 20 °C, how much energy is required to heat the water in a totally full pot to boiling point? (You may assume the specific heat capacity of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1 g cm^{-3}. You may also neglect the specific heat capacity of the pot.) [4]

(b) The explorer now goes up Mount Everest. She discovers that the boiling point of water decreases by 1 °C every 300 m. What physical effect causes this reduction in boiling point? [1]

(c) When she reaches 6000 m she uses her stove to make a cup of tea. Her mug only requires 100 g of water. How much energy will it take to boil the water and make the cup of tea (assuming it is 10 °C in her tent at 6000 m)? [2]

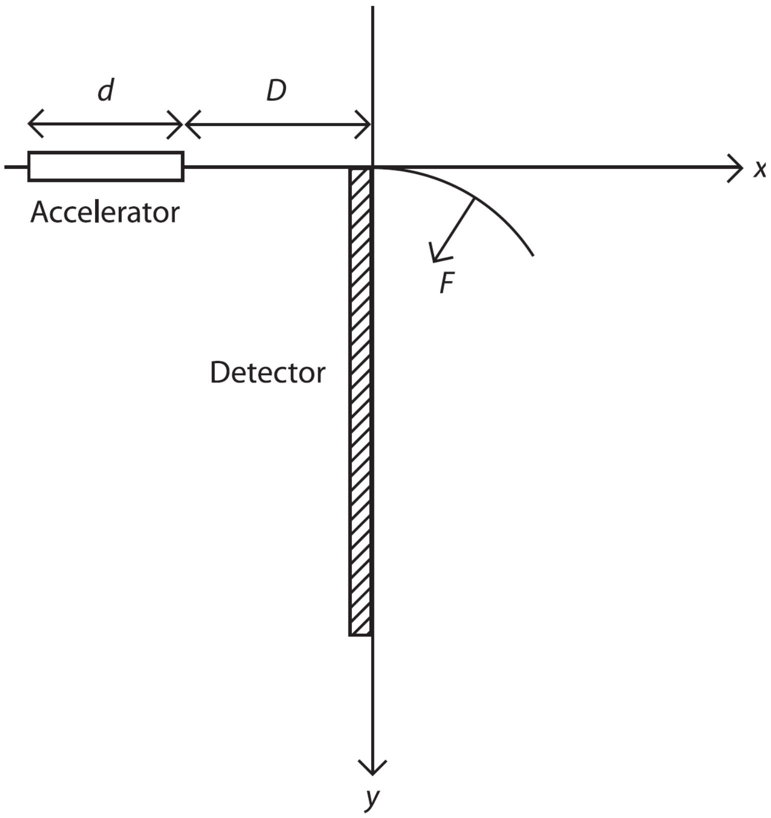
(d) She discovers there is a problem with her stove and it now only produces 50% of the power it did at sea level. If a full pot took 15 minutes to reach boiling point at sea level, how long will it take to boil the water for the cup of tea? [3]

[PAT, 2013Q21][M10]

A particle of mass m moves with a velocity v_0 along the positive x direction before entering a region of length d where it is accelerated by a constant force f acting along the direction x.

(a) What is the velocity of the particle as it leaves the region of the acceleration? [1]

After being accelerated, the particle travels a distance D and then enters a region where a force of a magnitude F proportional to its speed v acts on it, $F = \alpha v$ and $\alpha > 0$ is constant. As sketched below, the force acts in the plane of the figure, and is perpendicular to the velocity at every point on the particle trajectory. A detector is placed as shown below, extending downwards, from the point of the entry to the region where the force is acting, along the direction y. (There is no gravitational force involved.)



(b) Derive an expression for the y coordinate where the particle is detected. [4]

Now, instead of one particle, there are many particles, initially following the same path as the first particle with speeds v, ranging between v_1 and v_2, at the point of entry to the region where the force F is acting. The detector has a CCD like structure, meaning it is segmented into pixels of size Δy, the same for all y.

(c) What is the minimal spread of the speeds $\Delta v > 0$ such that v and $v + \Delta v < v_2$ are resolved by the detector? [2]

(d) For a given particle, how much work is done by both the forces involved from when the particle enters the accelerator until it strikes the detector? [3]

PAT 2014

PAT 2014
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 19 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10 \text{ m s}^{-2}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2014Q1][M4]

A jar contains buttons of four different colours. There are twice as many yellow as green, twice as many red as yellow and twice as many blue as red. What is the probability of taking from the jar:

a) a blue button

b) a red button

c) a yellow button

d) a green button

(You may assume that you are only taking one button at a time and replacing it in the jar before selecting the next colour.) [4]

[PAT, 2014Q2][M4]

What is the sum of the following terms:

$$1 + e^{-x} + e^{-2x} + \cdots$$

Over what range of x is the solution valid? [4]

[PAT, 2014Q3][M6]

Evaluate the integrals:

(a) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x} \mathrm{d}x$

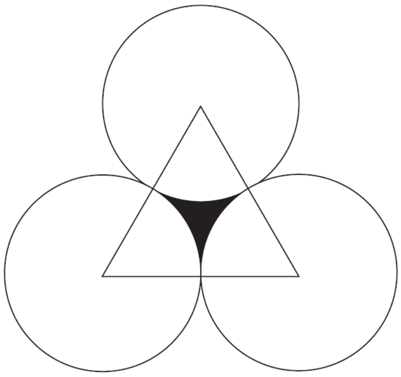
(b) $\int_0^2 \frac{x}{x^2+6x+8} \mathrm{d}x$. [6]

[PAT, 2014Q4][M4]

What is the coefficient of x^7 in the expansion of $(1+2x)^4(1-2x)^6$? [4]

[PAT, 2014Q5][M5]

Consider the shape shown below. What is the area of the equilateral triangle (with length of side $= 2r$) which is not enclosed within the circles (each with radius $= r$), and which is shown shaded black in the figure? [5]



[PAT, 2014Q6][M5]

You want to make a snowman out of modelling clay. The snowman consists of 2 spheres, where one sphere has a radius r, the other has a radius $2r$. The modelling clay comes in the form of a cylinder with radius $r/2$. What length of modelling clay is required to make the snowman? [5]

[PAT, 2014Q7][M7]

Sketch the region defined by:

$$y \leq x^2 \text{ and } 4 \geq y \geq 0 \text{ and } y \geq 2x - 4.$$

Evaluate the area defined by the above inequalities. [7]

[PAT, 2014Q8][M8]

If $f(x) = e^x$ for $x < 0$ and $f(x) = e^{-x} + 2x$ for $x \geq 0$, sketch the function $f(x)$ and its first, second and third derivatives. [8]

[PAT, 2014Q9][M7]

Find the equations of the tangents drawn from the point $(-4, 3)$ to the circle $x^2 + y^2 = 5$. [7]

[PAT, 2014Q10][M2]

Excluding Pluto, for the planets in our solar system, in order of increasing mean distance from the Sun, which of the following statements is/are correct?

i) the duration of the day on each planet increases

ii) the duration of the year on each planet increases

iii) the size/volume of the planets increases

iv) the number of moons of each planet increases

v) the planets change from rocky to gas giants

(A) statements i) and ii) and v)

(B) statement ii) only

(C) statements iii) and iv)

(D) statements ii) and v)

[PAT, 2014Q11][M2]

In which part of the Electromagnetic Spectrum do waves have a frequency of approx. 100 GHz?

(A) X rays

(B) visible light

(C) microwave

(D) radio wave

[PAT, 2014Q12][M2]

An object with small mass becomes detached from the International Space Station (ISS) while it orbits the Earth. Its relative velocity with respect to the ISS can be neglected. Would the object

(A) follow ISS in its orbit

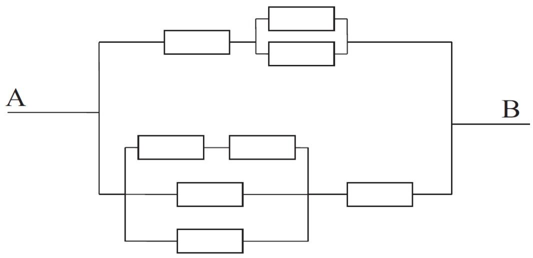
(B) go straight along a direction tangential to ISS orbit at the point when it became detached

(C) fall straight down towards the Earth

(D) stay still with respect to the Earth

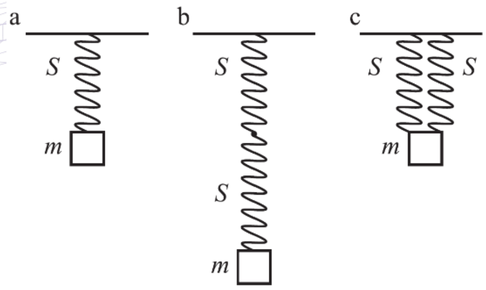
[PAT, 2014Q13][M5]

Given the circuit below, where all resistors have the same value ($= R$), what is the resistance between A and B? [5]



[PAT, 2014Q14][M4]

A mass m is attached to a spring S (as sketched in figure a below) and oscillates with a period T. What would be the period of the oscillation if two springs S are connected in series (figure b) or in parallel (figure c)? What would be the period of the oscillations in case (a) on a planet with surface gravity $2g$? [4]



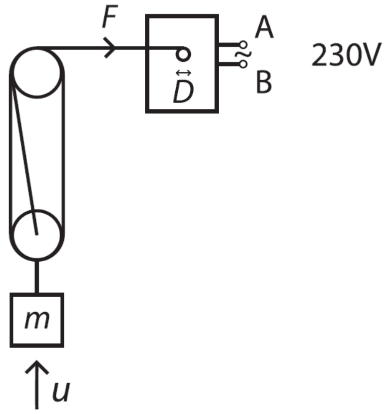
[PAT, 2014Q15][M5]

An electric motor is lifting a mass via a system of pulleys as sketched below. The motor is powered by a voltage source of 230 V. The diameter of the motor winding reel is $D = 5$ cm and a mass $m = 100$ kg is being lifted with a speed $u = 0.5$ m/s. The masses of the pulleys and the string can be neglected.

(a) What is the electric current driving the motor?

(b) What is the angular velocity of the motor's winding reel?

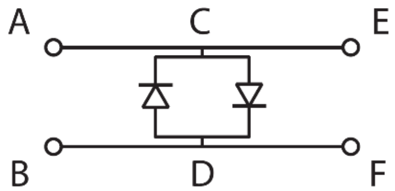
(c) What is the force F with which the motor is pulling? [5]



[PAT, 2014Q16][M4]

Two diodes are connected as sketched below. Sketch the current flowing between points C and D as a function of voltage applied between points C and D.

A sensitive amplifier is connected to terminals E and F to measure small electric signals from an instrument connected to terminals A and B. From time to time there are discharges in the instrument which might destroy the amplifier if the amplifier is connected to the instrument directly, without the diodes. Explain briefly how the diodes protect the amplifier. [4]



[PAT, 2014Q17][M6]

For this question you may assume that the electrostatic potential energy of two positively charged particles (with charge $+Q_1$ and $+Q_2$) separated by a distance x is given by

$$k\frac{Q_1 Q_2}{x}$$

where k is a constant.

Two charged particles are placed a distance d apart from each other. One has charge $= +Q$ and mass $= m$, whilst the other has charge $= +2Q$ and mass $= 2m$. The charges are initially held stationary, but are then released.

Find an expression for the maximum speed of the particle with mass $= 2m$. [6]

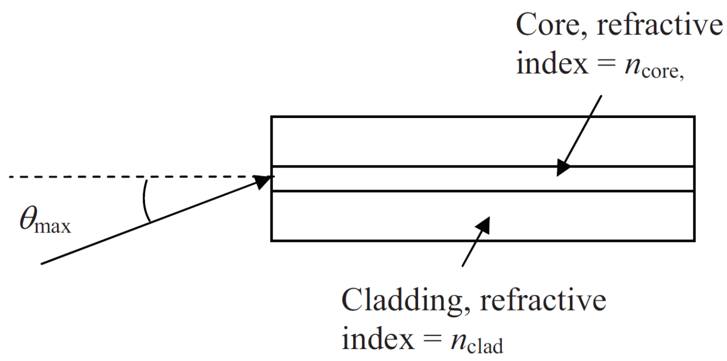
[PAT, 2014Q18][M10]

This question concerns total internal reflection, optical fibres and refraction. You may assume that the refractive index of glass is larger than that of water, and that the refractive index of water is larger than that of air.

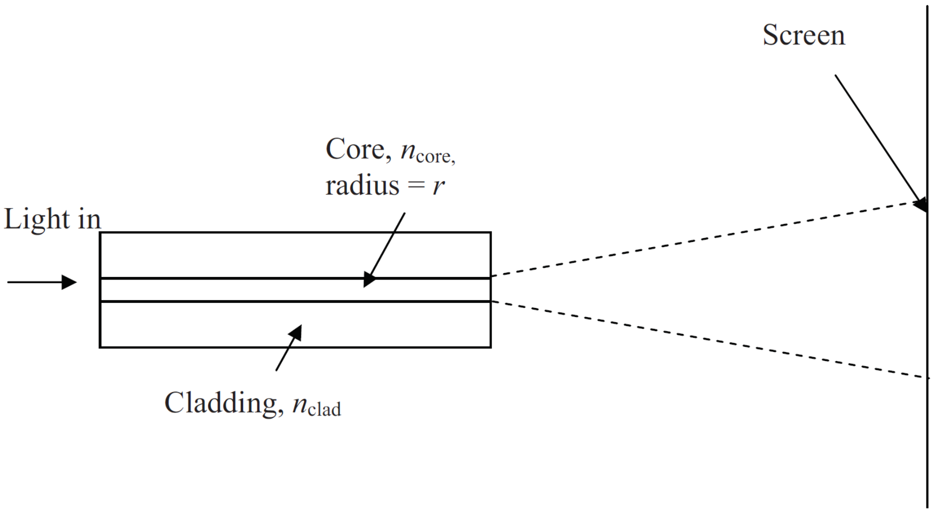
(a) Explain what is meant by the phrases "total internal reflection" and "critical angle". (You are encouraged to use a diagram to explain your answer.) [2]

(b) Derive an equation relating the critical angle and the refractive indices of two materials, n_1 and n_2 where $n_2 < n_1$. [2]

(c) An optical fibre is usually made of two materials - a core and a cladding as shown in the diagram below (not drawn to scale).



Light may only be transmitted along the fibre if the incident angle of the light is less than a maximum angle $\theta_{\max}$. By using your expression from part (b) and Snell's law, or otherwise, derive an expression for $\theta_{\max}$ in terms of the core and cladding refractive indices only. [3]

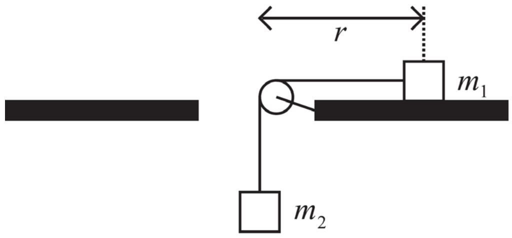


(d) In an experiment, light is transmitted along the glass fibre before leaving at a perfectly vertical end. A screen is placed a long distance away from the end of the fibre and a uniform circular spot is seen as shown above. A small glass tank containing water is placed in front of the beam so it perfectly touches the end of the fibre. You may assume the tank is parallel to the screen. On a copy of the above diagram, sketch how the extent of the spot on the screen changes. [2]

(e) The tank is kept in place and now white light is used instead of a laser. What will the image now look like? [1]

[PAT, 2014Q19][M10]

Two masses m_1 and m_2 are connected by a massless, non-extensible string supported by a massless pulley attached to a table with a hole in the middle; see sketch below.



(a) Assuming no friction, derive an expression for the acceleration of the masses and for the tension of the string. [2]

Now and for the rest of this question, consider friction acting on the table but not on the pulley. Friction force F_{fr} is proportional to the mass's weight; $F_{\text{fr}} = \mu_s mg$ or $F_{\text{fr}} = \mu_d mg$ depending whether the mass is at rest (μ_s = static friction coefficient) or in motion (μ_d = dynamic friction coefficient). Both coefficients are known.

(b) Derive expressions for the acceleration of the masses and for the tension of the string. What condition needs to be satisfied for m_1 to accelerate? [3]

(c) The table on which the mass m_1 is resting is now rotating about the vertical axis going through the middle of the table with angular speed ω. Assuming that the object with mass m_1 can be treated as point like, derive expressions for the minimal, r_{min}, and maximal, r_{max}, distance between m_1 and the axis of rotation such that for $r_{\text{min}} < r < r_{\text{max}}$, m_1 will not be moving radially. [5]

PAT 2015

PAT 2015
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 21 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2015Q1][M3]

Expand $(2x + x^2)^5$ in powers of x. [3]

[PAT, 2015Q2][M3]

Solve for x in the equation $\log_2 x + \log_4 16 = 2$. [3]

[PAT, 2015Q3][M4]

Evaluate the sum

$$\sum_{n=1}^{5} \left(\frac{1}{3}\right)^n.$$

What is the sum if the number of terms tends to infinity? [4]

[PAT, 2015Q4][M5]

Evaluate the integral

$$\int_4^6 (2x - 6)[(x - 4)(x - 2)]^{\frac{1}{2}}\,dx.$$

[5]

[PAT, 2015Q5][M4]

For what values of m does $4x^2 + 8x - 8 = m(4x - 3)$ have no real solutions? [4]

[PAT, 2015Q6][M4]

An unbiased coin is tossed 3 times. Each toss results in a "head" or "tail".

What is the probability

(a) of two or more tails in succession?

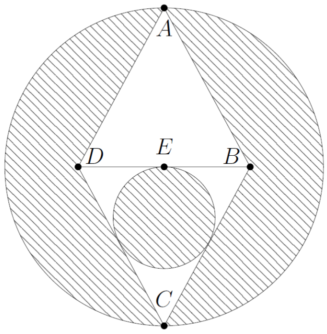
(b) that two consecutive toss results are the same?

(c) that if any one of the toss results is known to be a tail, that all of the tosses resulted in tails?

[4]

[PAT, 2015Q7][M6]

In the figure below, which is drawn only approximately to scale, the lengths of line segments AB, BC, CD, DA, and BD are equal. E is at the centre of the larger circle. The smaller circle is tangent to BC, CD, and BD, and has radius r. Derive an exact expression for the area of the shaded region in terms of r. [6]



[PAT, 2015Q8][M5]

Find the slopes and y-intercepts of the straight lines that are tangent and normal to the circle $(x+3)^2+(y-3)^2=17$ at the point $(1,2)$. [5]

[PAT, 2015Q9][M8]

By sketching the function below, or otherwise, find what values of y the function takes when x can take any real value.

$$y = -\left(\frac{8}{x^2 - 4}\right) - 3$$

[8]

[PAT, 2015Q10][M8]

For what values of x are the following inequalities satisfied?

$$-1 < \frac{3x+4}{x-6} < 1$$

[8]

[PAT, 2015Q11][M4]

A ball is thrown at an angle of 30° up from the horizontal, at a speed of 10 m/s, off the top of a cliff which is 10 metres high above a flat beach. How long does it take for the ball to hit the beach below? You may assume that the acceleration due to gravity is 10 m/s^2, and that air resistance can be neglected. [4]

[PAT, 2015Q12][M3]

An ice cube slides down a frictionless slope, which is at an angle α to the horizontal. The slope sits on a horizontal table of height H above the ground. If the ice cube is released from rest at height h above the ground, what is the speed of the cube when it is half way down the slope? [3]

[PAT, 2015Q13][M3]

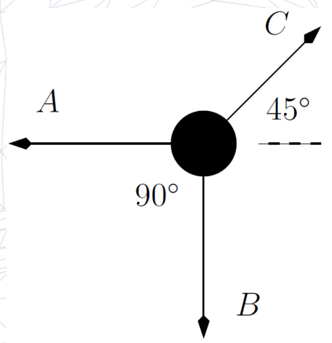
Using your knowledge of solar eclipses, estimate the radius of the Moon. You may assume the radius of the Sun is 700,000 km, the distance from the Earth to the Sun is 150,000,000 km, and the distance from the Earth to the centre of the Moon is 400,000 km. [3]

[PAT, 2015Q14][M3]

Write an expression relating the angle of incidence θ_1 and angle of refraction θ_2 of a ray of light travelling from one optically transparent material with index of refraction n_1 to another with index of refraction n_2. Sketch a diagram of the ray of light, clearly labelling the angles and indices of refraction. Under what conditions and for what angles of incidence is light reflected completely at the boundary? [3]

[PAT, 2015Q15][M3]

Consider a mass and three strings, all lying on a horizontal table. The strings exert forces outwards on the mass as shown below. The mass does not move. What is the force on string C in terms of the force on string A? What is the relationship between the force exerted by string A and the force exerted by string B? [3]



[PAT, 2015Q16][M4]

A non-rotating ball of mass 2 kg slides on a smooth, frictionless, horizontal surface at a speed of 1 m/s. It collides elastically and head-on with a stationary ball of mass 1 kg. What are the speeds of the two balls after the collision? [4]

[PAT, 2015Q17][M4]

A small boat floats on the sea. It encounters waves of the form

$$y(x,t) = A\sin(kx - \omega t)$$

where $y(x,t)$ is the height of the wave at position x and time t, and k and ω are constants. The waves have a wavelength of 10 metres, amplitude of 0.5 metres, and travel at a horizontal speed of 2 metres per second. What is the maximum vertical velocity of the boat? [4]

[PAT, 2015Q18][M5]

A garden hose with a cross sectional area A ejects water at a rate of x, usually measured in units of $m^3\ s^{-1}$.

(a) What is the speed of the water leaving the nozzle?

(b) The water hits a wall close to the end of the garden hose, perpendicular to the direction of flow. What is the force on the wall if:

(i) the water falls to the ground when it hits the wall, or

(ii) the water rebounds horizontally?

You may assume the density of water is ρ, usually measured in units of kg m^{-3}. [5]

[PAT, 2015Q19][M6]

Consider a circular orbit of radius r around the Earth. If the Earth's mass is M and radius is R, and Newton's gravitational constant is G, derive an expression for the speed that is required for a stable orbit.

Assuming that the radius of the Earth is 6400 km, and the gravitational acceleration g at the surface of the Earth is 10 m/s^2, calculate the speed required for a stable orbit around the equator at sea level. [5]

What physical reasons make it difficult for a satellite to maintain a circular orbit around the equator at sea level, even if one can ignore the atmosphere? [1]

[PAT, 2015Q20][M7]

The energy levels of hydrogen are given by the expression

$$E_n = -\left(\frac{R}{n^2}\right)$$

where R is the Rydberg constant, and n is a positive integer.

Neutral hydrogen atoms are excited to a state with $n = 10$. The atoms then de-excite, emitting light before settling to the ground state ($n = 1$).

(a) What is the shortest wavelength of light emitted?

(b) What is the longest wavelength emitted?

(c) How many emission lines may be observed?

[Leave answers as fractions multiplied by powers of R, Planck's constant h, and the speed of light c as needed.] [7]

[PAT, 2015Q21][M8]

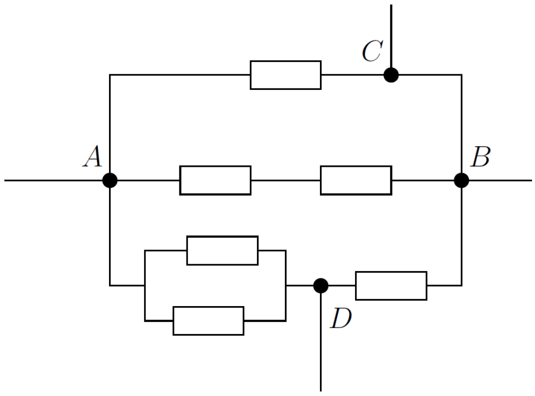
Consider the resistor array below. All the resistors are identical, with resistance R.

(a) Calculate the total resistance between A and B.

(b) If a potential difference V is applied between A and B, calculate the power dissipated in the resistor between B and D.

(c) Calculate the total resistance between C and D.

[Leave answers as fractions multiplied by powers of R and V as needed.] [8]



PAT 2016

PAT 2016
On-line Exam

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Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 21 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2016Q1][M2]

Differentiate the expression $x \sin x^2$ with respect to x. [2]

[PAT, 2016Q2][M5]

Find all values of θ between 0 and 2π which satisfy the equation

$$\sqrt{3}\tan^2\theta - 2\tan\theta - \sqrt{3} = 0.$$

[5]

[PAT, 2016Q3][M5]

Given the two equations

$$\log_4\left(\frac{64^x}{16^y}\right) = 13$$
$$\log_{10} 10^x + \log_3 3^y = 1,$$

find x and y. [5]

[PAT, 2016Q4][M3]

What is the term independent of x in the expansion $\left(x - \frac{1}{x^2}\right)^{12}$? [3]

[PAT, 2016Q5][M3]

How many numbers greater than 5000 may be formed by using some or all of the digits 3, 4, 5, 6, and 7 (but only those) without repetition? [3]

[PAT, 2016Q6][M4]

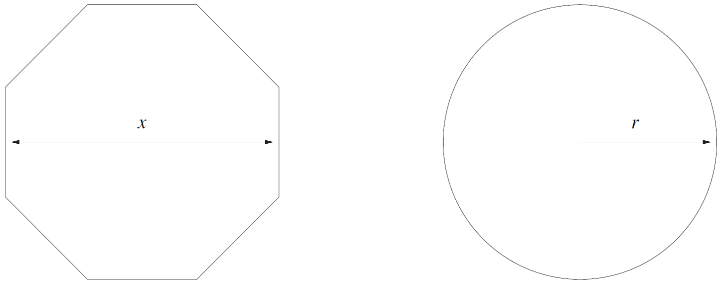
A seed is planted. After one month, there is one twig containing two leaves. In the following month, the twig grows two further twigs, each new twig again containing two leaves. If in successive months each new twig produces two further twigs with their leaves, how many leaves will be on the plant after 10 months? You may assume that no leaves fall off the plant, and once a new twig has produced two twigs it does not produce further twigs in subsequent months.) [4]

[PAT, 2016Q7][M4]

In a dice game, a player throws a fair 6-sided die n times. To win the game, the player needs to throw the following exact sequence within the n throws: 6, 5, 4, four 3's, two 2's, and 1. What is the probability that the player will win if the die is thrown (a) 8 times, (b) 10 times, and (c) 12 times? (You may leave your answer in terms of powers.) [4]

[PAT, 2016Q8][M6]

Compare a regular octagon with a circle of radius r. The distance between two parallel sides of the octagon is x.



What should x be (in terms of r only) if the two shapes are to have the same area? [6]

[PAT, 2016Q9][M6]

For what range(s) of x is the following inequality satisfied?

$$5 - 3x < \frac{2}{x}$$

[6]

[PAT, 2016Q10][M6]

Find the area of the region enclosed between the following curves and including the origin:

$$y = 2 - x$$
$$y = 2 + x$$
$$y = x^2 - 4.$$

[6]

[PAT, 2016Q11][M6]

A cylinder of dough is squashed such that its height h decreases linearly with time t as

$$h(t) = h_0 - \alpha t$$

for $t < h_0/\alpha$. Assume that the volume V of the dough remains constant, and it retains a cylindrical shape. Find an expression for the *rate of change* of the radius of the cylinder as a function of time and the parameters h_0, α, and V.

Does the rate of change increase or decrease with time? [6]

[PAT, 2016Q12][M3]

A ball of mass 100 g bounces on a hard surface. Every time it hits the floor, it loses a quarter of its kinetic energy. If the ball is released from a height of 1 m, after how many bounces will the ball bounce no higher than 0.25 m? [3]

[PAT, 2016Q13][M3]

For a range of temperatures around room temperature and above, sketch how the resistance of the following components vary with temperature:

(a) an ideal wire,

(b) a filament light bulb, and

(c) a thermistor. [3]

[PAT, 2016Q14][M4]

Assume that the moons Io and Europa travel in circular orbits around Jupiter. The radius of Europa's orbit is 1.6 times that of Io's. In what ratio are the moons' orbital periods? Give the answer to 2 significant figures. [4]

[PAT, 2016Q15][M3]

A rower wants to cross a river to a point on the opposite river bank directly opposite from where she will start. The distance to cover is 100 m, and she wants to cross in 10 seconds. The river flows uniformly at 7.5 m/s. How fast must she row her boat, and at what angle, relative to the flowing water? Give your answers as exact expressions in simplest terms. [3]

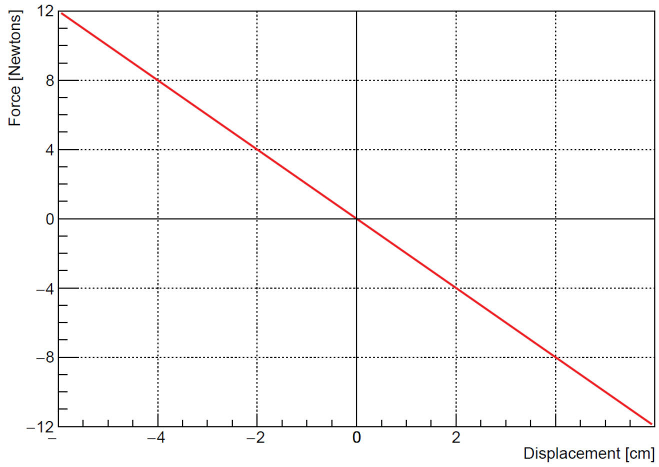
[PAT, 2016Q16][M6]

A negative point charge, with charge $-q$ (with $q > 0$ C), lies on a line between two fixed positive point charges with charges Q_1 and Q_2. Assume that charge Q_1 lies at $x = 0$, and charge Q_2 lies at $x = a$. Find all positions where the charge $-q$ experiences no net force from the positive charges, for cases $Q_1 = Q_2$ and $Q_1 \neq Q_2$.

In the latter case, what is the physical significance of your unused solution for the position of the charge $-q$? [6]

[PAT, 2016Q17][M7]

The diagonal line in the graph below shows the relationship between force and displacement for a certain frictionless physical system:



(a) Initially, a mass of 20 grams is placed at a displacement of 5 cm and released. What work is done in moving the mass to a displacement of 0 cm?

(b) What is the speed of the mass when it reaches a displacement of 0 cm?

(c) Describe the mass's motion after it reaches 0 cm. [7]

[PAT, 2016Q18][M4]

The drag force F on a sphere is related to the radius of the sphere (r), the velocity of the sphere (v), and the coefficient of viscosity of the fluid the drop is falling through (η) by the formula

$$F = kr^x\eta^y v^z$$

where k is a dimensionless constant, and x, y, and z are integers. By considering the units of the equation, work out the values of x, y, and z. (The coefficient of viscosity has units of kg m^{-1} s^{-1}.) [4]

[PAT, 2016Q19][M6]

Light of wavelength 625 nm shines onto a metal surface with a work function of 1.0 eV. Electrons are emitted from the metal.

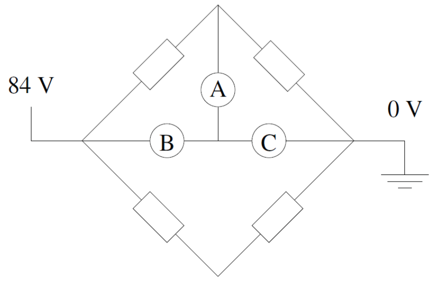
(a) What is the maximum speed of the emitted electrons?

(b) The electrons are then accelerated through a potential difference of 5.0 keV towards a fluorescent screen. What is the final speed of the electrons when they hit the screen?

[You may assume that Planck's constant is 6.0×10^{-34} J s, the speed of light is 3.0×10^{8} m/s, the mass of the electron is 1.0×10^{-30} kg, and the charge on the electron is 1.6×10^{-19} C.] [6]

[PAT, 2016Q20][M8]

In the resistor network shown below, each rectangle represents a resistor with fixed resistance 12 Ω, and each circle a water heater with fixed resistance 6 Ω. Each water heater contains 1 kg of water, initially at 20 °C, and you may assume that all the power dissipated by the water heater heats the water.



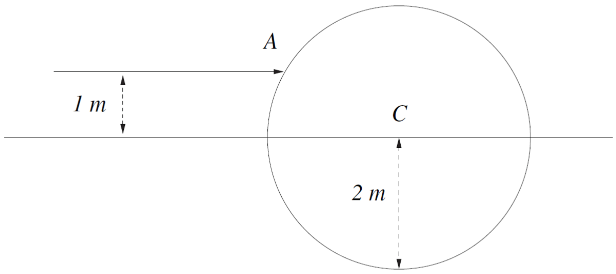
A voltage 84 V is applied as indicated in the figure. How long does it take for each individual water heater to raise their waters' temperature to 27 °C?

[The specific heat capacity of water is approximately 4200 J kg^{-1} $°C^{-1}$.] [8]

[PAT, 2016Q21][M6]

A sphere of radius $R = 2$ m is made of a material with the same refractive index as vacuum, and is supported in a fluid with refractive index n by a straight wire going through its centre C. A laser is directed toward the sphere with its beam parallel to the wire but offset by 1 m, and strikes the sphere at point A.

What refractive index n would be needed for the refracted beam to travel at 45° relative to the line AC (which includes points A and C)?



Another beam, not necessarily parallel with the wire, strikes the sphere at point A. At what angle, relative to the line AC, should the beam be directed such that it would be completely reflected by the sphere? [6]

PAT 2017

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Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 23 questions [100 marks].

Calculator

No calculators may be used.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2017Q1][M2]

Differentiate $y = 2x \cos x$ with respect to x:

(A) $-2 \sin x$

(B) $2 \cos x$

(C) $2 \cos x + 2x \sin x$

(D) $2 \cos x - 2x \sin x$

(E) $-2x \sin x$

[PAT, 2017Q2][M2]

Which equation has the same solutions as $2x^2 - 2x - 12 = 0$?

(A) $(x + 2)(x - 3) = 0$

(B) $2(x - 2)(x + 3) = 0$

(C) $(x - 6)(x + 1) = 0$

(D) $\left(x - 1 - 2\sqrt{10}\right)\left(x - 1 + 2\sqrt{10}\right) = 0$

(E) $\left(x - 2 - \sqrt{10}\right)\left(x - 2 + \sqrt{10}\right) = 0$

[PAT, 2017Q3][M2]

Evaluate the following sum:

$$\sum_{n=0}^{10} (-e^{-1})^n$$

(A) $\frac{1}{1+e^{-1}}$

(B) $\frac{1-e^{-10}}{1-e^{-1}}$

(C) $\frac{1+e^{-10}}{1+e^{-1}}$

(D) $\frac{1+e^{-9}}{1-e^{-1}}$

(E) $\frac{1+e^{-11}}{1+e^{-1}}$

[PAT, 2017Q4][M2]

If $a^{3-x}b^{5x} = a^{x+5}b^{3x}$ with a and b both real and positive, and $a \neq b$, what is x?

(A) $\frac{2\log(a-b)}{\log b}$

(B) $2\log a - \log b$

(C) $\frac{2\log b}{\log a - \log b}$

(D) $\frac{\log a}{\log b - \log a}$

(E) $\frac{\log a + \log b}{\log a}$

[PAT, 2017Q5][M2]

Which of the following integrals are equal to zero?

$$I_1 = \int_{-1}^{1} x^3 \, \mathrm{d}x$$
$$I_2 = \int_{-\infty}^{\infty} e^{-x^2} \, \mathrm{d}x$$
$$I_3 = \int_{-\pi}^{\pi} x \sin x \, \mathrm{d}x$$
$$I_4 = \int_{-\pi/2}^{\pi/2} x \cos x \, \mathrm{d}x$$

(A) None of these

(B) I_1 and I_4

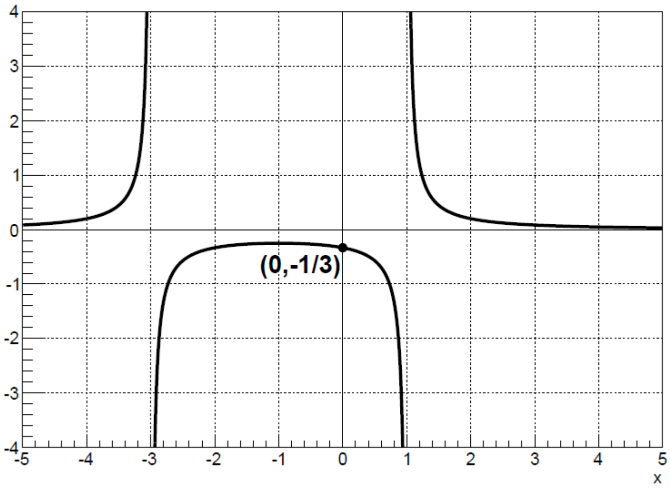
(C) I_1, I_2 and I_4

(D) I_2, I_3 and I_4

(E) All of these

[PAT, 2017Q6][M2]

The graph below could represent which of the following functions?



(A) $\frac{1}{x-1}+\frac{2}{x+3}$

(B) $\frac{-1}{x^2-2x+3}$

(C) $\frac{1}{x^2+2x-3}$

(D) $\frac{1}{x^2-1}+\frac{2}{x+3}$

(E) $\frac{3}{x^2-9}$

[PAT, 2017Q7][M2]

An astronaut on the surface of the Moon lightly tosses a ball of mass m upward. What happens to the ball?

(A) The ball enters an orbit around the Earth.

(B) The ball eventually falls toward the Earth, burning up in the atmosphere.

(C) The ball falls to the surface of the Moon.

(D) The ball rises slowly until it hovers above the astronaut.

(E) The ball enters an orbit around the Moon.

[PAT, 2017Q8][M2]

In which of the following lists are the parts of the electromagnetic spectrum ordered correctly from shortest wavelength (at the top) to longest wavelength (at the bottom)?

(A) ultraviolet, X-ray, visible, radio, infrared

(B) X-ray, ultraviolet, visible, radio, infrared

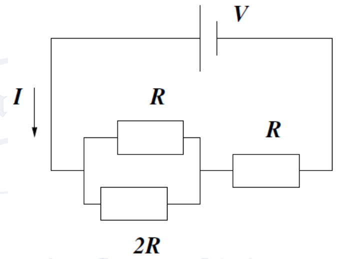
(C) ultraviolet, X-ray, visible, infrared, radio

(D) infrared, radio, visible, ultraviolet, X-ray

(E) X-ray, ultraviolet, visible, infrared, radio

[PAT, 2017Q9][M2]

What is the value of the current I in the circuit below?



(A) $\frac{V}{4R}$

(B) $\frac{3V}{5R}$

(C) $\frac{4V}{3R}$

(D) $\frac{5V}{3R}$

(E) $\frac{3V}{4R}$

[PAT, 2017Q10][M2]

A capacitor is constructed with two conducting plates of equal area A separated by an insulator. The capacitance is measured to be C.

The conducting plates are then shrunk to half the original area. What is the capacitance now?

(A) $\frac{C}{2}$

(B) C

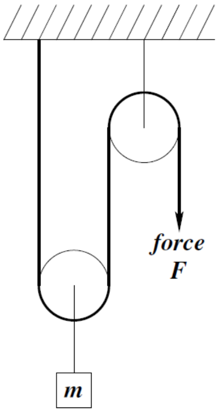
(C) $2C$

(D) C^2

(E) $\frac{1}{C^2}$

[PAT, 2017Q11][M2]

Consider the pulley system below supporting an object with mass m. Assume gravitational acceleration to be g, that the pulleys are massless and frictionless, and the string is massless and inextensible. With how much force F must the string be pulled to keep the mass at the same height?



(A) $\frac{mg}{3}$

(B) $\frac{mg}{2}$

(C) mg

(D) $2mg$

(E) $3mg$

[PAT, 2017Q12][M2]

A particle with charge q and initial speed v is stopped by a potential difference V in a distance d and time t. What was its initial momentum?

(A) $\frac{qVt}{d}$

(B) $\frac{qV}{v}$

(C) $\frac{qVd}{t}$

(D) $2qVv$

(E) $\frac{qV}{2v}$

[PAT, 2017Q13][M3]

Expand $(3+2x)^5$ as a sum of powers of x. [3]

[PAT, 2017Q14][M4]

Person A is busy for 50% of the week, Person B 75%, and Person C 20%. If a time for a meeting is picked at random, what is the probability that

(a) all three people are busy,

(b) all three people can attend the meeting? [4]

[PAT, 2017Q15][M5]

A spring with spring constant k and natural length L joins two blocks of mass m and M. The two blocks lie on a horizontal table, initially L apart. The maximum force for static friction between a block and the table is given by the coefficient of static friction μ_s multiplied by the block's weight.

How far must the mass M be displaced to cause mass m to move? [5]

[PAT, 2017Q16][M4]

A cone has a height equal to the diameter of a sphere. If the volumes of the two objects are equal, and the radius of the sphere is r, what is the radius of the base of the cone? [4]

[PAT, 2017Q17][M7]

A parachutist jumps out of a plane at height h. She is subject to air resistance with a force of $-\alpha v^2$. The equation of her motion is given by

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - \alpha v^2$$

(a) What are the units of α?

(b) Calculate the terminal velocity of the parachutist.

(c) Estimate how much work is done by the air resistance as she falls, assuming that she is falling at near terminal velocity by the time she reaches the ground. [7]

[PAT, 2017Q18][M8]

Consider two sound waves travelling with the same speed and amplitude but having similar but slightly different wavelengths, λ_1 and λ_2, and angular frequencies, ω_1 and ω_2. The two waves are described with the functions

$$y_1(x,t) = A\cos\left(\frac{2\pi x}{\lambda_1} - \omega_1 t\right)$$
$$y_2(x,t) = A\cos\left(\frac{2\pi x}{\lambda_2} - \omega_2 t\right).$$

(a) What is the speed v in terms of the angular frequencies and wavelengths?

(b) Sketch $y_1 + y_2$ as a function of x, at some time t.

(c) If you stood in the path of these sound waves, what frequency would you hear (assuming you can hear it)? What is the distance between points where the sound disappears?

[Hint: you can use the formula $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$.] [8]

[PAT, 2017Q19][M9]

A curve is defined parametrically:

$$x = a(\omega t - \sin \omega t)$$
$$y = a\left(\sqrt{3} - 2\cos \omega t\right)$$

with non-zero constants a and ω. At what values of x is y equal to zero? [9]

[PAT, 2017Q20][M9]

In a certain binary star system, two stars with identical mass m have equal and opposite velocities v_2 on the opposite sides of the same circular orbit with radius R.

In another system with three identical stars of the same mass m as before, it is observed that all three stars are equally spaced around a circular orbit with the same radius R as before. What is the speed v_3 of these stars in terms of v_2?

[Hint: consider the direction and magnitude of the force exerted on one star by the other two.]

[9]

[PAT, 2017Q21][M9]

Evaluate the following expression:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^{2t^2} (xt)^4 \,\mathrm{d}x$$

[9]

[PAT, 2017Q22][M9]

The equation of circle C_1 is

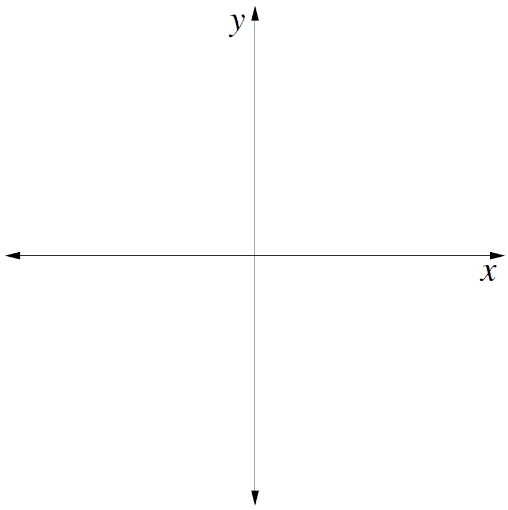
$$4x^2 + 24x + 4y^2 - 16y + 43 = 0$$

while the equation of circle C_2 is

$$4x^2 - 40x + 4y^2 - 8y + 79 = 0.$$

Sketch a diagram of these circles on the axes below, along with all lines which are tangent to both circles.

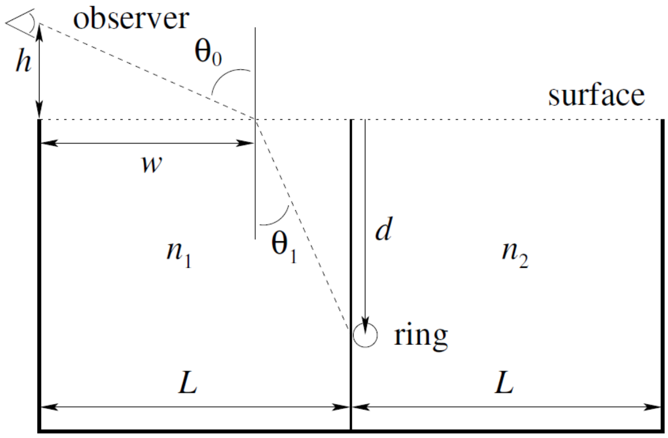
For each line, calculate the length of the line segment joining the tangent points. [9]



[PAT, 2017Q23][M9]

An experimental setup consists of two deep tanks, each of width L, separated by a thin, transparent membrane, as shown in the figure below. The left tank is filled with a transparent liquid with refractive index n_1, and the right tank with a transparent liquid with refractive index n_2. The membrane has refractive index n_1. Assume that the refractive index of air is 1, and $1 < n_2 < n_1$.

A gold ring is dropped in the right pool (with refractive index n_2), near the membrane, and drops straight down. An observer, at height h above the left edge of the experimental setup, watches the ring drop. The dashed line in the figure indicates the path of a light ray from the ring to the observer, with lengths and angles indicated.



At a certain apparent depth, the ring will appear to the observer to stop descending. At what apparent depth does this happen? [9]

PAT 2018

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Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 23 questions [100 marks].

Calculator

Only calculators meeting the specifications for PAT are allowed.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2018Q1][M2]

What is the next number in the sequence? 4, 5, 9, 14, 23

(A) 28

(B) 34

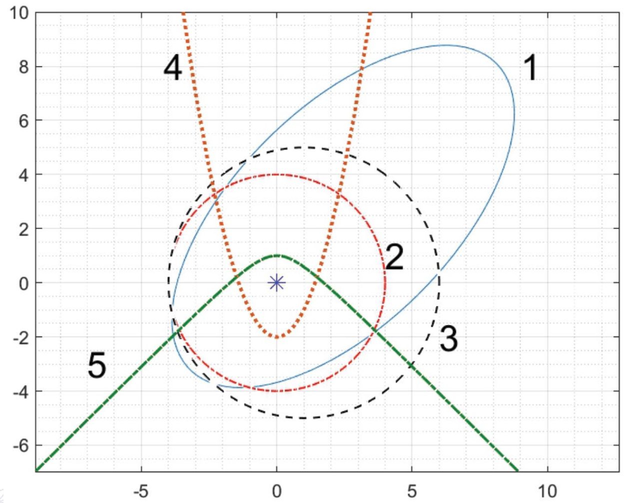
(C) 37

(D) 39

(E) 42

[PAT, 2018Q2][M2]

Which of the curves shown in the figure is not a trajectory in the gravitational field of a central star at the point $(x = 0, y = 0)$ marked by the star symbol?



(A) All are trajectories

(B) none of these is a trajectory

(C) 5 is not a trajectory

(D) 4 and 5 are not trajectories

(E) 3 is not a trajectory

[PAT, 2018Q3][M2]

Which combination of units is the odd one out?

(A) C V m^{-1}

(B) A T m

(C) kg m s^{-2}

(D) J m^{-1}

(E) C m s^{-1}

[PAT, 2018Q4][M2]

90 people enter a maze. At each junction a third will go left and two thirds will go right. After three such junctions, what is the most likely combination of turns people will have taken?

(A) Gone right three times

(B) Gone left three times

(C) Gone right twice and once left

(D) Gone twice left and once right

(E) It is impossible to tell

[PAT, 2018Q5][M2]

A person drinks many cups of tea. The first cup the person drinks is filled completely. They don't want to drink too much tea in total so the second cup is filled with only a fraction (α) of the tea in the first cup, the third cup contains the same fraction α of the second cup and so on. What is the maximum value of α so that the person drinks no more than 3 times the amount of tea in the first cup however many drinks they take?

(A) $\alpha = \frac{1}{3}$

(B) $\alpha = \frac{1}{2}$

(C) $\alpha = \frac{2}{3}$

(D) $\alpha = \frac{1}{4}$

(E) $\alpha = \frac{3}{4}$

[PAT, 2018Q6][M2]

A stationary wave is set up on a string of length L. If the centre and the ends of the string are held fixed with zero displacement, what wavelengths (λ_m) can this stationary wave have? In the table below m denotes any positive non zero integer.

(A) $\lambda_m = \frac{2L}{m}$

(B) $\lambda_m = \frac{L}{m}$

(C) $\lambda_m = \frac{L}{2m}$

(D) $\lambda_m = mL$

(E) $\lambda_m = 2mL$

[PAT, 2018Q7][M2]

A car of mass m is traveling along a straight and narrow road at speed u. A cat walks into the road a distance d in front of the car and stops in the middle of the road. What constant force must be applied to the car so that it does not hit the cat? The road is too narrow for the car to drive past the cat without hitting it.

(A) $F = \frac{-mu^2}{2d}$

(B) $F = \frac{-u^2}{2d}$

(C) $F = \frac{-mu^2}{2}$

(D) $F = \frac{-mu^2}{d}$

(E) the car cannot avoid hitting the cat

[PAT, 2018Q8][M2]

What is the value of x for which y has a minimum in the function $y = (x-3)(x+1)$?

(A) $x = 1$

(B) $x = -1$

(C) $x = 3$

(D) $x = 0$

(E) $x = -3$

[PAT, 2018Q9][M2]

What is the equation of the line which intersects $y = 2x - 2$ at right angles and at position $x = 1$?

(A) $y = -\frac{1}{2}x$

(B) $y = -\frac{1}{2}x + \frac{1}{2}$

(C) $y = \frac{1}{2}x - \frac{1}{2}$

(D) $y = x$

(E) $y = 2x$

[PAT, 2018Q10][M2]

For which range of x is the inequality $x^3 - x^2 - x + 1 \geq 0$ satisfied?

(A) $x \geq 1$

(B) $x \leq 1$

(C) $x \geq 1$ and $x \leq -1$

(D) $-1 \leq x \leq 1$

(E) $x \geq -1$

[PAT, 2018Q11][M2]

A rectangular building with sides 50 m and 100 m long has a flat roof on top of it. The roof has a mass per unit area of 100 kg m^{-2}. The walls are 10 cm thick and can take a maximum stress of $S_{\max}$ which depends on the material of the wall. The values of $S_{\max}$ for five building materials are:

moist wood: $S_{\max} = 25.5$ N mm^{-2}

dry wood: $S_{\max} = 48.3$ N mm^{-2}

concrete: $S_{\max} = 300$ N mm^{-2}

solid cardboard: $S_{\max} = 15$ N mm^{-2}

brick: $S_{\max} = 7$ N mm^{-2}

Which materials could be used so that the walls will support the building, assuming that the mass of the walls is negligible?

(A) only concrete

(B) only brick

(C) any of the materials above

(D) concrete or dry wood

(E) concrete, dry or moist wood

[PAT, 2018Q12][M2]

A light ray passes through an infinite stack of thin transparent plates. The refractive index of these plates increases slightly by a constant factor from one plate to the next. The ray enters the first plate making an angle θ_0 with the surface normal. The ray's angle to the normal changes at each new interface. At the ith interface between plates the ray makes an angle θ_i to the normal. Find the limiting value θ_∞ for the angle θ_i when the ray has traversed an infinite number of plates.

(A) $\theta_\infty = \theta_0$

(B) $\theta_\infty = -\theta_0$

(C) $\theta_\infty = 0°$

(D) $\theta_\infty = 90°$

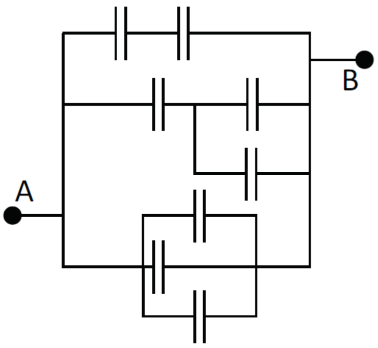
(E) $\theta_\infty = 180°$

[PAT, 2018Q13][M4]

Consider the network of capacitors shown below. All capacitors in the network have the same capacitance C. Assume that capacitances combine as follows:

- Two capacitors in series combine their capacitance in the same way that two resistors in parallel combine their resistance.
- Two capacitors in parallel combine their capacitance in the same way that two resistors in series combine their resistance.

Find the total capacitance C_{tot} between the points A and B. [4]



[PAT, 2018Q14][M4]

Solve the following equation for x: $\log_x 25 = \log_5(x)$ [4]

[PAT, 2018Q15][M8]

A car is initially stationary. At time t_0 it starts to move at speed v_1 for a time Δt_1. Immediately after this it moves at speed $v_2 > v_1$ for a time $\Delta t_2 < \Delta t_1$. After this time it is stationary again.

(a) Using this information sketch a graph of speed v versus time t. Label all axes sufficiently so that the graph is quantitatively accurate. [1]

(b) From your graph determine the average speed $\langle v \rangle_t$ of the car while it is moving. Express $\langle v \rangle_t$ as a formula using the symbols v_1, Δt_1, v_2, Δt_2. $\langle v \rangle_t$ is known as the time weighted average. Compute a numerical value for $\langle v \rangle_t$ given the following: $v_1 = 1\ \mathrm{m\ s^{-1}}$, $\Delta t_1 = 2\ \mathrm{s}$, $v_2 = 2\ \mathrm{m\ s^{-1}}$, $\Delta t_2 = 1\ \mathrm{s}$, $t_0 = 0\ \mathrm{s}$. [2]

(c) Now draw a graph of speed v versus distance s using the numerical values for speeds and times given in section (b) and label the axes as before. [1]

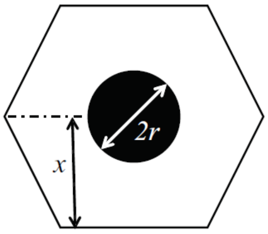
(d) From the second graph find the average speed $\langle v \rangle_t$ known as the distance weighted average. Again express $\langle v \rangle_t$ as a formula using the symbols v_1, Δs_1, v_2, Δs_2, where Δs_1 and Δs_2 are the distances that the car travels at v_1 and v_2. Also provide a numerical value for $\langle v \rangle_s$ using the values given in section (b). [2]

(e) Which of the two averages $\langle v \rangle_t$ or $\langle v \rangle_s$ is equivalent to the conventional definition of average speed which we assume to be $\langle v \rangle_c = \frac{\text{total distance travelled}}{\text{total time taken}}$. [1]

(f) State the mathematical operation that should be applied to the curve of $v(t)$ to compute $\langle v \rangle_c$ if we assume v to be any function of time or distance. [1]

[PAT, 2018Q16][M6]

Consider the cross section of a pencil, where a concentric circle of radius r sits at the centre of regular hexagon. If the black area is a quarter of the total area of the hexagon, what is the length of the dimension x in terms of r? [6]



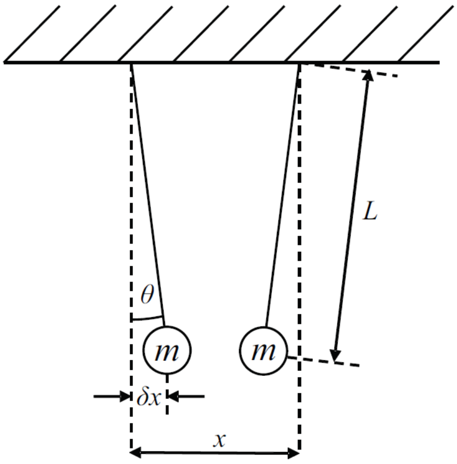
[PAT, 2018Q17][M7]

To which order should you expand $(1+x)^9$ in the integral $\int_0^{0.1}(1+x)^9\,\mathrm{d}x$ so that the approximate integral agrees with the exact result to better than 10%? [7]

[PAT, 2018Q18][M9]

The diagram shows two spheres of mass m suspended from wires of negligible mass. The length between the pivot points and the centres of each sphere is L. The two suspension points lie on a horizontal line and are separated by a distance x. The system is in equilibrium. Take the total acceleration due to gravity to be g.

Hint: You may assume that $\delta x \ll L$ as well as $\delta x \ll x$.



(a) Show that the small deflection δx by which the gravitational force F_g between the spheres will deflect them from the vertical is given by:

$$\delta x \approx \frac{x}{8} - \sqrt{\left(\frac{x}{8}\right)^2 - \frac{LmG}{4xg}}$$

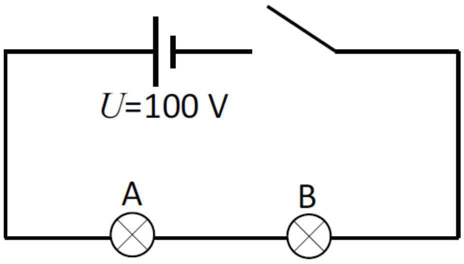
where G is Newton's gravitational constant. Clearly state all assumptions you make and show all your workings. [7]

(b) The spheres are now both electrically charged with the same positive charge Q. How big must Q be so that the wires hang vertically again? Hint: The force between two spheres, electrically charged with charges Q_1 and Q_2 is given by:

$$F_Q = k\frac{Q_1 Q_2}{r^2}$$

where k is a constant and r is the separation of their centres. [2]

[PAT, 2018Q19][M7]



Consider the above circuit. The filament lamp A consumes an electrical power $P_A = 100$ W when it alone is connected to a voltage of $U = 100$ V. Filament lamp B consumes a power $P_B = 20$ W when it alone is connected to a voltage of $U = 100$ V.

(a) When the switch is closed, which lamp will be brighter and why?

Find the ratio of the levels of brightness of the two lamps assuming their resistances are constant. Assume further that the brightness of a lamp is proportional to the power it consumes. [5]

(b) How would your answer change if the lamps were wired in parallel rather than series?[2]

[PAT, 2018Q20][M7]

In a crystalline material, planes of symmetry are labeled using triplets of integer numbers of the form (hkl), e.g. (100) is equivalent to $h = 1$, $k = 0$, $l = 0$. For this question it is not important how this labeling scheme works exactly or what h, k and l mean. By performing certain experiments, it is possible to determine these (hkl) triplets for multiple planes of symmetry. From the knowledge of several such triplets the microscopic structure of the material can be inferred using the following rules:

- If for all planes all positive integer values of h, k and l are possible the material has a simple cubic lattice.
- If for all planes $h + k + l = 2n$ where n is any positive integer then the material has a body centred cubic lattice.
- If for all planes h and k and l are either all even or all odd the material has a face centred cubic lattice.

Note: zero is an even number

During the analysis of the data from such an experiment the following four planes of symmetry were found: (111), (200), (220), (311).

(a) What type of lattice did the material in these experiments have? [2]

(b) In a material with some form of cubic lattice the distance d between the planes of symmetry is given by:

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

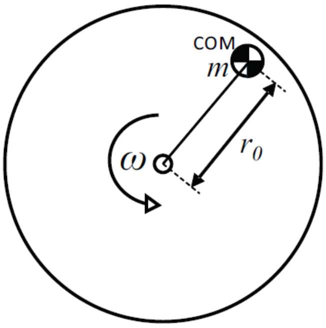
where a is a constant length that is usually of the order of a fraction of a nanometre. Using the formula for d and the data in the table below determine the best estimate for a. [3]

d (nm)	Plane
0.224	(111)
0.195	(200)
0.137	(220)
0.117	(311)

(c) A cube of the material has a length L in each dimension where $L = Na$ and N is a very large number. Pressure is applied to two opposite sides of the cube. The volume of the cube remains constant but one of the cube's dimensions reduces to a length of $L' = \frac{2}{3}Na$. What is the new length of the deformed object in the other two dimensions? [2]

[PAT, 2018Q21][M10]

A child of mass m stands on a platform which rotates freely at angular speed ω. The child's centre of mass (COM) is initially at a radius of r_0 from the axis of the platform as shown in the figure below. The child now slowly pulls itself closer to the centre of the platform.



(a) State the minimal force $F(r)$ needed to pull the child closer to the centre as a function of the radius r of the child's COM. [1]

(b) Find a mathematical expression for the work done by the child when it has reached the centre of the platform.

Hint: You may take the work done along a path to be $W = \int F(s)\,\mathrm{d}s$ where F is the force in the direction of motion and s is the distance. If your expression contains an integral you do not have to perform the integration yet. [1]

(c) Into which form of energy is the child's work converted and what will qualitatively happen to the angular speed ω as the child approaches the centre? [2]

(d) The angular momentum J of the system of child and platform is a conserved quantity during the above process. It is defined to be $J = I\omega$ where I is the moment of inertia of the system which is given by $I = mr^2 + I_p$ where I_p is the constant moment of inertial of the platform. Find an expression for the angular speed $\omega(r)$ as a function of the radius r of the child's COM. [2]

(e) Find the total work done by the child when it has reached $r = 0$.

Hint: $\int \frac{x}{(a+x^2)^2}\mathrm{d}x = \frac{-1}{2(a+x^2)}$ [4]

[PAT, 2018Q22][M9]

Determine the area inside the circle defined by:

$$x^2 + y^2 - 8x + 4y + 4 = 0$$

but outside the triangle bounded by the three lines below.

$$y = x - 7$$
$$y = \frac{1}{5}(2x - 29)$$
$$x = 7$$

[9]

[PAT, 2018Q23][M5]

Given that

$$f(x) = \frac{\sqrt{x^2 - 2}}{\ln(3x + 10)}$$

Determine the range of x for which $f(x)$ is real and finite. [5]

PAT 2019

PAT 2019
On-line Exam

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TIME ALLOWED: 2 HOURS

Questions and Score

Total 24 questions [100 marks].

Calculator

Only calculators meeting the specifications for PAT are allowed.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2019Q1][M2]

What is the next number in the sequence? $-972, 324, -108, 36, -12$

(A) -4

(B) -3

(C) 3

(D) 4

(E) 9

[PAT, 2019Q2][M2]

Which values of x and y solve the following equations simultaneously:

$$\log x + 2\log y = \log 32$$
$$\log x - \log y = -\log 2$$

(A) $x = 2, y = 4$

(B) $x = -2, y = -4$

(C) $x = 2, y = -4$

(D) $x = -2, y = 4$

(E) no solution exists

[PAT, 2019Q3][M2]

Consider a system of many interacting particles. Let each particle have a potential energy $V(r)$ with respect to any other particle, where $V(r) \propto r^n$ where r is the distance to another particle and n is an integer. For such systems the Virial Theorem relates the time averaged total kinetic energy of all particles $\langle T_{\text{tot}} \rangle$ to the time averaged total potential energy $\langle V_{\text{tot}} \rangle$ as follows:

$$2\langle T_{\text{tot}} \rangle = n\langle V_{\text{tot}} \rangle$$

If the particles in our system interact only via gravity, what is the time averaged total energy $\langle E_{\text{tot}} \rangle$ of the system?

(A) $\langle E_{\text{tot}} \rangle = 0$

(B) $\langle E_{\text{tot}} \rangle = 2\langle V_{\text{tot}} \rangle$

(C) $\langle E_{\text{tot}} \rangle = \langle V_{\text{tot}} \rangle/2$

(D) $\langle E_{\text{tot}} \rangle = -\langle V_{\text{tot}} \rangle$

(E) $\langle E_{\text{tot}} \rangle = -2\langle V_{\text{tot}} \rangle$

[PAT, 2019Q4][M2]

The acceleration g due to gravity on a spherical planet in any universe is given by:

$$g = \frac{GM}{R^2}$$

where M is the mass, R the radius of the planet and G is the gravitational constant in that planet's universe.

In a different universe the gravitational constant is G' and has twice the value of the gravitational constant in our Universe G.

Find the ratio $\frac{g_{\text{planet}}}{g_{\text{Earth}}}$ for a planet in the different universe which has half the radius and twice the density of the Earth.

(A) $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 2$

(B) $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 1$

(C) $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \frac{1}{2}$

(D) $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 4$

(E) $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \frac{1}{4}$

[PAT, 2019Q5][M2]

In which range of α does the following equation have real solutions?

$$\sec^2\theta + \alpha\tan\theta = 0$$

(A) $\alpha \le -2$ or $\alpha \ge 2$

(B) $\alpha \le -2$

(C) $\alpha \ge 2$

(D) $\alpha \ge -0$

(E) $\alpha \le 0$

[PAT, 2019Q6][M2]

A bag contains b blue balls and r red balls. If two balls are picked at random and removed from the bag, what is the probability P that they are different colours?

(A) $\frac{2br}{(b+r)(b+r-1)}$

(B) $\frac{br}{(b+r)(b+r-1)}$

(C) $\frac{br}{(b+r)^2}$

(D) $\frac{2br}{(b+r)^2}$

(E) $2br$

[PAT, 2019Q7][M2]

We wish to represent integer numbers by using our ten fingers. A finger is assumed to be either stretched out or curled up. How many different integers can we represent with our fingers?

(A) 10

(B) 512

(C) 1000

(D) 20

(E) 1024

[PAT, 2019Q8][M2]

Without explicit calculation state which integrals are non-zero.

$$I_1 = \int_{-3\pi}^{3\pi} x^2 \sin(x)\,\mathrm{d}x \qquad (1)$$

$$I_2 = \int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x \qquad (2)$$

$$I_3 = \int_{3\pi/2}^{-3\pi/2} \cos^2(x)\,\mathrm{d}x \qquad (3)$$

$$I_4 = \int_{-\infty}^{\infty} xe^{-x^2}\,\mathrm{d}x \qquad (4)$$

(A) 2 and 3

(B) 1 and 4

(C) 1 and 3

(D) 2 and 4

(E) all

[PAT, 2019Q9][M2]

A long, thin, straight wire carrying an electric current I causes a magnetic field of flux density B at a perpendicular distance r from the wire. The magnitude of this flux density is given by the following relation:

$$B = \frac{\alpha I}{r}$$

where α is a constant. The magnetic field points circumferentially around the wire. A second, identical wire is placed parallel to the first one at a distance D. Find the current I_2 that has to flow in the second wire if the flux density at a line half way between and parallel to the wires is to double, compared to the flux density from only one wire at current I.

(A) $I_2 = I$

(B) $I_2 = 2I$

(C) $I_2 = -2I$

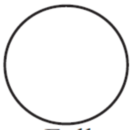
(D) $I_2 = -I$

(E) $I_2 = -I/2$

[PAT, 2019Q10][M2]

When the phase of the Moon as seen from the Earth is Full, what phase of the Earth is seen by an observer on the Moon?

The symbols below show phases of the Earth as seen from the Moon.

(A)  Full

(B)  Gibbous

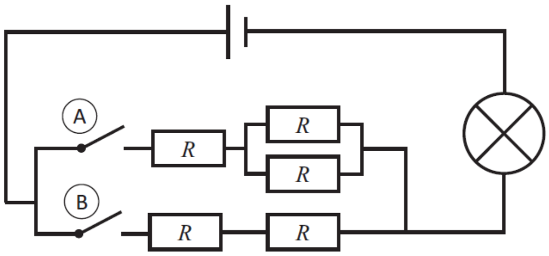
(C)  Quarter (or 'half')

(D)  Crescent

(E)  New

[PAT, 2019Q11][M2]

In the circuit shown below all resistors have the same resistance R and the light bulb has a fixed resistance. You wish to change the state of the switches so that the brightness of the bulb increases from its minimum to its maximum. Which sequence of switch states will achieve this?



(A) 1. both closed 2. only A closed 3. only B closed

(B) 1. both closed 2. only B closed 3. only A closed

(C) 1. only B closed 2. only A closed 3. both closed

(D) 1. only A closed 2. only B closed 3. both closed

(E) all states give the same brightness

[PAT, 2019Q12][M2]

An organ pipe is open at one end and closed at the other. The lowest note you can play on this pipe has frequency f_{min}. If the speed of sound in the pipe is v, what is the length L of the pipe?

(A) $L = \frac{v}{2f_{\text{min}}}$

(B) $L = \frac{v}{4f_{\text{min}}}$

(C) $L = \frac{v}{f_{\text{min}}}$

(D) $L = \frac{2v}{f_{\text{min}}}$

(E) $L = \frac{4v}{f_{\text{min}}}$

[PAT, 2019Q13][M6]

(a) Sketch the graphs of $y = (1 + x)^n$ for integer values of n from 0 to 3, each on a separate set of axes. Which point(s) are common to all the graphs? [4]

(b) Describe two of the further features common to the graphs for integer $n > 1$. [2]

[PAT, 2019Q14][M6]

A radioactive sample contains two isotopes, A and B. Isotope A decays to isotope B with a half life of $t_{1/2}$. Isotope B is stable.

(a) The number of atoms of A left after a time t is given by:

$$N_A(t) = N_{A0}e^{-\lambda t}$$

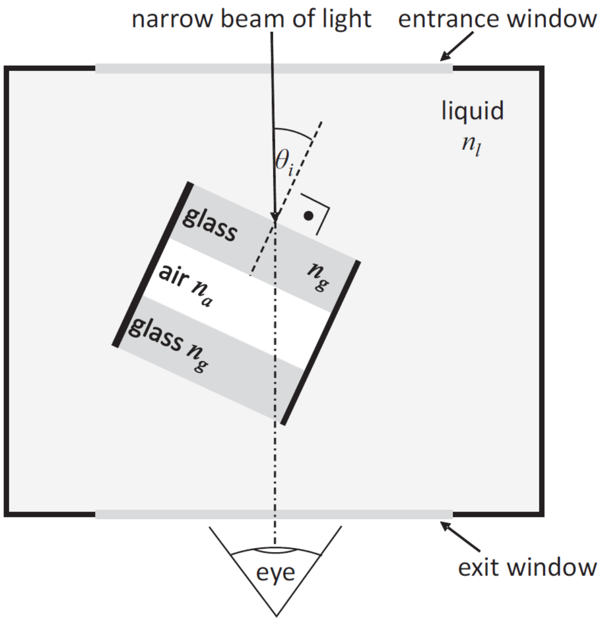
where N_{A0} is the initial number of atoms of A. Derive an expression for λ in terms of $t_{1/2}$. [2]

(b) Initially the number of B atoms in the sample is $N_B(t = 0) = N_{B0}$. Let $N_B(t)$ be the time dependent number of B atoms in the sample. Write down an expression for $N_B(t)$ in terms of λ, N_{B0}, N_{A0} and t. [1]

(c) At the start there are x times as many A atoms in the sample as there are B atoms. How long does it take until this ratio is reversed? [3]

[PAT, 2019Q15][M8]

The diagram below shows an air-cell refractometer.



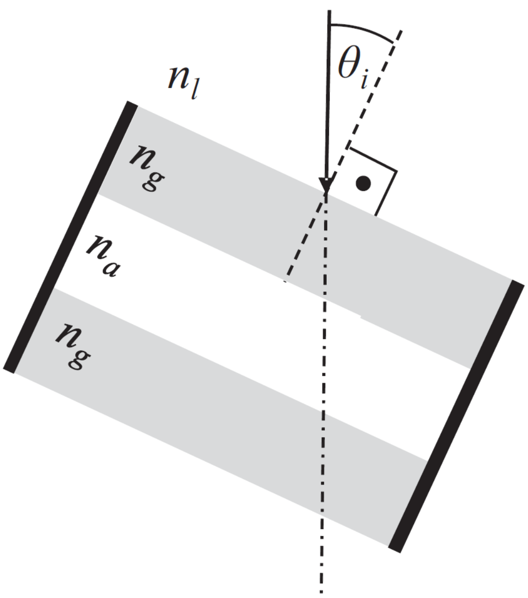
A narrow beam of light enters the entrance window at normal incidence and you can observe the light leaving the exit window by eye. The outer box is filled with a liquid of unknown refractive index n_l. The glass of the air cell has refractive index n_g and the cell is filled with air of refractive index n_a. The air-cell in the liquid filled vessel makes angle θ_i with respect to the incoming beam of light, as shown in the diagram.

This angle can be precisely adjusted and measured.

(a) On the diagram provided on the next page, draw the refracted path of the beam through the air cell. For this diagram you should assume $n_g > n_l > n_a$. [3]

(b) Describe qualitatively what you will observe at the exit window as you increase θ_i from zero and hence explain how this instrument could be used to determine the refractive index of the liquid n_l in the chamber. Find the relation between n_l and a special value of θ_i. [3]

(c) Suggest with reasons, a way to modify the apparatus or its use to improve the measurement. [2]



[PAT, 2019Q16][M7]

The energy levels of the electron in a hydrogen atom are characterised by a quantum number n:

$$E_n = -\frac{hcR}{n^2}$$

where h is Planck's constant, c is the speed of light and R is the Rydberg constant.

(a) State a formula that relates the wavelength of light λ to h, c and E which is the energy of a photon. [1]

(b) Let p and q be the quantum numbers of the upper and lower energy levels of an electron transition in hydrogen. Find a formula that relates the wavelength of light emitted in such transitions to p and q. [2]

(c) For each of the three hydrogen emission line sets shown in the table below, identify the quantum number of its lower energy level q. Each set (column) of five emission lines has the same lower energy level. The first column shows the quantum number of the upper energy level q relative to the lower level. [4]

p	Set-A λ [nm]	Set-B λ [nm]	Set-C λ [nm]
$q+1$	121.57	4051	7460
$q+2$	102.57	2625	4654
$q+3$	97.254	2166	3741
$q+4$	94.974	1944	3297
$q+5$	93.780	1817	3039

You may assume that $p < 6$ and $R = 10973731.6\ \mathrm{m^{-1}}$

[PAT, 2019Q17][M7]

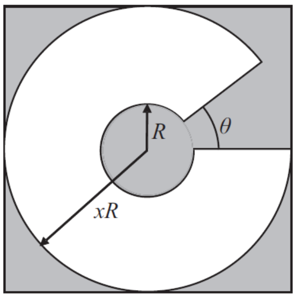
(a) Find an expression for the angle θ for which the grey area A_g is f times the area of the outer square A_S. Your expression for θ should take the form:

$$\theta = B - \frac{C(f)x^2}{x^2 - 1}$$

where B is a constant and $C(f)$ is a function of f. State B and $C(f)$ explicitly.

You may assume that $x > 1$, $f < 1$ and $f > 0$. The value of xR indicates the radius of the outer circle. [6]

(b) Find the numerical value for θ to five significant figures when $x = 3$ and $f = 1/2$. [1]



[PAT, 2019Q18][M3]

Solve the following equation for x:

$$\frac{e^x + 9}{e^{-x} + 5} = 2.$$

[3]

[PAT, 2019Q19][M8]

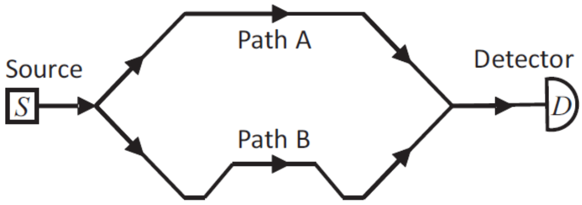
A firework rocket is launched vertically. At the moment of explosion it is moving with a vertical speed of $v_0 = 2\ \text{m s}^{-1}$ upwards. The explosion releases an energy of $E_{\text{exp}} = 1\ \text{J}$ and the rocket bursts into four pieces with masses of $m_1 = 1\ \text{g}$, $m_2 = 2\ \text{g}$, $m_3 = 3\ \text{g}$ and $m_4 = 4\ \text{g}$. The piece with mass m_4 moves vertically upwards with a speed of $v_4 = 1\ \text{m s}^{-1}$. The pieces of mass m_3 and m_2 move horizontally and the piece of mass m_1 moves vertically. All velocities and directions in this question are given relative to the ground and your answer should do the same.

(a) Obtain the speeds of all the pieces after the explosion. [5]

(b) Higher speed pieces can be obtained if the directions of movement of the pieces are different from those in part (a). Under which choice of directions would the maximum speed of one of the pieces be achieved? [3]

[PAT, 2019Q20][M5]

The diagram below shows an interferometer with two paths (Path A and Path B) which a wave can take from its source S to a detector D.



The lengths of the paths differ by an amount L which can change with time. The intensity at the detector I is measured and varies as a function of L as follows:

$$I = I_p + I_q \cos(kL)$$

In the above k is the wavenumber of the wave which relates to the wavelength λ via $k = \frac{2\pi}{\lambda}$. I_p and I_q are constants.

(a) Sketch the intensity as a function of L in the range from 0 to 2λ. Label both axes and identify I_p and I_q in the sketch. [3]

We wish to use the interferometer to measure how the path length difference L changes with time by measuring the intensity at the detector as a function of time. The change in path length difference is ΔL.

(b) Indicate on your sketch the biggest ΔL you can infer unambiguously from a measurement of intensity. [2]

[PAT, 2019Q21][M7]

You wish to build an adjustable delay line using electrical switches as shown in the diagram below.

Its purpose is to adjust the delay of an electrical signal through the delay line by switching different amounts of delay into the signal path.

The delay line should use the minimum amount of switches.

The delay line should have a minimal delay of $L_{\min}$ and a maximal delay of $L_{\max} \leq L_{\min} + \Delta L$. We refer to ΔL as the delay range.

The delay should be adjustable in increments of δL so that the line can achieve an evenly spaced set of delays between $L_{\min}$ and $L_{\max}$ with a resolution of δL.

For all segments in the line you have to determine a common, small delay l which is active when its switch is in the lower position.

You further have to determine a larger, individual delay of $L_i + l \gg l$ for each segment which is active when its switch is in the upper position.

For a line with n segments to satisfy the demands on the minimum number of switches, minimum delay $L_{\min}$, delay range ΔL and delay resolution δL:

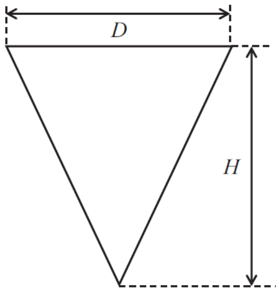
(a) Find a value for l in terms of $L_{\min}$ and n. [1]

(b) Find the delays of each segment L_i in terms of δL. [3]

(c) Find the minimum necessary n in terms of ΔL and δL. [3]

[PAT, 2019Q22][M4]

A conical cup has dimensions as shown in the diagram of its cross-section below. The cup can hold a maximum volume V when filled to its full depth H. Find an expression for the depth h to which you have to fill the cup so that it contains a volume of liquid equal to $V/2$. Your expression for h should only depend on the dimensions of the cup. [4]

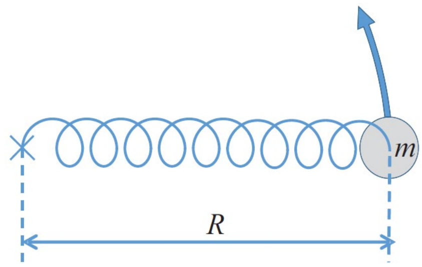


[PAT, 2019Q23][M5]

In an imaginary water filtration process a fraction of $1/n$ of an impurity is removed in the first pass of the water through the system. In each succeeding pass, the amount of impurity removed is $1/n$ of the amount removed in the preceding pass. Show that if $n = 2$ the water can be made arbitrarily pure but if $n = 3$, at least half of the impurity will remain. [5]

[PAT, 2019Q24][M10]

The sketch below shows a ball of mass m on a spring of unextended length R_0 and spring constant k. The spring is pivoted on the left on a central axis marked with a cross. The axis is perpendicular to the plane of the sketch. The ball and spring rotate around the central axis on a smooth horizontal table as indicated by the arrow in the sketch. The spring will break if it is stretched with a force larger than F_{max}.



(a) Find the equilibrium extension R of the system when it rotates with angular frequency ω. [3]

(b) Find the equilibrium angular frequency ω_c at which the spring will break. [1]

(c) Sketch ω_c against F_{max} in the range from zero to one Newton for the following parameters $m = 1$ kg, $R_0 = 1$ m, $k = 1$ N m^{-1}. Label your axes. [2]

(d) Under some conditions the system can only achieve a maximum angular frequency $\omega_i < \omega_c$. Find a relationship between k, m and ω_i and explain what happens to the system as the angular frequency increases to ω_i. [4]

PAT 2020

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Applicants

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[PAT, 2020Q1][M2]

The stable isotopes of carbon, nitrogen and oxygen are represented symbolically below:

$${}^{12}_{6}\text{C},\ {}^{13}_{6}\text{C},\ {}^{14}_{7}\text{N},\ {}^{15}_{7}\text{N},\ {}^{16}_{8}\text{O},\ {}^{17}_{8}\text{O},\ {}^{18}_{8}\text{O}$$

Which of the following statements are true?

1. ${}^{13}_{6}\text{C}$ has a larger number of protons than ${}^{12}_{6}\text{C}$.
2. ${}^{15}_{7}\text{N}$ has a larger mass than ${}^{14}_{7}\text{N}$.
3. ${}^{16}_{8}\text{O}$ has a larger nuclear charge than ${}^{15}_{7}\text{N}$.
4. ${}^{18}_{8}\text{O}$ has a larger mass per unit charge than ${}^{12}_{6}\text{C}$.
5. ${}^{14}_{7}\text{N}$ has a larger number of neutrons than ${}^{13}_{6}\text{C}$.

(A) 1, 3, 4

(B) 3, 4, 5

(C) 2, 3, 4

(D) 1, 2, 3

(E) 2, 3, 5

[PAT, 2020Q2][M2]

A triangle ABC has vertices at points in two-dimensional Cartesian co-ordinates $A:(0,1)$, $B:(1,2)$, and $C:(-1,2)$. It is reflected in the line $y=x$ and then rotated around the origin by 90 degrees in a clockwise direction. Which single transformation maps the initial triangle to the final state of the above transformations?

(A) reflection in $x=0$

(B) reflection in $y=0$

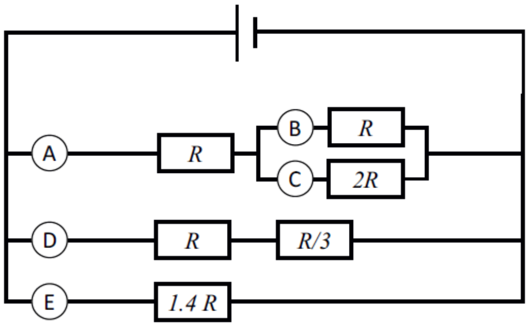
(C) rotation by 180° anti-clockwise around the origin

(D) rotation by 90° anti-clockwise around $(2,0)$

(E) scale factor of -1

[PAT, 2020Q3][M2]

Which ammeter A, B, C, D, E gives the highest reading?



(A) A

(B) B

(C) C

(D) D

(E) E

[PAT, 2020Q4][M2]

Solve $\log_2 x + \log_2(2x+3) = 1$ for x.

(A) $x = -2$

(B) $x = \frac{1}{2}$

(C) $x = 1$

(D) $x = -2$ and $\frac{1}{2}$

(E) $x = 0$

[PAT, 2020Q5][M2]

If the gravitational field strength at the Earth's surface is $g_E = 10$ N/kg, and at a distance $R > R_E$ from its centre the field strength is $g_R = 2$ N/kg, what is the radius of the Earth R_E in terms of R?

(A) $\frac{R}{25}$

(B) $\frac{R}{5}$

(C) $\frac{R}{\sqrt{10}}$

(D) $\frac{R}{\sqrt{5}}$

(E) $\frac{R}{\sqrt{2}}$

[PAT, 2020Q6][M2]

Consider the function $y(x) = \sin\left(\frac{100}{x}\right)$. The angle is in degrees, so that $\sin(180) = 0$. How many maxima of $y(x)$ occur for $x > 0.1$?

(A) 0

(B) 1

(C) 3

(D) 14

(E) ∞

[PAT, 2020Q7][M2]

What is the order, from shortest to longest, of the wavelengths of the peak electromagnetic emission from each of the following objects?

1. an electric torch
2. a microwave oven
3. a radioactive source
4. a hot cooking stove
5. a short-wave radio transmitter.

(A) 31425

(B) 52413

(C) 34152

(D) 31245

(E) 54213

[PAT, 2020Q8][M2]

A particle of type X decays with equal probability either to a pair of particles of type Y or a pair of particles of type Z. Both Y and Z particles are stable.

The decays of two X particles are observed. A pair of Y particles is found among the decay products. What is the probability that a pair of Z particles is among these decay products?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) 1

[PAT, 2020Q9][M2]

Ten students need to complete their compulsory practicals for their high school examinations as detailed in the table below:

No. of students	No. of different practicals to complete
2	1
4	2
4	3

The school only has one laboratory in which several different experiments can be set up simultaneously. A maximum of six students are allowed in the school's laboratory for a lesson. Each practical takes one lesson. What is the minimum number of lessons required to complete all the practicals?

(A) 3

(B) 4

(C) 5

(D) 6

(E) 10

[PAT, 2020Q10][M2]

What is the next number in the sequence? 37, 41, 43, 47, 53, 59

(A) 61

(B) 62

(C) 64

(D) 65

(E) 67

[PAT, 2020Q11][M2]

A stone of average diameter 10 cm is hit with a hammer and splits into pieces. Every time the stone or one of its pieces is hit, it splits into three further pieces of equal volume and similar shape. How many hits will it take before a piece reaches the size of a typical atom?

(A) 9

(B) 12

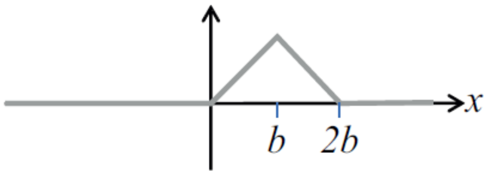
(C) 22

(D) 56

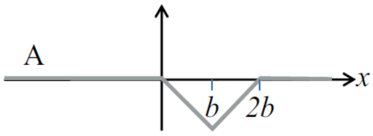
(E) 81

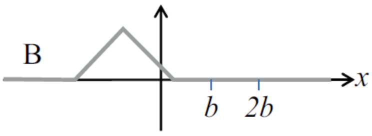
[PAT, 2020Q12][M2]

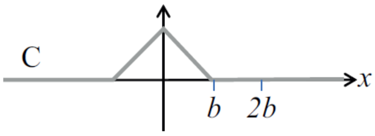
The graph below shows a function $f(x)$.

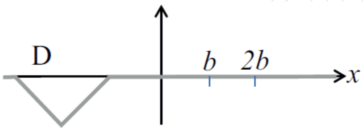


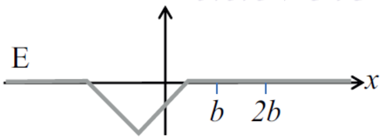
If a is a constant such that $0 < a < b$, identify the sketch of $g(x) = -f(a - x)$ from the sketches below.

(A) 

(B) 

(C) 

(D) 

(E) 

[PAT, 2020Q13][M4]

Consider a set of masses of three different values m_a, m_b and m_c. Each of the following three combinations have the same total mass.

1. two masses of m_a plus three masses of m_b
2. five masses of m_a plus one mass of m_c
3. two masses of m_a plus one mass of m_b plus one mass of m_c

Find m_b and m_c in terms of m_a. [4]

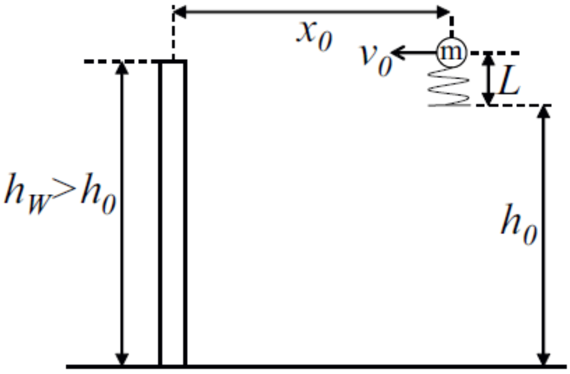
[PAT, 2020Q14][M3]

Find all solutions of the following equation in the range $0 \leq \theta \leq 360°$.

$$4\cos^2(\theta) + 2\left(\sqrt{3} - 1\right)\sin(\theta) = 4 - \sqrt{3}$$

[3]

[PAT, 2020Q15][M9]



The above figure depicts a ball of mass m which is equipped on its lower side with a massless spring of uncompressed length L_0 and spring constant k. The spring is initially compressed to a length $L < L_0$, as shown in the figure.

The ball-spring system is either dropped vertically or launched with a horizontal speed v_0 from a height of h_0 at a distance x_0 from a wall of height h_W.

The spring remains compressed until it touches the ground. When it touches the ground the spring is released (by a mechanism not shown on the figure) and expands very quickly back to its uncompressed length L_0 and is then held fixed at length L_0.

The bounce of the ball-spring system on the ground is assumed to be totally elastic.

(a) When the ball is dropped vertically, find the maximum height $h_{\max}$ the ball-spring system could reach after its bounce in terms of the spring's compression $\Delta L = L_0 - L$, and the parameters m, g, k and h_0. In analogy to h_0 in the figure, $h_{\max}$ should be the vertical distance between the floor and the lower end of the spring after the bounce. You may take $g = 10$ m s^{-1}. [3]

(b) Find the optimal value for the distance x_0 from which the ball would bounce over a wall with maximum value of h_W. [6]

[PAT, 2020Q16][M4]

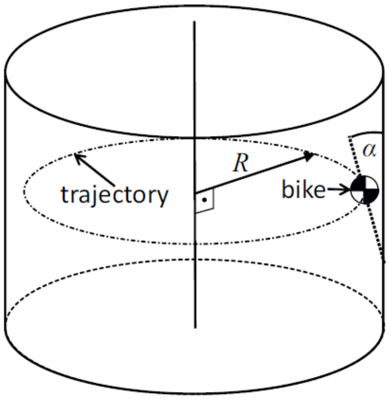
Given that

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = -2x - x^{\frac{1}{2}} + \frac{1}{3}$$
$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} = f(x),$$

determine $g(x)$ such that $f(1) = -1$ and $g(1) = 0$. [4]

[PAT, 2020Q17][M6]

As indicated in the figure, a motor bike of total mass m (m is the mass of bike plus rider) is ridden along a horizontal trajectory of radius R on the inside of a cylindrical cage.



The bike and the rider are inclined at an angle α to the wall. The angles α that the bike can make with respect to the walls of the cage are limited by its handle bars to a certain minimum $\alpha_{\text{min}} > 0$. The tyres of the bike have a very high coefficient of friction with the cage so that the tyres can only roll but not slip along the cage.

(a) Show in a diagram the forces acting on the bike if it is to maintain a horizontal trajectory as shown in the figure. [2]

(b) At what minimum speed v_{min} must the bike travel if it is not to fall down? [3]

(c) Given that $\alpha_{\text{min}} = 30°$, $R = 4$ m, $g = 10$ m s^{-2}, $m = 250$ kg find a numerical value for v_{min}. [1]

[PAT, 2020Q18][M6]

For which x is the following inequality satisfied?

$$\frac{2x^2 + 3x - 2}{2x^2 - 3x - 2} > 0$$

[6]

[PAT, 2020Q19][M4]

The Euler number E_u has no units. It is used in fluid flow calculations. It depends on the pressure P, density ρ, and fluid velocity v such that:

$$E_u = P^a \rho^b v^c$$

where a, b and c are constants.

Find the ratio of $a : b : c$ in its simplest form, where a, b and c are positive or negative integers.

[4]

[PAT, 2020Q20][M5]

Find the coordinates of the point(s) at which the line $y = m(3x - 2)$ is tangent to the curve $y = 9x^2 + 6x - 7$. In the above m is a real constant. [5]

[PAT, 2020Q21][M5]

Mars' moons Phobos and Deimos are in equatorial, near-circular orbits around Mars. They both orbit in the direction of the planet's rotation. Phobos has an orbital period of $\frac{1}{3}$ of a Martian day, and Deimos has an orbital period of $\frac{5}{4}$ Martian days. An astronomer on Mars, near the equator, observes both moons during the course of a Martian night.

(a) Would the two moons appear to move in the same direction in the sky? Explain your answer. [2]

(b) Describe qualitatively how the phases of the two moons might vary during the night. [2]

(c) Would it be possible for the astronomer to see Phobos both rise and set within a single night? [1]

[PAT, 2020Q22][M5]

A number can be represented using base N as follows:

$$(x \ldots cba)_N = a \cdot N^0 + b \cdot N^1 + c \cdot N^2 + \cdots + x \cdot N^w$$

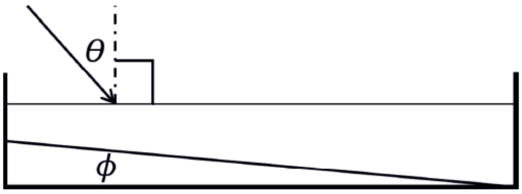
In which base less than 10 is the following equation true?

$$(1101)_N - (313)_N = (344)_N$$

[5]

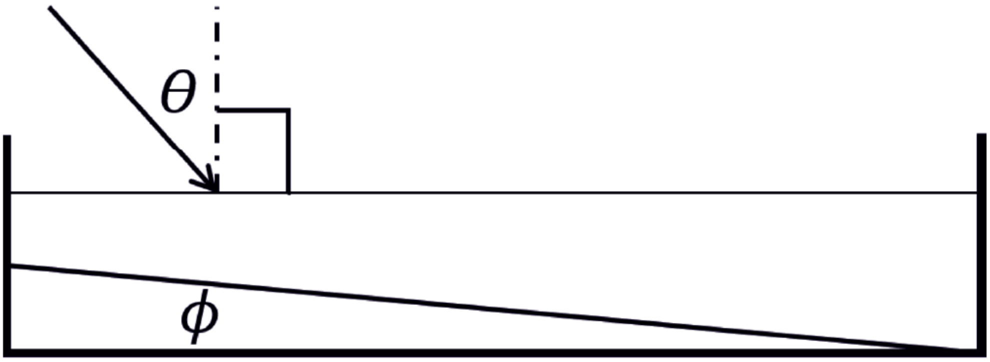
[PAT, 2020Q23][M5]

A monochromatic beam of light travels through air of refractive index n_a and strikes a liquid of refractive index n_l at an angle of incidence θ as shown in the figure below. At the bottom of the tank which contains the liquid is a plane mirror at angle Φ to the horizontal. The tank can be considered infinitely long.



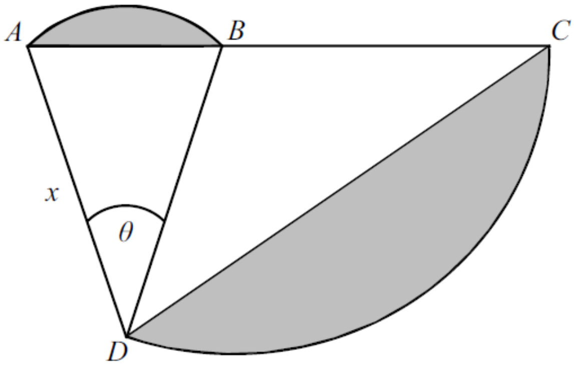
Beyond a certain value of θ the light no longer leaves the tank after the first reflection in the mirror. Find the value of θ for this case given $\Phi = 10°$, $n_a = 1$ and $n_l = \frac{4}{3}$. [5]

You may wish to use the larger diagram below to draw a ray diagram.



[PAT, 2020Q24][M5]

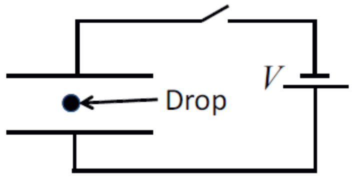
The points A, B and C lie on a straight line. The length of AD is x.



D is the centre of the arc AB, and B is the centre of the arc CD. Find the total shaded area in terms of x and θ. [5]

[PAT, 2020Q25][M5]

In Millikan's oil drop experiment, shown below, a spherical oil drop with charge $+Q$, radius r and density ρ_{oil} falls between two parallel conducting plates.



(a) Initially the switch is open and the drop is falling with terminal velocity v_t in air of density ρ_{air} and viscosity η. We now measure its terminal velocity.

Write down an equation that relates all the forces on the drop while it falls at terminal velocity. [1]

Hint: You may assume that the drag force f_D on the drop is given by $f_D = 6\pi\eta r v_t$ and that the drop also experiences a buoyant force or upthrust f_B equal to the weight of the air displaced by the drop.

(b) The switch is now closed. A uniform electric field E is applied to the drop and it becomes stationary. Write down a new equation relating the forces on the drop. [1]

(c) Derive an equation for Q which does not depend on the radius r. [3]

[PAT, 2020Q26][M10]

(a) On the same axes, sketch the functions $y = x^2 + 1$, $y = 2/x$, and $y = 3x + 1$. [3]

(b) Determine the exact x coordinates at which any of the graphs intersect and mark these on the x-axis. [3]

(c) Find the exact area enclosed between $y = 3x + 1$ and $y = x^2 + 1$ that is also below the curve $y = 2/x$. [4]

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[PAT, 2021Q1][M2]

What is the next number in the sequence? 1, 32, 243, 1024, 3125.

(A) 5040

(B) 6225

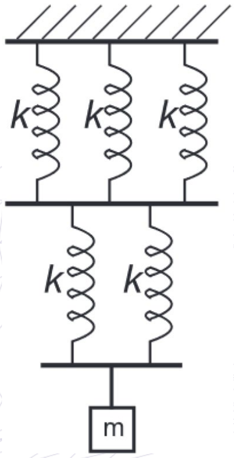
(C) 7164

(D) 7776

(E) 8192

[PAT, 2021Q2][M2]

What is the effective spring constant of the combination of springs shown in the diagram, if each spring has spring constant k?



(A) $\frac{5}{6}k$

(B) k

(C) $\frac{6}{5}k$

(D) $2k$

(E) $5k$

[PAT, 2021Q3][M2]

Evaluate $\sum_{n=1}^{10}\left(2-\frac{n}{2}+2^n\right)$.

(A) $2^{10}-\frac{11}{2}$

(B) $2^{12}-\frac{19}{2}$

(C) $2^{11}-\frac{19}{2}$

(D) $2^{10}-\frac{11}{4}$

(E) $2^{11}-\frac{11}{2}$

[PAT, 2021Q4][M2]

Five different ions are accelerated from rest by the same potential difference. Which will have the smallest final velocity?

(A) ${}^{6}_{3}\mathrm{Li}^{2+}$

(B) ${}^{7}_{3}\mathrm{Li}^{2+}$

(C) ${}^{7}_{3}\mathrm{Li}^{3+}$

(D) ${}^{9}_{4}\mathrm{Be}^{3+}$

(E) ${}^{9}_{4}\mathrm{Be}^{4+}$

[PAT, 2021Q5][M2]

Gravity on the Moon satisfies

$$g_{\text{Moon}} = \frac{1}{6} g_{\text{Earth}}.$$

A ball dropped on Earth from a height h takes a time t to reach the ground. From which height should it be dropped on the Moon so that it takes the same time t to reach the surface? You can neglect all effects of air resistance.

(A) $\frac{1}{36}h$

(B) $\frac{1}{6}h$

(C) $\frac{1}{\sqrt{6}}h$

(D) h

(E) $6h$

[PAT, 2021Q6][M2]

Two unbiased dice are rolled and the numbers obtained are added. If the probability of getting the sum S is $P(S)$, which of the following statements are true?

1. $P(10) + P(11) = P(6)$
2. $P(6) > P(8)$
3. $P(2) + P(3) + P(4) > P(7)$
4. $P(7) = \frac{3}{2} P(5)$
5. $P(11) = P(3)$

(A) 1, 2, 4

(B) 3, 4, 5

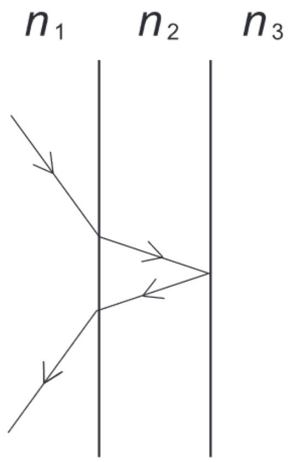
(C) 2, 3, 4

(D) 1, 3, 5

(E) 1, 4, 5

[PAT, 2021Q7][M2]

A light ray follows a path through three media separated by plane boundaries as shown in the diagram, with refractive indices n_1, n_2 and n_3.



Which of the following sequences puts the refractive indices in order of increasing value?

(A) n_1, n_2, n_3

(B) n_2, n_1, n_3

(C) n_1, n_3, n_2

(D) n_3, n_1, n_2

(E) n_3, n_2, n_1

[PAT, 2021Q8][M2]

If $f(x) = x^2$ and $g(x) = x + 3$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ where $y = f\big(g(x)\big) - g\big(f(x)\big)$.

(A) 6

(B) $2x + 5$

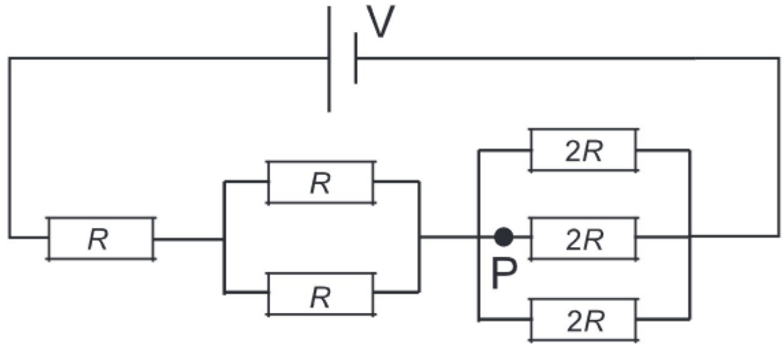
(C) $2x - 1$

(D) $6x + 6$

(E) 2

[PAT, 2021Q9][M2]

What is the current at the point P in the diagram?



(A) $\frac{2V}{13R}$

(B) $\frac{2V}{11R}$

(C) $\frac{V}{9R}$

(D) $\frac{6V}{13R}$

(E) $\frac{6V}{11R}$

[PAT, 2021Q10][M2]

Which of these represents a simpler form for $\cos(\sin^{-1}(x))$?

(A) $\sqrt{1-x^2}$

(B) $\sqrt{1+x^2}$

(C) $\sqrt{1-x}$

(D) $1-x^2$

(E) $\sqrt{x-1}$

[PAT, 2021Q11][M2]

Consider the following five graphs.

1. Force (y-axis) against distance (x-axis)
2. Force (y-axis) against time (x-axis)
3. Velocity (y-axis) against time (x-axis)
4. Mass (y-axis) against velocity squared (x-axis)
5. Voltage (y-axis) against charge (x-axis)

For which graph could the area under the graph potentially be a measurement of energy?

(A) 1, 4, 5

(B) 1, 5

(C) 1, 4

(D) 1, 3, 4

(E) All of them.

[PAT, 2021Q12][M2]

Which of the following integrals are equal to zero (you do **not** need to evaluate the integrals explicitly)?

1. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(3x)\ \mathrm{d}x$
2. $\int_{-\sqrt{7}}^{\sqrt{7}} \frac{1}{9}x^5 - \frac{1}{7}x^3 + \frac{1}{21}x\ \mathrm{d}x$
3. $\int_0^1 \cos x\ \mathrm{d}x$
4. $\int_{-2}^{2} \frac{1}{4}x^4 + \frac{1}{12}x^2\ \mathrm{d}x$
5. $\int_{-90}^{90} \sin(5x) - \frac{1}{2}\sin x\ \mathrm{d}x$

(A) 1, 2

(B) 3, 4

(C) 1

(D) 1, 2, 5

(E) 1, 5

[PAT, 2021Q13][M3]

A toilet roll is made up of an inner cardboard tube, diameter x cm, with toilet paper wrapped around it to give an overall diameter of y cm. The length of the cardboard tube is z cm. Suppose the diameter of the inner tube is reduced from x cm to $(x-1)$ cm, but the volume of toilet *paper* is kept the same. What is the difference in the *total* volume of the roll? [3]

[PAT, 2021Q14][M4]

A machine requires 10 kW of electrical power when operating at 100 V.

(i) If power is delivered through a cable with resistance 100 Ω, how much power is lost to the cable's resistance? [2]

(ii) In order to reduce power loss, a transformer is used between the cable and the machine. What ratio of turns is required on the transformer in order to reduce power loss on the cable by a factor of 10^4? [2]

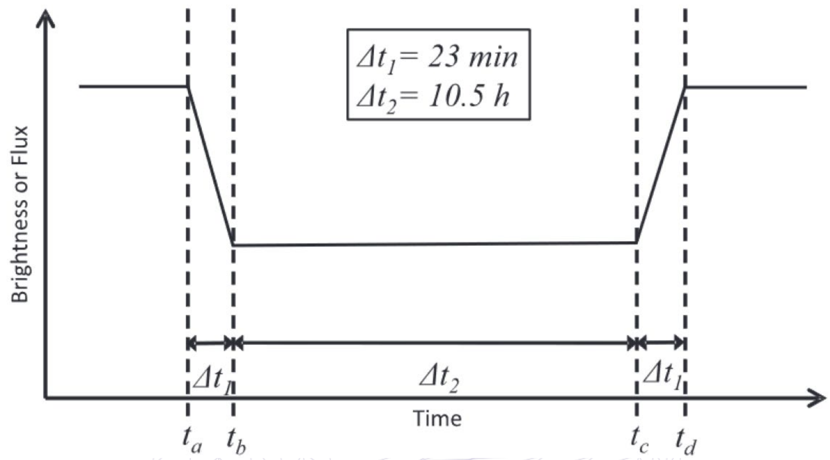
[PAT, 2021Q15][M10]

The observed brightness of a Sun-like star shows periodic dips. The time period between these dips $T = 225$ days.

(i) If these dips are interpreted as being due to the transits of a planet in a circular orbit around the star, estimate the radius R of that orbit.

You may assume that the star has the same mass as the Sun and that the planet's mass is much smaller that the mass of the star. You can take the mean radius of the Earth's orbit around the Sun to be approximately $R_E = 1.5 \times 10^{11}$ m. [5]

(ii) A model of a transit and its effect on the star's brightness is shown in the figure below. Assuming that we observe the system in the plane of the planet's orbit, draw the relative positions of the planet and the star at the times t_a to t_d and calculate the radii of the planet, R_P and the star, R_S. [5]



[PAT, 2021Q16][M10]

A vehicle travels in a fixed direction at a velocity $v(t)$ that varies with time as follows:

$$\begin{array}{rl} t < t_1 & v(t) = At^2 \\ t_1 < t < t_2 & v(t) = C - B(t - t_2)^2 \\ t > t_2 & v(t) = v_2 \end{array}$$

Here A, B, C and v_2 are constants.

(i) Find A, B and C such that both the velocity and acceleration are continuous at $t = t_1$ and $t = t_2$. [3]

(ii) Sketch the velocity and acceleration as a function of time. [3]

(iii) Find the total distance travelled between $t = 0$ and $t = t_3$, where $t_3 > t_2$. [4]

[PAT, 2021Q17][M7]

In Einstein's theory of gravity, light passing a star of mass M, at a distance R from the centre of the star, is bent by an angle θ (measured in radians, 2π radians $= 360°$) as shown in the diagram.

(i) Assume that θ depends on the gravitational constant G and also depends on the mass of the star M, the distance R and the speed of light $c = 3 \times 10^8$ m s^{-1}, as

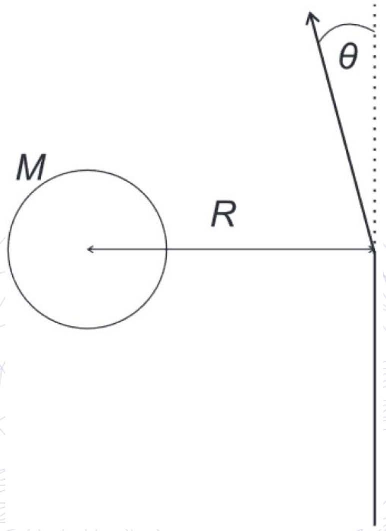
$$\theta = \lambda G M^{\alpha} R^{\beta} c^{\gamma},$$

Where λ is an undetermined dimensionless constant.

By considering the dimensions of these quantities determine α, β and γ. [4]

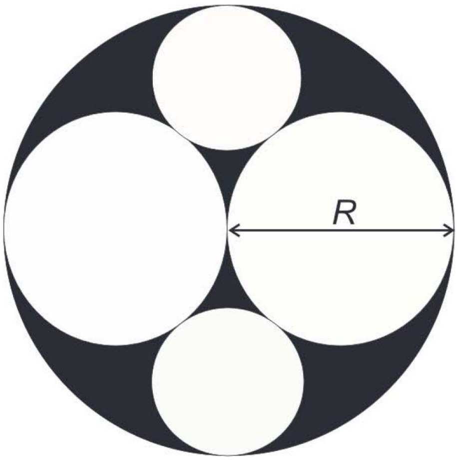
(ii) In SI units the numerical value of G is 6.67×10^{-11}.

Neglecting any spatial extent to the star, and taking $\lambda = 1$ and $M = 2 \times 10^{30}$ kg, determine the distance R such that light is bent so much that it falls directly into the star. [3]



[PAT, 2021Q18][M5]

Find the area of the dark regions of this figure in terms of the overall radius of the outer circle (denote R). [5]



[PAT, 2021Q19][M5]

Sand is poured at a constant rate onto a flat horizontal surface. It forms a pile in the shape of a cone with a constant slope.

(i) If r is the radius of the base of the cone, find how r varies with t, given that $r(0) = 0$. [3]

(ii) If it takes a time t_1 for r to reach r_1, how long does it take for the radius to reach $2r_1$ (in terms of t_1)? [2]

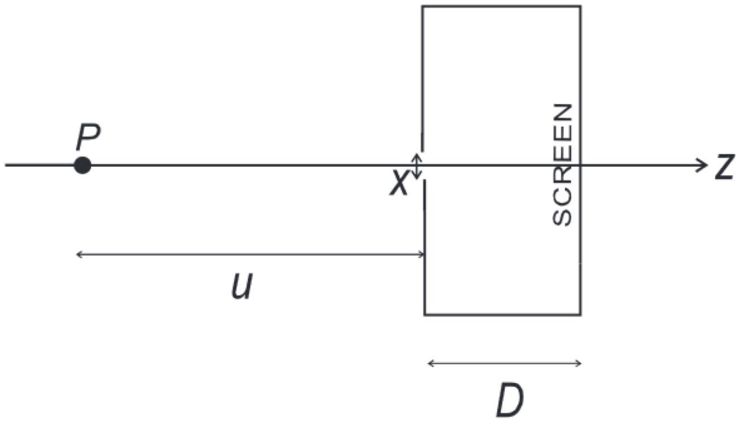
[PAT, 2021Q20][M4]

Expand $(a + bx)^c$, where c is a positive integer, as a polynomial in x up to and including terms involving x^2.

If the coefficient of the x^0 term is equal to $\frac{1}{16}$, $b = \frac{1}{4}$ and the coefficient of the x^2 term is $\frac{3}{32}$, find a and c. [4]

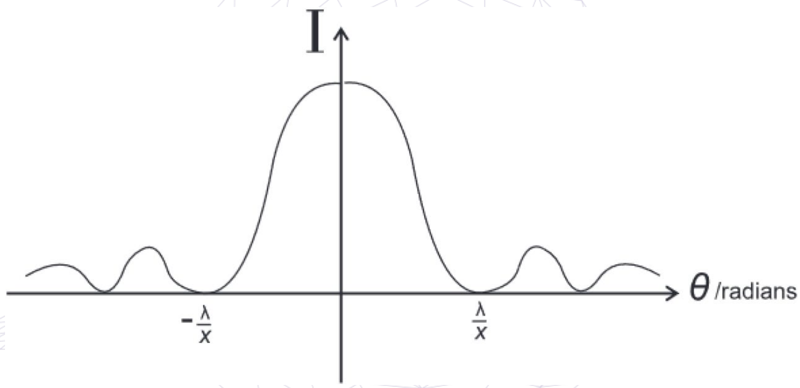
[PAT, 2021Q21][M8]

A pinhole camera consists of a box with a small aperture (of diameter x). A point light source P is at a distance u from the aperture outside the box. It projects an image on the screen, which is inside the box at a distance D from the aperture.



(i) If diffraction is ignored, what is the diameter s_1 of the spot projected on the screen? [2]

(ii) If $D \gg x$, diffraction of light of wavelength λ by the aperture produces a distribution of intensity that varies with angle θ (measure in radians from the z axis marked on the diagram; 2π radians = 360 degrees) approximately as shown below:



What is the approximate diameter, s_2, of the central peak of the diffraction pattern projected on the screen? [2]

(iii) Assume that the effects in part (i) and (ii) can be combined to give a total diameter s_3, where

$$s_3^2 = s_1^2 + s_2^2.$$

Find the pinhole size that produces the sharpest image of the light source on the screen. [4]

[PAT, 2021Q22][M10]

(i) Sketch the functions $f(x) = x^2 + 2x - 7$ and $g(x) = \frac{2}{x}$ on the same graph. [2]

(ii) Determine the coordinates of the points of intersection of the two curves and label their exact values. You are **not** required to determine coordinates of intersections with x or y axes. [4]

(iii) Evaluate the finite area enclosed between the two curves. [4]

You are expected to show clear working for all parts.

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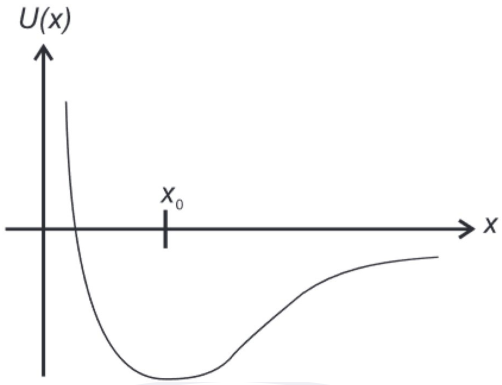
[PAT, 2021Q23][M4]

The potential energy for a system of two atoms separated by a distance x is $U(x)$ (shown below).

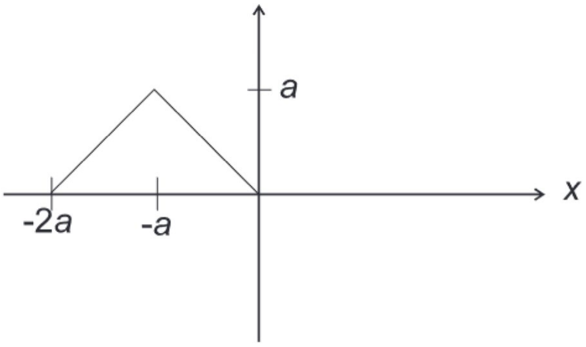
Sketch $-\frac{\mathrm{d}U}{\mathrm{d}x}$. [2]

What does this new curve represent physically and what does the point x_0 represent? [2]

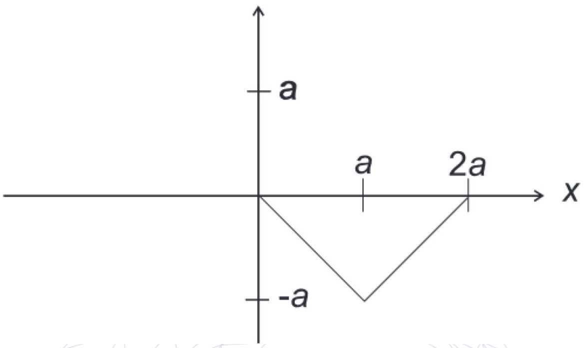


[PAT, 2021Q24][M6]

If $f(x)$ is



and $g(x)$ is



then sketch $f(x-ct)+g(x+ct)$ for $t=\frac{a}{2c},\frac{3a}{4c}$ and $\frac{a}{c}$. Each sketch should be done separately with its own set of axes. [6]

PAT 2022

PAT 2022
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 23 questions [100 marks].

Calculator

Only calculators meeting the specifications for PAT are allowed.

Formulas and constants

No tables or formula sheets may be used.

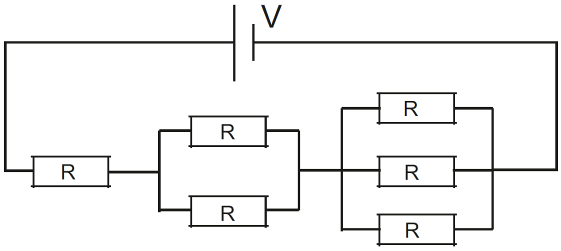
You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \text{m s}^{-2}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2022Q1][M2]

What is the total resistance of the circuit?



(A) $\frac{11R}{6}$

(B) $6R$

(C) $\frac{6R}{11}$

(D) $3R$

(E) $\frac{R}{3}$

[PAT, 2022Q2][M2]

For which values of x is $(24 - 14x - 3x^2)^{-1}$ positive?

(A) $x < -\frac{4}{3}$ and $x > 6$

(B) $x < -6$ and $x > \frac{4}{3}$

(C) $-\frac{4}{3} < x < 6$

(D) $-6 < x < \frac{4}{3}$

(E) $-\infty < x < \infty$

[PAT, 2022Q3][M2]

Molecules of oxygen in the atmosphere absorb solar radiation in bands centred at about 80 nm, 650 nm and 1000 nm. In which parts of the electromagnetic spectrum are these absorption bands?

(A) Visible, Infrared and Microwave

(B) Visible and Infrared

(C) Ultraviolet and Infrared

(D) Ultraviolet, Visible and Infrared

(E) X-ray, Ultraviolet and Visible

[PAT, 2022Q4][M2]

Which of these polynomial functions has the largest second derivative at $x = 0$?

(A) $5x^5 - x^3 + 4x$

(B) $3x^4 + x^2 + 16$

(C) $4x^6 + x^2 - 1$

(D) $x^3 + 2x^2 - 5x + 10$

(E) $10x^5 + 3x^3 - 7x + 2$

[PAT, 2022Q5][M2]

An asteroid of mass 10^3 kg is moving towards a space station at 1 ms^{-1}. It is proposed to stop it by firing a 1 MW laser at it. For how long must the laser be fired? You may assume that the surface of the asteroid is perfectly reflective, all photons are incident perpendicular to the surface of the asteroid, and a photon's momentum is related to its energy by $p = \frac{E}{c}$, where $c = 3 \times 10^8$ ms^{-1} is the speed of light.

(A) 3×10^{-3} s

(B) 7.5×10^4 s

(C) 1.5×10^5 s

(D) 3×10^5 s

(E) 3×10^{11} s

[PAT, 2022Q6][M2]

Which expression correctly represents the sum $\sum_{k=0}^{n} ar^{2k}$?

(A) $\frac{a}{1-r^2}k$

(B) $\frac{a(1-r^{2n})}{1-r}$

(C) $\frac{a(1-r^{2n})}{1-r^2}$

(D) $\frac{a(1-r^{2n+2})}{1-r}$

(E) $\frac{a(1-r^{2n+2})}{1-r^2}$

[PAT, 2022Q7][M2]

In a cathode ray tube, an electron (mass 9.1×10^{-31} kg, charge -1.6×10^{-19} C) is accelerated from rest by a uniform electric field of strength 20 kVm^{-1}. How much time does it take to travel 50 cm?

(A) 1.1×10^{-18} s

(B) 2.8×10^{-16} s

(C) 1.7×10^{-8} s

(D) 5.3×10^{-7} s

(E) 3.2×10^{-5} s

[PAT, 2022Q8][M2]

If a function $y = f(x)$ has a stationary point at (x_0, y_0), what are the co-ordinates of the corresponding stationary point of the function $y = af(bx + c)$?

(A) $\left(\frac{x_0}{b} - c, ay_0\right)$

(B) $(bx_0 + c, ay_0)$

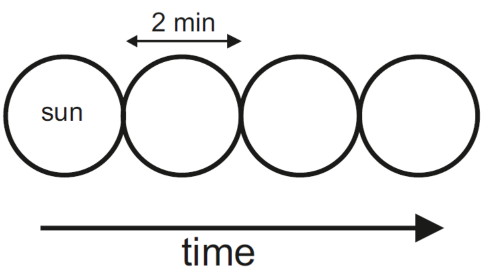
(C) $\left(\frac{x_0 - c}{b}, ay_0\right)$

(D) $\left(x_0 - \frac{c}{b}, ay_0\right)$

(E) $\left(\frac{x_0 + c}{b}, ay_0\right)$

[PAT, 2022Q9][M2]

As it appears to move across the sky, the Sun moves through an angle equal to that subtended by its diameter in about two minutes, as in the diagram. In a solar eclipse, the Moon covers the Sun almost exactly in the sky. Using this, what is the approximate ratio of the Moon's radius to its orbital distance from Earth?



(A) 0.0014

(B) 0.0022

(C) 0.0028

(D) 0.0044

(E) 0.0056

[PAT, 2022Q10][M2]

What is the next number in the sequence $0, \frac{3}{4}, \frac{3}{8}, \frac{9}{16}, \frac{15}{32}, \frac{33}{64}$?

(A) $\frac{51}{128}$

(B) $\frac{53}{128}$

(C) $\frac{63}{128}$

(D) $\frac{65}{128}$

(E) $\frac{71}{128}$

[PAT, 2022Q11][M2]

Two moons occupy circular orbits around a planet. The smaller moon has mass 1.5×10^{15} kg and orbital radius 2.3×10^4 km. The larger moon has mass 1.1×10^{16} kg and orbital radius 9.4×10^3 km. If the gravitational force exerted by the planet on the smaller moon is 10^{14} N, what force does the planet exert on the larger moon?

(A) 2.4×10^{14} N

(B) 6.0×10^{14} N

(C) 7.3×10^{14} N

(D) 1.8×10^{15} N

(E) 4.4×10^{15} N

[PAT, 2022Q12][M2]

What is the derivative of $y = x^6 + 6x^5 + 12x^4 + 8x^3$?

(A) $(3x + 3)(x^2 + 2x)^2$

(B) $(2x + 2)(x^2 + 2x)^2$

(C) $(6x + 6)(x^2 + 2x)$

(D) $(2x + 2)(x^2 + 2x)^3$

(E) $(6x + 6)(x^2 + 2x)^2$

[PAT, 2022Q13][M8]

(a) Draw the functions $y_1(x) = x^2 - 1$, $y_2(x) = 4x - 2$ and $y_3(x) = -\frac{x}{2} - 2$ on a common set of axes. Label where they cross the axes. [3]

(b) Work out the x-values of the intersection points of these three functions. [3]

(c) Write down a single integral which describes a finite area bounded by two of the three functions. You do *not* need to evaluate the integral. [2]

[PAT, 2022Q14][M6]

The Trojan asteroids share Jupiter's orbit around the Sun: approximately circular with a mean radius 5.2 AU (1 AU $= 1.5 \times 10^{11}$ m is the mean radius of the Earth's orbit around the Sun). The Trojans are clustered around two points labelled L4 and L5, where the L4 point is 60° ahead of Jupiter in its orbit and the L5 point is 60° behind Jupiter in its orbit.

(a) Determine the mean distance between the asteroids *588 Achilles* (at the L4 point) and *617 Patroclus* (at the L5 point). [2]

(b) A spacecraft travels in a straight line between the two asteroids, accelerating at 10 ms^{-2} until the half-way point between the asteroids, and decelerating at 10 ms^{-2} from there to the end-point. Assuming that the asteroids are approximately stationary on the timescale of the journey, and neglecting any gravitational effects of Jupiter or the Sun, find the total travel time. [3]

(c) Explain why the assumption that the asteroids are approximately stationary during the journey is well-justified. [1]

[PAT, 2022Q15][M10]

A projectile is launched at speed v and angle θ (as measured from the horizontal) outwards from the top of a high cliff.

(a) Sketch the trajectory of the projectile for launch angles $\theta = 5°, 45°$ and $85°$. Use $x(t)$ for the horizontal displacement from the launch point and $y(t)$ for the vertical displacement from the launch point. [2]

(b) Using separate axes, now sketch the absolute distance, $r(t) = \sqrt{x(t)^2 + y(t)^2}$, from its launch point as a function of time for all of the three launch angles above. [2]

(c) Obtain an expression for $r(t)$. For which angles does $r(t)$ have a stationary point? [5]

(d) For angles below these, what happens to $r(t)$ as time increases? [1]

[PAT, 2022Q16][M4]

Suppose $f(t) = 4t$ and $g(x) = \frac{3}{2}(3x - x^2)$. Consider the inequality

$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} > \int_{\frac{3}{2}}^{x} f(t)\,\mathrm{d}t.$$

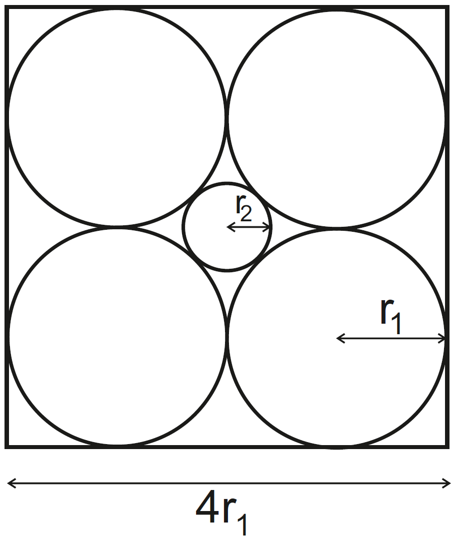
For which values of x is this inequality satisfied? [4]

[PAT, 2022Q17][M6]

Four circles of radius r_1 are inscribed inside a square of side $4r_1$ as shown in the diagram below.

(a) What is the radius r_2 of the largest circle that can fit in the space at the centre of the square, bounded by the outer circles? [3]

(b) If 8 spheres of radius r_1 are now similarly arranged inside a cube of edge length $4r_1$, what is the radius r_3 of the largest sphere that can fit in the space at the centre of the cube? [3]



[PAT, 2022Q18][M8]

Consider the function

$$f(x) = -\frac{P}{x^3} + \frac{Q}{x^2} - \frac{R}{x}$$

in the region $x > 0$, where P, Q and R are all positive constants.

(a) Find an inequality satisfied by P, Q and R in order for $f(x)$ to have at least one real root. [2]

(b) Find a relationship between P, Q and R in order for $f(x)$ to have exactly one stationary point. [3]

(c) If the relationship of the previous part holds, so that exactly one stationary point exists, what is the nature of that stationary point and at what value of x (expressed in terms of P, Q and R) is it? *It is not necessary to work out a second derivative to answer this.* [3]

[PAT, 2022Q19][M10]

Following Bohr, we assume that a hydrogen-like atom may be modelled as a single electron (mass m and charge $-e$) in a circular orbit around a much more massive nucleus (charge $+Ze$).

(a) By balancing forces, find the speed v of the electron in terms of its orbital radius. [3]

(b) Show that the total energy of the electron is equal to the negative of its kinetic energy. You may assume that its potential energy U is given by (where r is the radius of the orbit) [3]

$$U = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

(c) Assuming that for the electron the product $mvr = n\hbar$, where n is an integer and $\hbar$ (pronounced h-bar) is a constant, find an expression for the electron energy in terms of n (and which does not depend on either v or r). [2]

(d) If $E(n=1) = -13.6$ eV for hydrogen, what is $E(n=3)$ for once-ionised helium (He^+)? [2]

[PAT, 2022Q20][M7]

Two unbiased dice are rolled. The numbers obtained are multiplied.

(a) What is the probability that the product is even? [1]

(b) Which product has a probability of $\frac{1}{12}$ to occur? [1]

(c) What is the probability that the product is greater than 28? [1]

(d) Which product(s) has(ve) the highest probability to occur? [2]

(e) If the product is known to be even, what is the probability that it is also divisible by 4? [2]

[PAT, 2022Q21][M7]

A ball of mass $2m$ slides along a frictionless track with speed u. Starting from a long distance away, it collides elastically with a stationary ball of mass m.

(a) Calculate the final speeds of both balls (you may neglect any rotation of the balls). [5]

(b) If both balls were now positively electrically charged, describe qualitatively either how the results would change or why you would leave the results unaltered. [2]

[PAT, 2022Q22][M5]

Consider the following set of equations:

$$2x + y = z,$$
$$x^2 = y,$$
$$z + 2y = 2x^3.$$

Find the possible values of x which satisfy these equations. [5]

[PAT, 2022Q23][M5]

The number of atoms N_x in a sample of a radioactive substance x decays with time according to the equation,

$$N_x(t) = N_x(0)e^{-\lambda_x t},$$

where $N_x(0)$ is the number of atoms at time $t = 0$ and λ_x is a constant for substance x.

The half-life of a substance is defined as the time taken for N_x to reach half of its initial value. Substance a has a half-life of 1 hour. 36% of its decays emit an alpha particle and 64% of its decays emit a beta particle. Substance b has a half-life of 15 minutes. 56% of its decays emit an alpha particle and 44% of its decays emit a beta particle.

If the total particle emission rate of substance x (where $x = a, b$) is $\lambda_x N_x(t)$ and $N_a(0) = N_b(0)$, what time in minutes passes before the beta particle emission rates from the two samples are equal? [5]

PAT 2023

PAT 2023
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 26 questions [100 marks].

Calculator

Only calculators meeting the specifications for PAT are allowed.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

Applicants

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science.

[PAT, 2023Q1][M2]

What speed does a bull elephant (mass 4900 kg) have to move at to have the same kinetic energy as a cyclist (mass 100 kg) moving at 30 kmh^{-1}?

(A) 0.6 ms^{-1}

(B) 1.2 ms^{-1}

(C) 4.2 ms^{-1}

(D) 8.3 ms^{-1}

(E) 16.6 ms^{-1}

[PAT, 2023Q2][M2]

A seed packet contains 100 seeds. When planted, 75 will successfully become plants, but of these only a third will have flowers, and of these only one fifth will produce fruit. How many seeds produce fruiting plants?

(A) 5

(B) 10

(C) 15

(D) 20

(E) 25

[PAT, 2023Q3][M2]

Two black holes orbit each other and emit gravitational waves arising from the periodic nature of the orbit. The orbital separation is around10 km, the relative speeds of the black holes are close to the speed of light, and gravitational waves travel at the speed of light. Which of the following would best describe the frequency of the emitted radiation?

(A) 10^{-2} Hz

(B) 10 Hz

(C) 10^{4} Hz

(D) 10^{7} Hz

(E) 10^{10} Hz

[PAT, 2023Q4][M2]

What is the next number in the sequence $\frac{1}{5}, \frac{3}{25}, \frac{7}{125}, \frac{3}{125}, \frac{31}{3125}$?

(A) $\frac{7}{125}$

(B) $\frac{27}{3125}$

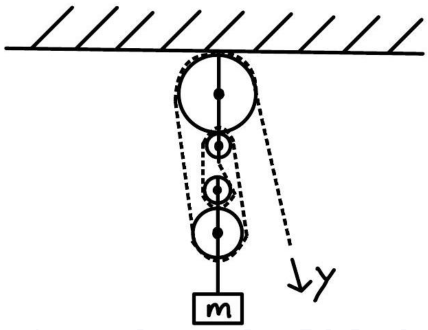
(C) $\frac{59}{3125}$

(D) $\frac{59}{15625}$

(E) $\frac{63}{15625}$

[PAT, 2023Q5][M2]

Consider the pulley system in the diagram, containing 4 wheels. If you pull the free end a distance y, how far will m rise by?



(A) $\frac{y}{16}$

(B) $\frac{y}{4}$

(C) $\frac{y}{2}$

(D) $2y$

(E) $4y$

[PAT, 2023Q6][M2]

Consider $f(x) = x^2$. You want to transform the function so you get a new function $g(x)$ stretched by a vertical scale factor of 2, with a line of symmetry about $x = 1$ and which is never positive. $g(x)$ would be equal to which of the following functions?

(A) $-2f(x-1)$

(B) $-f(x-1)$

(C) $-2f(x+1)$

(D) $-f(x+1)$

(E) $-f(2x-2)$

[PAT, 2023Q7][M2]

If $y = \left(2 + \frac{x}{2}\right)^4$, which of the following is $\frac{\mathrm{d}y}{\mathrm{d}x}$?

(A) $4 + 2x + \frac{3x^2}{4} + \frac{x^3}{4}$

(B) $8 + 6x + \frac{3x^2}{2} + \frac{x^3}{8}$

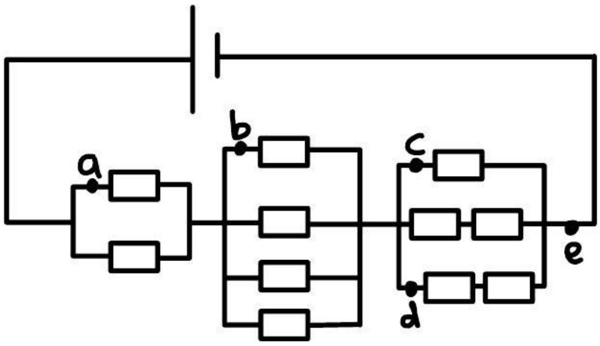
(C) $32 + 24x + 6x^2 + \frac{x^3}{2}$

(D) $16 + 12x + 3x^2 + \frac{x^3}{4}$

(E) $2 + x + \frac{3x^2}{8} + \frac{x^3}{8}$

[PAT, 2023Q8][M2]

All resistors in the circuit below have the same value. If an ammeter is placed in the circuit in turn at points (a) through to (e), which of the following sets of points will give the same reading?



(A) a, b

(B) a, c

(C) b, e

(D) c, d

(E) a, b, c

[PAT, 2023Q9][M2]

If $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + \frac{1}{x^3}$ and $y = 0$ when $x = 1$, what is $\int_1^3 y\, \mathrm{d}x$?

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) $\frac{20}{3}$

(D) 8

(E) $\frac{22}{3}$

[PAT, 2023Q10][M2]

A particle of mass m, travelling freely at an initial speed v, can be stopped in a distance d by a constant retarding force F. What magnitude of force (applied in a direction perpendicular to the motion) would be needed to change the trajectory of the same particle (at the same speed v) into a circular arc of radius d ?

(A) $\frac{F}{2}$

(B) $\frac{F}{\sqrt{2}}$

(C) F

(D) $\sqrt{2}F$

(E) $2F$

[PAT, 2023Q11][M2]

What is the (integer) m such that $\sum_{n=1}^{m}(3+2n)=140$?

(A) 6

(B) 8

(C) 10

(D) 12

(E) 14

[PAT, 2023Q12][M2]

A device uses 3 kW of power at a voltage of 60 V. It is connected to a power supply via an ideal transformer. The transformer has N turns on the winding connected to the device and $20N$ turns on the winding connected to the power supply. What current flows in the winding connected to the power supply?

(A) 1 mA

(B) 0.4 A

(C) 2.5 A

(D) 50 A

(E) 1 kA

[PAT, 2023Q13][M6]

You use some measuring scales to discover the following relationships between masses of apples (each of mass m_A), bananas (m_B) and carrots (m_C):

$$2m_A + 3m_B + 4m_C = 4m_A + 3m_B + 3m_C$$
$$m_A + 4m_B + m_C = 8m_B$$

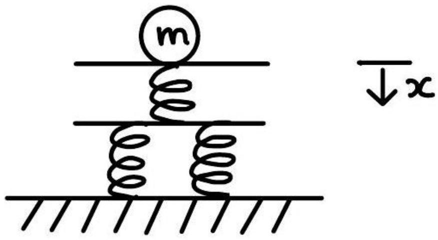
Find all combinations of apples and/or bananas that have the same mass as 5 carrots (note that only whole numbers of apples and bananas are allowed). [6]

[PAT, 2023Q14][M3]

If $2^{x+2y} = 16$ and $xy = 2$, find x and y. [3]

[PAT, 2023Q15][M6]

A ball of mass m sits in equilibrium on top of a set of three identical springs of spring constant k as in the diagram (you can assume that the springs are stiff and that the ball is light). The ball is pressed down by a distance x and then released. Assuming that 90% of the stored energy is transferred to the ball, how high will the ball go above its point of release (in terms of m, k, x and g, where g is the acceleration due to gravity)? [6]



[PAT, 2023Q16][M4]

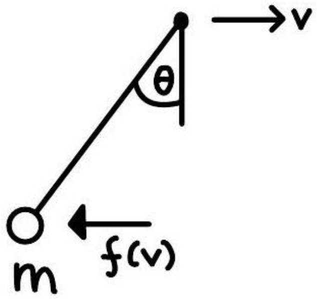
In astrophysics, the Jeans length λ_J is a measure of the size of a cloud of gas in which internal pressure just supports the cloud against collapse under gravity. It depends on the speed of sound in the gas, c_s, the gravitational constant $G = 6.67 \times 10^{-11}$ kg^{-1}m^{3}s^{-2} and the mass density of the cloud ρ. The dependences may be expressed in the form $\lambda_J = c_s^{\alpha} G^{\beta} \rho^{\gamma}$. What values of α, β and γ are required for λ_J to have the correct units (or dimensions) of length? [4]

[PAT, 2023Q17][M4]

A mass m on the end of a rigid rod of negligible mass hangs from a (pivot) point. The pivot point moves horizontally at speed v and the mass experiences a drag force $f(v)$ in the direction opposite to its velocity. At speed v, the rod makes a constant angle θ to the vertical. [4]

(a) Find an expression for $f(v)$ in terms of the angle θ that the pendulum makes to the vertical.

(b) Sketch θ as a function of v in the case that $f(v)$ is proportional to v.



[PAT, 2023Q18][M7]

A quartic polynomial function $f(x)$ has the following properties:

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 0 \qquad \text{at } x = 1 \text{ and } x = 3 \textbf{ only}$$
$$\frac{\mathrm{d}x}{\mathrm{d}x} = 0 \qquad \text{at } x = 2$$
$$f(0) = 0$$
$$f(1) = 3.$$

Find $f(x)$. [7]

[PAT, 2023Q19][M3]

Solve the following equation for real x, [3]

$$6e^{2x} + e^{x} = 15.$$

[PAT, 2023Q20][M4]

A diver 5 metres under the surface of the sea looks up. They see a circle of light directly above them, where they can see what is on the surface, but outside of this circle the diver only sees a reflection of what is under the water. Explain why there is such a circle and calculate its radius. You may assume $n_{\text{air}} = 1$ and $n_{\text{water}} = 1.33$. [4]

[PAT, 2023Q21][M5]

Find all values of x that satisfy the equation. [5]

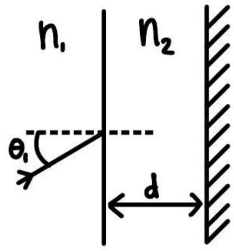
$$4\sin x\,(\sin x + \cos^2 x) = 3 + \sin x.$$

[PAT, 2023Q22][M6]

Two identical spacecraft of mass m are in stable circular orbits around the Earth - one at height R_E and the other at height $2R_\mathrm{E}$ above the surface of the Earth. What is the difference in the total energy between the two spacecraft? The radius of the Earth is R_E. [6]

[PAT, 2023Q23][M9]

A beam of light in a medium with refractive index n_1 is incident at an angle θ_1 on a slab of material of thickness d with refractive index $n_2 > n_1$ as shown in the figure. The rear surface of the slab is mirrored and perfectly reflective.



(a) What distance, l, does the beam transmitted into the slab travel before reemerging from it? Express your answer in terms of n_1, n_2, θ_1 and d. [3]

(b) What are the limiting values of l at large and small θ_1? [3]

(c) Now consider the case in which light of wavelength (in medium 1) λ_1 is incident normally. For what value(s) of d would light reflected from the two surfaces interfere constructively? Ignore any phase changes that might occur at reflections. [3]

[PAT, 2023Q24][M5]

A ship floating at anchor moves vertically only, as waves on the surface of the sea cause the surface height to vary with position x and time t as

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right),$$

where A, λ and v are positive constants.

(a) What is the period P of the ship's vertical oscillations? [1]

(b) What total vertical distance does the ship move through during a time interval equal to P? [1]

(c) Sketch curves for the ship's kinetic and potential energies as functions of time on the same plot, from $t = 0$ to $t = 2P$. [3]

[PAT, 2023Q25][M7]

Sketch $y = x^4 - 2x^3$ and $y = 2x - x^2$ on the same axes, showing clearly the natures of the stationary points and labelling their coordinates. Write down an integral expression for the finite area enclosed between the two curves (you do not need to evaluate the integral). [7]

[PAT, 2023Q26][M7]

Two separate pairs of unbiased dice are rolled. One pair consists of two eight-sided dice (with faces numbered 1-8). The other pair consists of one six-sided die (with faces numbered 1-6) and one ten-sided die (with faces numbered 1-10).

(a) Which pair is most likely to show a total of 16? [1]

(b) Are any totals equally likely to be rolled using the two pairs of dice? [1]

(c) What is the smallest total that is more probable when using the pair consisting of 8-sided dice? [1]

(d) Which pair is more likely to give a total that is divisible by 3? [2]

(e) Given that at least one of the eight-sided dice has landed 5, is a total of 11 or 10 more likely? Give a reason for your answer. [2]

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On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 24 questions [100 marks].

Calculator

Only calculators meeting the specifications for PAT are allowed.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.

UEIE PAT Mock 2024

PAT Mock 2024
On-line Exam

Scan the QR code or click on the link to take an on-line exam.
Full solutions can be accessed after submission.

TIME ALLOWED: 2 HOURS

Questions and Score

Total 25 questions [100 marks].

Calculator

Only calculators meeting the specifications for PAT are allowed.

Formulas and constants

No tables or formula sheets may be used.

You may take the gravitational field strength on the surface of earth to be $g \approx 10\ \mathrm{m\ s^{-2}}$.