# CAMBRIDGE ENGINEERING ADMISSIONS ASSESSMENT 

## ENGAA

# PAST PAPERS 2016-2023 

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On-line Complete Solutions

## UE INTERNATIONAL EDUCATION

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Answer Keys ENGAA 2020 S1

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UEIE ESAT Mock 2024

## Introduction

"ENGAA Past Papers" is presented by UE International Education (ueie.com), which is designed as a companion to the ESAT Standard Course and the ESAT Question Practice. It aims to help students to prepare the Engineering and Sciences Admissions Test. It is also a useful reference for teachers who are teaching ESAT.

All questions in this collection are reproduced from the official past papers released by the University of Cambridge, with a few typos from the source files corrected. The 2024 Edition collects a total of 488 ENGAA questions from 2016 to 2023.

In addition, subscribed users can access one more on-line ESAT mock papers, which are made up by our professional teachers based on the latest research on ENGAA/NSAA questions.

## How to Access Full Solutions

Although this document is free for everyone to use, the detailed solutions to all questions are only available for subscribed users who have purchased one of the following products of the UE Oxbridge-Prep series (click on the link to learn more):

## ESAT Standard Course

## ESAT Question Practice

At least one of the official solution, hand-written solution or video solution is provided for each question. Hand-written solutions are provided if official solutions are unavailable. There are video solutions for some questions.

All solutions can be accessed ON-LINE ONLY.

## ENGAA Score Conversions

You may look up ENGAA score conversions through the following page:

## Cambridge ENGAA Score Conversions

Statistics of Solutions

| Year | Number of <br> Questions | Official <br> Solutions | Handwritten <br> Solutions | Video <br> Solutions |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 0 1 6}$ | $\mathbf{5 8}$ | 0 | 58 | 9 |
| $\mathbf{2 0 1 7}$ | $\mathbf{5 8}$ | 0 | 58 | 9 |
| $\mathbf{2 0 1 8}$ | $\mathbf{7 2}$ | 0 | 72 | 14 |
| $\mathbf{2 0 1 9}$ | $\mathbf{6 0}$ | 0 | 60 | 8 |
| $\mathbf{2 0 2 0}$ | $\mathbf{6 0}$ | 0 | 60 | 10 |
| $\mathbf{2 0 2 1}$ | $\mathbf{6 0}$ | 0 | 60 | 1 |
| $\mathbf{2 0 2 2}$ | $\mathbf{6 0}$ | 0 | 60 | 0 |
| $\mathbf{2 0 2 3}$ | $\mathbf{6 0}$ | 0 | 60 | 0 |
| Total | $\mathbf{4 8 8}$ | $0(0 \%)$ | $488(100 \%)$ | $51(10.5 \%)$ |

简介
《ENGAA 历年真题集》由优易国际教育（ueie．com）出品，是 ESAT 标准课程和 ESAT刷题训练的配套资料之一。其主要用途是帮助学生提高备考 ESAT 考试的效率，以及为教授 ESAT 考试的同行老师提供参考。

真题集中的所有真题均由剑桥大学官方发布的真题重新排版制作而成，并修订了源文件中的若干印刷错误。2024版收录了 2016 年至 2023 年共 488 道 ENGAA 真题。

此外，我们还为付费订阅用户提供一套线上 ESAT 模考题。这些模考题是由我们的专业教师团队依据近几年 ENGAA／NSAA 考试命题趋势而命制的。

## 真题解析在哪里可以看到

所有用户均可免费使用真题集，但所有题目的解析仅向购买以下任意优易牛剑备考系列产品之一的付费用户开放：

## ESAT 标准课

## ESAT 刷题训练

所有真题都有详细解析，解析形式为官方解析，手写解析或视频讲解中的一种或多种。如果没有官方解析，则提供手写解析。部分题目提供视频讲解。

所有解析均只能在线查看。

## ENGAA 分数转换

你可以通过下方页面查询 ENGAA 分数转换关系：

剑桥 ENGAA 分数转换

解析数量统计

| 年份 | 真题数量 | 官方解析题量 | 手写解析题量 | 视频講解题量 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 0 1 6}$ | $\mathbf{5 8}$ | 0 | 58 | 9 |
| $\mathbf{2 0 1 7}$ | $\mathbf{5 8}$ | 0 | 58 | 9 |
| $\mathbf{2 0 1 8}$ | $\mathbf{7 2}$ | 0 | 72 | 14 |
| $\mathbf{2 0 1 9}$ | $\mathbf{6 0}$ | 0 | 60 | 8 |
| $\mathbf{2 0 2 0}$ | $\mathbf{6 0}$ | 0 | 60 | 10 |
| $\mathbf{2 0 2 1}$ | $\mathbf{6 0}$ | 0 | 60 | 1 |
| $\mathbf{2 0 2 2}$ | $\mathbf{6 0}$ | 0 | 60 | 0 |
| $\mathbf{2 0 2 3}$ | $\mathbf{6 0}$ | 0 | 60 | 0 |
| 总计 | $\mathbf{4 8 8}$ | $0(0 \%)$ | $488(100 \%)$ | $51(10.5 \%)$ |

## 簡介

《ENGAA 歷年真題集》由優易國際教育（ueie．com）出品，是 ESAT 標準課程和 ESAT刷題訓練的配套資料之一。其主要用途是幫助學生提高備考 ESAT 考試的效率，以及為教授 ESAT 考試的同儕老師提供參考。

真題集中的所有真題均由劍橋大學官方發布的真題重新排版製作而成，並修訂了源文檔中的若干印刷錯誤。2024 版收錄了 2016 年至 2023 年共 488 道 ENGAA 真題。

此外，我們還為付費訂閱用戶提供一套在線 ESAT 模擬題。這些模擬題系我們的專業教師團隊依據近幾年 ENGAA／NSAA 考試命題芻勢而命製的。

## 真題解析在哪裡可以看到

所有用戶均可免費使用真題集，但所有題目的解析僅向購買以下任意優易牛劍備考系列產品之一的付費用戶開放：

## ESAT 標準課

## ESAT 刷題訓練

所有真題都有詳細解析，解析形式為官方解析，手寫解析或視訊講解中的一種或多種。如果沒有官方解析，則提供手寫解析。部分題目提供影片講解。

所有解析均只能線上查看。

## ENGAA 分數換算

你可以透過下方頁面查詢 ENGAA 分數換算關係：

劍橋 ENGAA 分數換算

解析數量統計

| 年份 | 真題數量 | 官方解析題量 | 手寫解析題量 | 影片講解題量 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 0 1 6}$ | $\mathbf{5 8}$ | 0 | 58 | 9 |
| $\mathbf{2 0 1 7}$ | $\mathbf{5 8}$ | 0 | 58 | 9 |
| $\mathbf{2 0 1 8}$ | $\mathbf{7 2}$ | 0 | 72 | 14 |
| $\mathbf{2 0 1 9}$ | $\mathbf{6 0}$ | 0 | 60 | 8 |
| $\mathbf{2 0 2 0}$ | $\mathbf{6 0}$ | 0 | 60 | 10 |
| $\mathbf{2 0 2 1}$ | $\mathbf{6 0}$ | 0 | 60 | 1 |
| $\mathbf{2 0 2 2}$ | $\mathbf{6 0}$ | 0 | 60 | 0 |
| $\mathbf{2 0 2 3}$ | $\mathbf{6 0}$ | 0 | 60 | 0 |
| 總計 | $\mathbf{4 8 8}$ | $0(0 \%)$ | $488(100 \%)$ | $51(10.5 \%)$ |

## Answer Keys

## ENGAA 2016-2023

Only keys to multiple-choice questions are provided.
Full solutions can be accessed on-line by the links or scanning the QR codes provided.

| $2016 \text { S1 }$ <br> Answer Keys |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q1 | G | Q21 | A | Q41 | D |
| Q2 | D | Q22 | A | Q42 | A |
| Q3 | B | Q23 | D | Q43 | G |
| Q4 | F | Q24 | G | Q44 | G |
| Q5 | C | Q25 | D | Q45 | E |
| Q6 | C | Q26 | D | Q46 | E |
| Q7 | E | Q27 | E | Q47 | C |
| Q8 | D | Q28 | G | Q48 | A |
| Q9 | C | Q29 | C | Q49 | B |
| Q10 | D | Q30 | C | Q50 | C |
| Q11 | B | Q31 | C | Q51 | A |
| Q12 | E | Q32 | C | Q52 | E |
| Q13 | F | Q33 | C | Q53 | C |
| Q14 | H | Q34 | A | Q54 | B |
| Q15 | E | Q35 | A |  |  |
| Q16 | F | Q36 | H |  |  |
| Q17 | B | Q37 | D |  |  |
| Q18 | F | Q38 | B |  |  |
| Q19 | C | Q39 | G |  |  |
| Q20 | D | Q40 | B |  |  |


| 2016 S2 |  |
| :---: | :---: |
| Answer Keys |  |
| Q1(a) | A |
| Q1(b) | B |
| Q1(c) | A |
| Q1(d) | A |
| Q2(a) | F |
| Q2(b) | A |
| Q2(c) | D |
| Q2(d) | B |
| Q3(a) | C |
| Q3(b) | F |
| Q3(c) | A |
| Q4(a) | E |
| Q4(b) | B |
| Q4(c) | C |
| Q4(d) | D |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |


| Answer Keys |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q1 | F | Q21 | B | Q41 | D |
| Q2 | B | Q22 | C | Q42 | A |
| Q3 | E | Q23 | C | Q43 | E |
| Q4 | G | Q24 | C | Q44 | A |
| Q5 | B | Q25 | F | Q45 | A |
| Q6 | D | Q26 | D | Q46 | B |
| Q7 | G | Q27 | G | Q47 | C |
| Q8 | D | Q28 | E | Q48 | H |
| Q9 | E | Q29 | E | Q49 | A |
| Q10 | B | Q30 | B | Q50 | B |
| Q11 | D | Q31 | C | Q51 | H |
| Q12 | F | Q32 | A | Q52 | B |
| Q13 | D | Q33 | B | Q53 | B |
| Q14 | A | Q34 | C | Q54 | A |
| Q15 | F | Q35 | C |  |  |
| Q16 | A | Q36 | E |  |  |
| Q17 | C | Q37 | F |  |  |
| Q18 | E | Q38 | D |  |  |
| Q19 | B | Q39 | D |  |  |
| Q20 | C | Q40 | F |  |  |
|  |  |  |  |  |  |


| 2017 S2 |  |
| :--- | :---: |
| Answer Keys |  |
| Q1(a) | B |
| Q1(b) | A |
| Q1(c) | A |
| Q1(d) | C |
| Q1(e) | E |
| Q2(a) | C |
| Q2(b) | B |
| Q2(c) | D |
| Q2(d) | C |
| Q2(e) | B |
| Q3(a) | $D$ |
| Q3(b) | C |
| Q3(c) | B |
| Q3(d) | D |
| Q4(a) | B |
| Q4(b) | D |
| Q4(c) | E |
|  |  |
|  |  |
|  |  |
|  |  |


| $\begin{gathered} 2018 \text { S1 } \\ \text { Answer Keys } \end{gathered}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q1 | E | Q21 | C | Q41 | C |
| Q2 | B | Q22 | B | Q42 | G |
| Q3 | B | Q23 | C | Q43 | C |
| Q4 | B | Q24 | H | Q44 | F |
| Q5 | H | Q25 | G | Q45 | D |
| Q6 | G | Q26 | C | Q46 | A |
| Q7 | D | Q27 | B | Q47 | D |
| Q8 | E | Q28 | A | Q48 | A |
| Q9 | E | Q29 | A | Q49 | E |
| Q10 | C | Q30 | E | Q50 | E |
| Q11 | C | Q31 | D | Q51 | A |
| Q12 | A | Q32 | F | Q52 | F |
| Q13 | E | Q33 | C | Q53 | C |
| Q14 | A | Q34 | D | Q54 | B |
| Q15 | D | Q35 | D |  |  |
| Q16 | C | Q36 | F |  |  |
| Q17 | C | Q37 | E |  |  |
| Q18 | E | Q38 | B |  |  |
| Q19 | E | Q39 | B |  |  |
| Q20 | C | Q40 | D |  |  |


| 2018 S2 <br> Answer Keys |  |
| :---: | :---: |
| Q1 | B |
| Q2 | B |
| Q3 | C |
| Q4 | A |
| Q5 | F |
| Q6 | F |
| Q7 | A |
| Q8 | G |
| Q9 | E |
| Q10 | D |
| Q11 | B |
| Q12 | D |
| Q13 | F |
| Q14 | B |
| Q15 | G |
| Q16 | A |
| Q17 | D |
| Q18 | B |
|  |  |
|  |  |


| $\begin{gathered} 2019 \text { S1 } \\ \text { Answer Keys } \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Q1 | F | Q21 | C |
| Q2 | F | Q22 | C |
| Q3 | E | Q23 | B |
| Q4 | A | Q24 | E |
| Q5 | C | Q25 | B |
| Q6 | A | Q26 | C |
| Q7 | E | Q27 | D |
| Q8 | A | Q28 | C |
| Q9 | D | Q29 | E |
| Q10 | F | Q30 | C |
| Q11 | B | Q31 | B |
| Q12 | C | Q32 | D |
| Q13 | F | Q33 | A |
| Q14 | C | Q34 | F |
| Q15 | D | Q35 | E |
| Q16 | E | Q36 | D |
| Q17 | E | Q37 | D |
| Q18 | B | Q38 | A |
| Q19 | E | Q39 | F |
| Q20 | B | Q40 | B |


| $\begin{gathered} 2019 \\ \mathrm{~S} 2 \\ \hline \end{gathered}$ | Answer Keys |
| :---: | :---: |
| Q1 | A |
| Q2 | A |
| Q3 | C |
| Q4 | B |
| Q5 | E |
| Q6 | A |
| Q7 | E |
| Q8 | C |
| Q9 | G |
| Q10 | E |
| Q11 | C |
| Q12 | A |
| Q13 | B |
| Q14 | C |
| Q15 | E |
| Q16 | F |
| Q17 | A |
| Q18 | B |
| Q19 | B |
| Q20 | C |


| 2020 S1 |  |  |  |
| :---: | :---: | :---: | :---: |
| Q1 | C | Q21 | B |
| Q2 | D | Q22 | B |
| Q3 | A | Q23 | G |
| Q4 | A | Q24 | E |
| Q5 | D | Q25 | H |
| Q6 | B | Q26 | C |
| Q7 | F | Q27 | D |
| Q8 | F | Q28 | A |
| Q9 | C | Q29 | B |
| Q10 | A | Q30 | D |
| Q11 | D | Q31 | F |
| Q12 | B | Q32 | E |
| Q13 | C | Q33 | C |
| Q14 | E | Q34 | A |
| Q15 | D | Q35 | C |
| Q16 | F | Q36 | E |
| Q17 | G | Q37 | B |
| Q18 | D | Q38 | D |
| Q19 | A | Q39 | D |
| Q20 | B | Q40 | C |
|  |  |  |  |


| $2020 \text { S2 }$ <br> Answer Keys |  |
| :---: | :---: |
| Q1 | E |
| Q2 | D |
| Q3 | C |
| Q4 | E |
| Q5 | G |
| Q6 | D |
| Q7 | B |
| Q8 | F |
| Q9 | F |
| Q10 | C |
| Q11 | D |
| Q12 | D |
| Q13 | F |
| Q14 | A |
| Q15 | E |
| Q16 | E |
| Q17 | D |
| Q18 | B |
| Q19 | C |
| Q20 | D |


| 2021 S1 |  |  |  |
| :---: | :---: | :---: | :---: |
| Q1 | E | Q21 | C |
| Q2 | E | Q22 | G |
| Q3 | D | Q23 | G |
| Q4 | F | Q24 | B |
| Q5 | C | Q25 | D |
| Q6 | C | Q26 | D |
| Q7 | A | Q27 | H |
| Q8 | D | Q28 | D |
| Q9 | D | Q29 | E |
| Q10 | E | Q30 | B |
| Q11 | F | Q31 | A |
| Q12 | H | Q32 | C |
| Q13 | F | Q33 | B |
| Q14 | B | Q34 | D |
| Q15 | B | Q35 | C |
| Q16 | C | Q36 | G |
| Q17 | A | Q37 | H |
| Q18 | D | Q38 | D |
| Q19 | H | Q39 | B |
| Q20 | E | Q40 | E |
|  |  |  |  |


| $2021 \text { S2 }$ <br> Answer Keys |  |
| :---: | :---: |
| Q1 | D |
| Q2 | C |
| Q3 | G |
| Q4 | D |
| Q5 | F |
| Q6 | A |
| Q7 | D |
| Q8 | B |
| Q9 | D |
| Q10 | C |
| Q11 | E |
| Q12 | F |
| Q13 | C |
| Q14 | E |
| Q15 | H |
| Q16 | E |
| Q17 | G |
| Q18 | C |
| Q19 | A |
| Q20 | C |


| $\begin{gathered} 2022 \text { S1 } \\ \text { Answer Keys } \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Q1 | E | Q21 | A |
| Q2 | A | Q22 | E |
| Q3 | D | Q23 | A |
| Q4 | G | Q24 | A |
| Q5 | D | Q25 | B |
| Q6 | A | Q26 | E |
| Q7 | C | Q27 | A |
| Q8 | G | Q28 | F |
| Q9 | B | Q29 | B |
| Q10 | D | Q30 | B |
| Q11 | G | Q31 | A |
| Q12 | C | Q32 | C |
| Q13 | G | Q33 | D |
| Q14 | D | Q34 | E |
| Q15 | E | Q35 | D |
| Q16 | D | Q36 | F |
| Q17 | E | Q37 | C |
| Q18 | F | Q38 | C |
| Q19 | D | Q39 | F |
| Q20 | E | Q40 | G |


| 2022 S2 <br> Answer Keys |  |
| :---: | :---: |
| Q1 | E |
| Q2 | C |
| Q3 | C |
| Q4 | E |
| Q5 | B |
| Q6 | F |
| Q7 | D |
| Q8 | D |
| Q9 | B |
| Q10 | F |
| Q11 | C |
| Q12 | G |
| Q13 | F |
| Q14 | C |
| Q15 | E |
| Q16 | E |
| Q17 | A |
| Q18 | A |
| Q19 | D |
| Q20 | B |


| $\begin{gathered} 2023 \text { S1 } \\ \text { Answer Keys } \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Q1 | A | Q21 | H |
| Q2 | F | Q22 | D |
| Q3 | C | Q23 | A |
| Q4 | F | Q24 | D |
| Q5 | F | Q25 | G |
| Q6 | B | Q26 | C |
| Q7 | D | Q27 | E |
| Q8 | B | Q28 | E |
| Q9 | D | Q29 | F |
| Q10 | E | Q30 | B |
| Q11 | B | Q31 | E |
| Q12 | A | Q32 | B |
| Q13 | H | Q33 | C |
| Q14 | C | Q34 | G |
| Q15 | B | Q35 | A |
| Q16 | D | Q36 | E |
| Q17 | E | Q37 | B |
| Q18 | B | Q38 | A |
| Q19 | C | Q39 | G |
| Q20 | A | Q40 | A |


| $\begin{gathered} 2023 \text { S2 } \\ \text { Answer Keys } \end{gathered}$ |  |
| :---: | :---: |
| Q1 | A |
| Q2 | D |
| Q3 | F |
| Q4 | E |
| Q5 | C |
| Q6 | D |
| Q7 | D |
| Q8 | E |
| Q9 | F |
| Q10 | B |
| Q11 | C |
| Q12 | D |
| Q13 | E |
| Q14 | G |
| Q15 | A |
| Q16 | E |
| Q17 | C |
| Q18 | E |
| Q19 | D |
| Q20 | E |

## ENGAA 2016 S1



## TIME ALLOWED: 80 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 80 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.
Part A Mathematics and Physics (28 questions)
Part B Advanced Mathematics and Advanced Physics (26 questions)
This paper contains 54 multiple choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 54 questions. Each question is worth one mark.

Questions ask you to show your choice between options. Choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators may NOT be used.

## Part A

[ENGAA, 2016S1Q1]
Find the complete set of solutions to $-8<6-\frac{x}{2}$.
(A) $x<4$
(B) $x>4$
(C) $x<20$
(D) $x>20$
(E) $x<22$
(F) $x>22$
(G) $x<28$
(H) $x>28$
[ENGAA, 2016S1Q2]
A nuclide ${ }_{82}^{214} \mathrm{~Pb}$ changes by radioactive decay into the nuclide ${ }_{82}^{210} \mathrm{~Pb}$.
Which combination of emissions produces this change?
(A) 3 alpha
(B) 2 alpha and 1 beta
(C) 2 alpha and 2 beta
(D) 1 alpha and 2 beta
(E) 3 beta
[ENGAA, 2016S1Q3]
Which one of the following is a simplification of $(\sqrt{3}-\sqrt{2})^{2}$ ?
(A) $1-2 \sqrt{3} \sqrt{2}$
(B) $5-2 \sqrt{2} \sqrt{3}$
(C) $2 \sqrt{3}-2 \sqrt{2}$
(D) 1
(E) $5-\sqrt{2} \sqrt{3}$
(F) $13-2 \sqrt{2} \sqrt{3}$
(G) $5+2 \sqrt{2} \sqrt{3}$
(H) 5
[ENGAA, 2016S1Q4]
The graph shown of quantity $y$ against quantity $x$ represents the motion of a body.

(The scales on both axes are in the appropriate S.I. units, and the gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$.)
Which two of the following could the graph represent?
1 kinetic energy against velocity for an object of mass 10 kg undergoing free-fall
2 potential energy against height for an object of mass 20 kg being lifted by a constant external force

3 velocity against time for an object of mass 20 kg being accelerated by a resultant force of 100 N

4 work done by an external force of 5 N against distance moved for an object of mass 12 kg being moved at constant speed by (and in the direction of) the external force
(A) 1 and 2
(B) 1 and 3
(C) 1 and 4
(D) 2 and 3
(E) 2 and 4
(F) 3 and 4
[ENGAA, 2016S1Q5]
The ratio of $Q: R$ is $5: 2$ and the ratio of $R: S$ is $3: 10$
Which one of the following gives the ratio $Q: S$ in its simplest form?
(A) $1: 2$
(B) $2: 1$
(C) $3: 4$
(D) $3: 25$
(E) $4: 3$
(F) $25: 3$
[ENGAA, 2016S1Q6]
A uranium- 235 nucleus can undergo fission to produce two smaller nuclei.
Which of the diagrams, if any, could represent this process?

(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2016S1Q7]
The mean age of the twenty members of a running club is exactly 28.
The mean age increases by exactly 2 years when two new members join.
What is the mean age of the two new members?
(A) 20 years
(B) 22 years
(C) 30 years
(D) 40 years
(E) 50 years
(F) 52 years
[ENGAA, 2016S1Q8]
A circuit consists of a $5.0 \Omega$ resistor and a variable resistor connected in series with a 24 V battery. The variable resistor has a minimum resistance of $3.0 \Omega$ and a maximum resistance of $15 \Omega$. The battery and the connecting wires have negligible resistance.
What is the maximum power dissipated in the $5.0 \Omega$ resistor?
(A) 7.2 W
(B) 18 W
(C) 27 W
(D) 45 W
(E) 72 W
(F) 75 W

## [ENGAA, 2016S1Q9]

A medical scanner is bought for $£ 15000$.
The value of the scanner depreciates by $20 \%$ every year.
By how much has the scanner reduced in value after 2 years?
(A) $£ 600$
(B) $£ 3000$
(C) $£ 5400$
(D) $£ 6000$
(E) $£ 9000$
(F) $£ 9600$
(G) $£ 12000$
[ENGAA, 2016S1Q10]
The total power $P$ radiated by a star is given by:

$$
P=k R^{2} T^{4}
$$

where $R$ is the radius of the star, $T$ is its surface temperature and $k$ is a constant.
The power currently radiated by the Sun is $4.0 \times 10^{26} \mathrm{~W}$. Towards the end of the Sun's life its radius will increase by a factor of a hundred and its surface temperature will decrease by a factor of two.
What will be the power radiated by the Sun when these changes have occurred?
(A) $2.5 \times 10^{27} \mathrm{~W}$
(B) $1.0 \times 10^{28} \mathrm{~W}$
(C) $2.0 \times 10^{28} \mathrm{~W}$
(D) $2.5 \times 10^{29} \mathrm{~W}$
(E) $1.0 \times 10^{30} \mathrm{~W}$
(F) $2.0 \times 10^{30} \mathrm{~W}$
(G) $2.5 \times 10^{33} \mathrm{~W}$
(H) $1.0 \times 10^{34} \mathrm{~W}$
[ENGAA, 2016S1Q11]
The point $A$ is 4 km due East of the point $B$.
The bearing of the point $C$ from $A$ is $330^{\circ}$ and the bearing of $C$ from $B$ is $060^{\circ}$.
Find the distance $B C$.
(A) 2 km
(B) $2 \sqrt{3} \mathrm{~km}$
(C) 4 km
(D) $2 \sqrt{5} \mathrm{~km}$
(E) $4 \sqrt{2} \mathrm{~km}$
[ENGAA, 2016S1Q12]
A transverse wave travelling through a medium has a frequency of 5.0 Hz , a wavelength of 4.0 cm and an amplitude of 3.0 cm .
What is the total distance travelled by a particle of the medium in one minute?
(A) 900 cm
(B) 1200 cm
(C) 1800 cm
(D) 2400 cm
(E) 3600 cm
(F) 4800 cm
[ENGAA, 2016S1Q13]
The quantities $x$ and $y$ are positive.
$x$ is inversely proportional to the square root of $y$.
When $x=8, y=9$.
What is the value of $y$ when $x=6$ ?
(A) $\frac{3}{2}$
(B) 2
(C) $\frac{81}{16}$
(D) $\frac{27}{14}$
(E) 12
(F) 16
[ENGAA, 2016S1Q14]
A motor is used to lift a mass of 5.0 kg using a pulley system as shown in the diagram. The pulley is secured to the roof using a coupling.


The motor needs to cause the mass to accelerate upwards at $0.80 \mathrm{~m} \mathrm{~s}^{-2}$.
What is the minimum tension force that the coupling must be able to withstand without breaking?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$. The pulley system is frictionless and has negligible mass. The rope has negligible mass and is inextensible.)
(A) 4.0 N
(B) 8.0 N
(C) 46 N
(D) 50 N
(E) 54 N
(F) 92 N
(G) 104 N
(H) 108 N
[ENGAA, 2016S1Q15]
In a trapezium $P Q R S$, the parallel sides are $P Q$ and $R S$.
$P Q=(x-1) \mathrm{cm}, R S=(x+5) \mathrm{cm}$ and the vertical height $Q R=x \mathrm{~cm}$.


The area of the trapezium is $120 \mathrm{~cm}^{2}$. What is the length of $R S$ ?
(A) 9 cm
(B) 10 cm
(C) 11 cm
(D) 12 cm
(E) 15 cm
(F) 17 cm
[ENGAA, 2016S1Q16]
A heater is connected in series with a resistor and a 6.0 V battery in the circuit shown.


The total resistance of the circuit is $15 \Omega$. In 3.0 minutes, 180 J of electrical energy is transferred into other forms in the heater.
How much charge flows through the heater in the 3.0 minutes and what is the voltage across the heater?

|  | Charge / C | Voltage / V |
| :---: | :---: | :---: |
| (A) | 1.2 | 150 |
| (B) | 1.2 | 216 |
| (C) | 7.5 | 0.041 |
| (D) | 7.5 | 24 |
| (E) | 72 | 0.40 |
| (F) | 72 | 2.5 |
| (G) | 450 | 0.40 |
| (H) | 450 | 2.5 |

[ENGAA, 2016S1Q17]
Make $b$ the subject of the formula:

$$
a=\frac{b^{2}+2}{3 b^{2}-1}
$$

(A) $b= \pm \sqrt{\left(\frac{a+2}{3 a+1}\right)}$
(B) $b= \pm \sqrt{\left(\frac{a+2}{3 a-1}\right)}$
(C) $b= \pm \sqrt{\left(\frac{2-a}{3 a+1}\right)}$
(D) $b= \pm \sqrt{\left(\frac{2-a}{3 a-1}\right)}$
(E) $b= \pm \sqrt{\left(\frac{3}{3 a+1}\right)}$
(F) $b= \pm \sqrt{\left(\frac{3}{3 a-1}\right)}$
[ENGAA, 2016S1Q18]
A cubic block has a hole through it with a square cross-section. The dimensions are shown on the diagram. The weight of the block is 30 N .


What is the density of the material from which the block is made?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$.)
(A) $0.30 \mathrm{~g} \mathrm{~cm}^{-3}$
(B) $0.40 \mathrm{~g} \mathrm{~cm}^{-3}$
(C) $0.60 \mathrm{~g} \mathrm{~cm}^{-3}$
(D) $1.2 \mathrm{~g} \mathrm{~cm}^{-3}$
(E) $3.0 \mathrm{~g} \mathrm{~cm}^{-3}$
(F) $4.0 \mathrm{~g} \mathrm{~cm}^{-3}$
(G) $6.0 \mathrm{~g} \mathrm{~cm}^{-3}$
(H) $12 \mathrm{~g} \mathrm{~cm}^{-3}$
[ENGAA, 2016S1Q19]
A thin rectangular sheet of metal 10 m by 5 m is made into an open ended cylinder by joining the edges $P S$ and $Q R$.
The height of the cylinder is 10 m .
What is the volume, in cubic metres, enclosed by this cylinder?

(A) $\frac{5}{2 \pi}$
(B) $\frac{25}{4 \pi}$
(C) $\frac{125}{2 \pi}$
(D) $62.5 \pi$
(E) $\frac{125}{\pi}$
(F) $250 \pi$

## [ENGAA, 2016S1Q20]

The diagram shows four solid steel balls $P, Q, R$ and $S$ which are of identical size.
Balls $P$ and $R$ have shiny surfaces. Balls $Q$ and $S$ have dull surfaces.
Balls $P$ and $Q$ are in a room at $20^{\circ} \mathrm{C}$. Balls $R$ and $S$ are in a room at $40^{\circ} \mathrm{C}$.
The temperature of each ball at a given moment in time is shown on the diagram.


Which two balls lose thermal energy by convection, and which ball emits thermal radiation at the greatest rate?

|  | lose thermal energy by <br> convection | greatest rate of emission of <br> thermal radiation |
| :---: | :---: | :---: |
| (A) | $P$ and $Q$ | $P$ |
| (B) | $P$ and $Q$ | $Q$ |
| (C) | $P$ and $Q$ | $R$ |
| (D) | $P$ and $Q$ | $S$ |
| (E) | $R$ and $S$ | $P$ |
| (F) | $R$ and $S$ | $Q$ |
| (G) | $R$ and $S$ | $R$ |
| (H) | $R$ and $S$ | $S$ |

[ENGAA, 2016S1Q21]
Which one of the following is a simplification of $4+\frac{4-x^{2}}{x^{2}-2 x}$ ?
(A) $3-\frac{2}{x}$
(B) $3+\frac{2}{x}$
(C) $4-\frac{2}{x}$
(D) $4+\frac{2}{x}$
(E) $5-\frac{2}{x}$
(F) $5+\frac{2}{x}$
[ENGAA, 2016S1Q22]
The diagram shows the velocity-time graph for an object travelling in a straight line over a period of 30 s .


What total distance did the object travel in the 30 s , how far from its starting position was it at the end of the 30 s , and what was its average speed over the 30 s ?

|  | total distance <br> travelled $/ \mathrm{m}$ | distance from <br> starting position / <br> m | average speed <br> $/ \mathrm{m} \mathrm{s}^{-1}$ |
| :---: | :---: | :---: | :---: |
| (A) | 90 | 70 | 3.0 |
| (B) | 90 | 70 | 5.0 |
| (C) | 90 | 90 | 3.0 |
| (D) | 90 | 90 | 5.0 |
| (E) | 180 | 140 | 5.0 |
| (F) | 180 | 140 | 6.0 |
| (G) | 180 | 180 | 5.0 |
| (H) | 180 | 180 | 6.0 |

## [ENGAA, 2016S1Q23]

During summer activities week 120 students each chose one activity from swimming, archery, and tennis.
46 of the students were girls.
36 of the students chose tennis, and $\frac{2}{3}$ of these were boys; 25 girls chose swimming, and 27 students chose archery.
A boy is picked at random. What is the probability that he chose swimming?
(A) $\frac{3}{20}$
(B) $\frac{9}{37}$
(C) $\frac{4}{15}$
(D) $\frac{16}{37}$
(E) $\frac{32}{57}$
[ENGAA, 2016S1Q24]
Bronze is a mixture of tin and copper.
A particular sample of bronze contains $10 \%$ tin by volume. (In other words, $10 \%$ of the total volume of the sample is tin and $90 \%$ of it is copper.)
What percentage of the mass of the sample is tin?
(Density of tin $=Y$ and density of copper $=X$.)
(A) $\frac{X}{9 X-Y} \times 100$
(B) $\frac{X}{9 Y-X} \times 100$
(C) $\frac{Y}{9 X-Y} \times 100$
(D) $\frac{Y}{9 Y-X} \times 100$
(E) $\frac{X}{9 X+Y} \times 100$
(F) $\frac{X}{9 Y+X} \times 100$
(G) $\frac{Y}{9 X+Y} \times 100$
(H) $\frac{Y}{9 Y+X} \times 100$
[ENGAA, 2016S1Q25]
Which one of the following expressions is equivalent to $\frac{9^{2 n+1} \times 3^{4-3 n}}{27^{2-n}}$ ?
(A) $3^{9}$
(B) $3^{-2 n}$
(C) $3^{2-2 n}$
(D) $3^{4 n}$
(E) $3^{6 n-2}$
(F) $3^{6}$
[ENGAA, 2016S1Q26]
When a stationary uranium- 238 nucleus decays by alpha emission it forms a nucleus of thorium-234. The total kinetic energy produced by the decay is $E$.


What is the kinetic energy of the alpha particle?
(A) $\frac{4 E}{238}$
(B) $\frac{4 E}{234}$
(C) $\frac{E}{2}$
(D) $\frac{234 E}{238}$
(E) $E$
[ENGAA, 2016S1Q27]
In the diagram below, $P Q R S$ is part of a regular polygon.
The polygon has $n$ sides.
The side $P Q$ is extended to $T$ such that $P Q T$ is a straight line.
The length of $R Q$ is the same as the length of $R T$.


Find an equation for $n$ in terms of $x$, where $x$ is the size of angle $\angle Q R T$ in degrees.
(A) $n=\frac{180}{x-90}$
(B) $n=\frac{180-x}{720}$
(C) $n=\frac{360-x}{90}$
(D) $n=\frac{360}{180-x}$
(E) $n=\frac{720}{180-x}$
(F) $n=\frac{720}{360-x}$
(G) $n=\frac{360}{360-x}$
[ENGAA, 2016S1Q28]
A student carries out an experiment to measure the speed of sound. A loudspeaker that emits sound in all directions is placed between two buildings that are 128 m apart as shown. The student and loudspeaker are 48 m from one of the buildings.


The loudspeaker is connected to a signal generator that causes it to emit regular clicks. The student notices that each click results in two echoes, one from each building. The rate at which the clicks are produced is gradually increased from zero until each echo coincides with a new click being emitted by the loudspeaker.
What is the frequency of emission of clicks when this happens?
(The speed of sound in air $=320 \mathrm{~m} \mathrm{~s}^{-1}$.)
(A) 2.0 Hz
(B) 2.5 Hz
(C) 3.3 Hz
(D) 4.0 Hz
(E) 5.3 Hz
(F) 6.7 Hz
(G) 10 Hz

## Part B

[ENGAA, 2016S1Q29]
When $x=2$ is substituted in the expression $x^{3}+p x^{2}+q x+p^{2}$ the result is 0 .
When $x=1$ is substituted into the same expression, the result is -3.5 .
Find all possible value(s) of $p$.
(A) $p=-1 \pm \frac{\sqrt{6}}{3}$
(B) $p=1$ or $p=-3$
(C) $p=1$
(D) $p=1 \pm \sqrt{7}$
(E) there are no values for $p$
[ENGAA, 2016S1Q30]
A parachutist is falling at terminal speed with his parachute open. The diagrams show, separately, the vertical forces acting on the parachute and the vertical forces acting on the parachutist.
The letters $L, M, N, P, Q$ and $R$ represent the magnitude of each force as indicated.
air resistance on parachute $(L)$

weight of
force from parachute ( $M$ ) parachutist ( $N$ )

weight of parachutist ( $R$ )

Consider the following equations:
Equation 1: $L=M+N$
Equation 2: $R=P+Q$
Equation 3: $L=Q$
Equation 4: $N=P$
Equation 5: $M+R=L+Q$
Which of these equations, if any, is/are the direct result of the application of Newton's Third Law to this situation?
(A) none of them
(B) 3 only
(C) 4 only
(D) 5 only
(E) 1 and 2 only
(F) 3 and 4 only
(G) 1, 2 and 5 only
(H) 1, 2, 3, 4 and 5
[ENGAA, 2016S1Q31]
A square $P Q R S$ is drawn above the $x$-axis with the side $P Q$ on the $x$-axis.
$P$ is the point $(-5,0)$ and $Q$ is the point $(1,0)$.
A circle is drawn inside the square with diameter equal in length to the side of the square.
Which one of the following is an equation of the circle?
(A) $x^{2}+y^{2}-4 x+6 y+4=0$
(B) $x^{2}+y^{2}-4 x+6 y+9=0$
(C) $x^{2}+y^{2}+4 x-6 y+4=0$
(D) $x^{2}+y^{2}+4 x-6 y+9=0$
(E) $x^{2}+y^{2}-6 x-4 y+9=0$
(F) $x^{2}+y^{2}-6 x+4 y+4=0$
(G) $x^{2}+y^{2}+6 x-4 y+4=0$
(H) $x^{2}+y^{2}+6 x+4 y+9=0$
[ENGAA, 2016S1Q32]
A crate has a total mass of 800 kg , including its contents. A helicopter of mass 4200 kg is carrying the crate using a light inextensible rope as shown:


The helicopter and crate are accelerating upwards at $2.0 \mathrm{~m} \mathrm{~s}^{-2}$.
What is the tension in the rope?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$; air resistance can be ignored.)
(A) 6400 N
(B) 8000 N
(C) 9600 N
(D) 18000 N
(E) 40000 N
(F) 42000 N
(G) 50000 N
(H) 60000 N
[ENGAA, 2016S1Q33]
The first term of a convergent geometric series is 8 .
The fifth term is 2.
The sixth term is real and positive.
What is the sum to infinity of this series?
(The sum to infinity of a convergent geometric series is given by $\frac{a}{1-r}$, where $a$ is the first term and $r$ is the common ratio.)
(A) $8(1+\sqrt{2})$
(B) $8(1-\sqrt{2})$
(C) $8(2+\sqrt{2})$
(D) $8(2-\sqrt{2})$
(E) 16
(F) $\frac{8 \sqrt[5]{4}}{\sqrt[5]{4}-1}$
(G) $\frac{8 \sqrt[5]{4}}{\sqrt[5]{4}+1}$
[ENGAA, 2016S1Q34]
A shopper pushes a supermarket trolley a distance of 15 m in a straight line across a level, horizontal surface. The shopper applies a constant force of 50 N at an angle of $37^{\circ}$ below the horizontal. The total weight of the trolley and its contents is 350 N .


What is the magnitude of the total vertical force that the surface exerts on the trolley and how much work is done by the pushing force?
(You may use the approximations $\sin 37^{\circ}=0.60$; $\cos 38^{\circ}=0.80$.)

|  | vertical force / N | work done / J |
| :---: | :---: | :---: |
| (A) | 380 | 600 |
| (B) | 380 | 750 |
| (C) | 390 | 450 |
| (D) | 390 | 750 |
| (E) | 400 | 450 |
| (F) | 400 | 600 |

[ENGAA, 2016S1Q35]
Tangents are drawn from a point $P$ to a circle of radius 10 cm .
The centre of the circle is $C$ and the distance $P C$ is 20 cm .


Which one of the following is an expression for the shaded area in square centimetres?
(A) $\frac{100}{3}(3 \sqrt{3}-\pi)$
(B) $\frac{100}{3}(3 \sqrt{5}-\pi)$
(C) $\frac{50}{3}(6 \sqrt{3}-\pi)$
(D) $\frac{50}{3}(6 \sqrt{5}-\pi)$
(E) $\frac{50}{3}(\sqrt{3}-2 \pi)$
(F) $\frac{50}{3}(2 \pi-\sqrt{3})$
[ENGAA, 2016S1Q36]
A plank of non-uniform density which has a mass of 15 kg is used to make a see-saw. A pivot is placed under the centre of the plank as shown on the diagram.


A boy of mass 35 kg sits at one end of the plank with his centre of gravity 1.20 m from the pivot. The see-saw balances when a woman of mass 60 kg sits on the plank on the other side of the pivot. Her centre of gravity is 0.80 m from the pivot.
Where is the centre of gravity of the plank and what is the magnitude of the force between the pivot and the plank?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$.)

|  | distance from pivot | force $/ \mathrm{N}$ |
| :---: | :---: | :---: |
| (A) | 0.40 m on left of pivot | 100 |
| (B) | 0.40 m on left of pivot | 1100 |
| (C) | at the pivot | 100 |
| (D) | at the pivot | 1100 |
| (E) | 0.20 m on right of pivot | 100 |
| (F) | 0.20 m on right of pivot | 1100 |
| (G) | 0.40 m on right of pivot | 100 |
| (H) | 0.40 m on right of pivot | 1100 |

[ENGAA, 2016S1Q37]
Given that $7 \cos \theta-3 \tan \theta \sin \theta=1$, which one of the following is true?
(A) $\cos \theta=-\frac{3}{5}$ or $-\frac{1}{2}$
(B) $\cos \theta=-\frac{3}{5}$ or $\frac{1}{2}$
(C) $\cos \theta=\frac{3}{5}$ or $\frac{1}{2}$
(D) $\cos \theta=\frac{3}{5}$ or $-\frac{1}{2}$
[ENGAA, 2016S1Q38]
A car of mass 200 kg on a fairground ride travels at a speed of $5.0 \mathrm{~m} \mathrm{~s}^{-1}$ at point $X$. The car is allowed to move down a sloping section of track without any energy input. The heights above the ground of points $X$ and $Y$ are shown. When the car reaches point $Y$ its speed is $9.0 \mathrm{~m} \mathrm{~s}^{-1}$.


How much energy is transferred in overcoming resistive forces as the car travels from $X$ to $Y$ ?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$.)
(A) 3900 J
(B) 6400 J
(C) 7900 J
(D) 10400 J
(E) 11200 J
[ENGAA, 2016S1Q39]
The complete set of values of $a$ for which the equation $3 x^{2}=(a+2) x-3$ has two real distinct roots is
(A) no values of $a$
(B) $-4 \sqrt{2}<a<4 \sqrt{2}$
(C) $a<-4 \sqrt{2}, a>4 \sqrt{2}$
(D) $-4<a<8$
(E) $a<-4, a\rangle 8$
(F) $-8<a<4$
(G) $a<-8, a\rangle 4$
(H) all values of $a$
[ENGAA, 2016S1Q40]
The diagram shows a uniform, solid, heavy cube with side $d$. The cube rests with one of its edges in contact with a table that is perfectly level. A horizontal force $P$ acts on another edge of the cube, and the cube is stationary.
[diagram not to scale]


Below are four statements about the forces on the cube.
1 It is possible that there is no frictional force between the cube and the table.
2 There must be a frictional force acting to the left between the cube and the table.
3 There must be a frictional force acting to the right between the cube and the table.
4 Force $P$ has a clockwise moment about the edge in contact with the table equal to $P \times d$.

Which of the statements is/are correct?
(A) 1 only
(B) 2 only
(C) 3 only
(D) 1 and 4 only
(E) 2 and 4 only
(F) 3 and 4 only
[ENGAA, 2016S1Q41]
The straight line with equation $y=m x+3$, where $m>0, m \neq 1$, is perpendicular to the line with equation $y=p x+2$.
The lines cut the x-axis at the points $L$ and $M$ respectively. The length of $L M$ is 5 units.
What is the value of $m+p$ given that $m>1$ ?

(A) $-\frac{8}{3}$
(B) $-\frac{13}{6}$
(C) $-\frac{5}{6}$
(D) $\frac{5}{6}$
(E) $\frac{13}{6}$
(F) $\frac{8}{3}$
[ENGAA, 2016S1Q42]
The diagram shows two identical blocks, each of mass $m$, in two different arrangements.


Which expression gives:
(gravitational potential energy of arrangement 2) - (gravitational potential energy of arrangement 1)?
( $g$ is the gravitational field strength.)
(A) $2 m g(a-b)$
(B) $2 m g(b-a)$
(C) $-m g(b+a)$
(D) $m g(a+b)$
(E) $\frac{3}{2} m g(a-b)$
(F) $\frac{1}{2} m g(a-b)$

## [ENGAA, 2016S1Q43]

$f(x)=x^{3}-a^{2} x$ where $a$ is a positive constant.
Find the complete set of values of $x$ for which $f(x)$ is an increasing function.
(A) $x \leq-a, x \geq a$
(B) $-a \leq x \leq a$
(C) $x \leq-a, 0 \leq x \leq a$
(D) $-a \leq x \leq 0, x \geq a$
(E) $x \leq-\frac{a}{3}, x \geq \frac{a}{3}$
(F) $-\frac{a}{3} \leq a \leq \frac{a}{3}$
(G) $x \leq-\frac{a}{\sqrt{3}}, x \geq \frac{a}{\sqrt{3}}$
(H) $-\frac{a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$
[ENGAA, 2016S1Q44]
An object is fired vertically upwards from the ground at time $t=0 \mathrm{~s}$ in still air at a speed of 8.0 $\mathrm{m} \mathrm{s}^{-1}$.
On the way up, what is the height of the object above the ground when it has a speed of 2.0 m $\mathrm{s}^{-1}$, and at what time does it reach this height on the way down?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$. Air resistance can be ignored.)

|  | height $/ \mathrm{m}$ | time $/ \mathrm{s}$ |
| :--- | :--- | :--- |
| (A) | 2.4 | 0.60 |
| (B) | 2.4 | 0.64 |
| (C) | 2.4 | 1.0 |
| (D) | 2.4 | 2.0 |
| (E) | 3.0 | 0.60 |
| (F) | 3.0 | 0.64 |
| (G) | 3.0 | 1.0 |
| (H) | 3.0 | 2.0 |

## [ENGAA, 2016S1Q45]

The curve $y=x^{2}$ is translated by the vector $\binom{4}{3}$ and then reflected in the line $y=-1$.
Which one of the following is an equation of the resulting curve?
(A) $y=-3-(x-4)^{2}$
(B) $y=-3+(x+4)^{2}$
(C) $y=3-(x+4)^{2}$
(D) $y=3+(x-4)^{2}$
(E) $y=-5-(x-4)^{2}$
(F) $y=-5+(x+4)^{2}$
(G) $y=5-(x+4)^{2}$
(H) $y=5+(x-4)^{2}$
[ENGAA, 2016S1Q46]
The diagram shows a ball $P$, of mass 4.0 kg , moving to the right at $10 \mathrm{~m} \mathrm{~s}^{-1}$ directly towards a stationary ball $Q$, of mass 2.0 kg .


The balls collide but do not join together. Immediately after the collision ball $Q$ moves at 10 m $\mathrm{s}^{-1}$ to the right.
What is the velocity of ball $P$ immediately after the collision, and how much kinetic energy in total is lost during the collision?

|  | velocity of ballP after <br> collision | kinetic energy lost <br> during collision $/ \mathrm{J}$ |
| :---: | :---: | :---: |
| (A) | 0 | 50 |
| (B) | 0 | 150 |
| (C) | $10 \mathrm{~m} \mathrm{~s}^{-1}$ to the left | 50 |
| (D) | $10 \mathrm{~m} \mathrm{~s}^{-1}$ to the left | 150 |
| (E) | $5.0 \mathrm{~m} \mathrm{~s}^{-1}$ to the right | 50 |
| (F) | $5.0 \mathrm{~m} \mathrm{~s}^{-1}$ to the right | 150 |

[ENGAA, 2016S1Q47]
The complete set of values of $x$ for which $2 x^{4}-9 x^{2}+4>0$ is
(A) $x<\frac{1}{2}, x>4$
(B) $\frac{1}{2}<x<4$
(C) $x<-2,-\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}, x>2$
(D) $-2<x<-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}<x<2$
(E) $-2<x<2$
[ENGAA, 2016S1Q48]
A point object of mass 2.0 kg is at rest on a level, horizontal surface.
The coefficient of friction between the object and the surface is 0.25 .
Two horizontal forces at right-angles to each other, with magnitudes 9.0 N and 12.0 N , are applied simultaneously to the object.
What is the magnitude of the acceleration of the object as it begins to move?
(The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$.)
(A) $5.0 \mathrm{~m} \mathrm{~s}^{-2}$
(B) $7.25 \mathrm{~m} \mathrm{~s}^{-2}$
(C) $7.5 \mathrm{~m} \mathrm{~s}^{-2}$
(D) $8.0 \mathrm{~m} \mathrm{~s}^{-2}$
(E) $10 \mathrm{~m} \mathrm{~s}^{-2}$
(F) $10.5 \mathrm{~m} \mathrm{~s}^{-2}$
[ENGAA, 2016S1Q49]
A cursor starts at the point $(0,10)$ and moves parallel to the $x$-axis in the negative direction.
What is the minimum distance parallel to the $y$-axis between the cursor and the graph of $y=$ $4 x^{3}-12 x^{2}-36 x-15$ ?
(A) 0
(B) 5
(C) 25
(D) 69
(E) 133
[ENGAA, 2016S1Q50]
An object of mass 20 kg is pulled up a rough plane inclined at $30^{\circ}$ to the horizontal by a light, inextensible cable attached via a frictionless pulley to a freely-falling 30 kg mass. The acceleration of the object along the plane is $2.5 \mathrm{~m} \mathrm{~s}^{-2}$.

$$
\begin{aligned}
& \cos 30^{\circ}=\sin 60^{\circ}=\frac{\sqrt{3}}{2} \\
& \sin 30^{\circ}=\cos 60^{\circ}=\frac{1}{2}
\end{aligned}
$$

frictionless


What is the frictional force between the object and the plane?
(Air resistance and the mass of the pulley can be ignored. The gravitational field strength $g$ is $10 \mathrm{~N} \mathrm{~kg}^{-1}$.)
(A) 25 N
(B) 50 N
(C) 75 N
(D) 100 N
(E) 150 N
(F) 175 N
(G) 250 N
[ENGAA, 2016S1Q51]
What is the area enclosed by the line $x=7$ and the curve $x=3(y-1)^{2}+4$ ?
(A) 4
(B) 8
(C) 10
(D) 11
(E) 14
(F) 20
[ENGAA, 2016S1Q52]
A spacecraft of initial total mass 4000 kg is travelling relative to the Earth at a constant speed of $7425 \mathrm{~m} \mathrm{~s}^{-1}$.
It ejects some fuel backwards in a sudden burst at a speed relative to the spacecraft of 1425 m $\mathrm{s}^{-1}$. As a result of this, the speed of the spacecraft immediately after the fuel is ejected increases to $7500 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the mass of fuel ejected?
(A) 22 kg
(B) 34 kg
(C) 40 kg
(D) 50 kg
(E) 200 kg
(F) 210 kg
[ENGAA, 2016S1Q53]
A curve has equation $y=3 x^{4}-4 x^{3}-12 x^{2}+20$.
What is the complete set of values of the constant $k$ for which the equation

$$
3 x^{4}-4 x^{3}-12 x^{2}+20=k
$$

has exactly four distinct real roots?
(A) no values of $k$
(B) $-12<k<15$
(C) $15<k<20$
(D) $k>20$
(E) $7<k<20$
(F) all values of $k$
[ENGAA, 2016S1Q54]
An object of weight 40 N hangs from the end of a light inextensible string of length 0.35 m , which is attached to the ceiling. A constant horizontal force of 30 N is applied to the object, causing it to move to a new equilibrium position with the string no longer vertical.
By how much has the gravitational potential energy of the object increased as a result of its change of position?
(A) 2.1 J
(B) 2.8 J
(C) 3.5 J
(D) 4.2 J
(E) 4.9 J
(F) 5.6 J

## ENGAA 2016 S2



## TIME ALLOWED: 40 MINUTES

This paper is Section 2 of 2 .
This question paper contains 15 multiple choice questions arranged into 4 groups. Some questions are connected to other questions.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 15 questions. The number of marks each question is worth is indicated. In total 29 marks are available.

Please complete this section in pencil. For each question circle the one option you consider correct. If you make a mistake, erase thoroughly and try again.

Unless otherwise indicated, marks will only be awarded for correct answers if these are accompanied by working or reasoning justifying the answer chosen. Such working or reasoning must be written in the spaces provided on the question paper.

You can use the blank inside front and back covers for rough working or notes, but no extra paper is allowed. Only answers in the spaces indicated in the paper will be marked.

Calculators may be used in this section. Please record your calculator model in the box below:

[^0][ENGAA, 2016S2Q1]
A series of 100 mm long samples have a square cross-section of $5 \mathrm{~mm} \times 5 \mathrm{~mm}$. Each sample is tested in an apparatus which applies increasing tension to the ends of the sample until it breaks. The length of the sample is measured and a plot of tension vs extension obtained. The results for five samples S1-S5 are shown in the figure below.

[ENGAA, 2016S2Q1(a)]
Which of the samples is the stiffest compared to the other samples?

## SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.

(A) S 1
(B) S 2
(C) S 3
(D) S 4
(E) S 5
[ENGAA, 2016S2Q1(b)]
Which sample does not obey Hooke's Law up to 2\% strain?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) S 1
(B) S 2
(C) S 3
(D) S 4
(E) S 5
[ENGAA, 2016S2Q1(c)]
What is the value of the Young's Modulus of sample S3, assuming that changes in its crosssection are negligible?

## SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

(A) 200 MPa
(B) 20 MPa
(C) 100 MPa
(D) 10 Mpa
[ENGAA, 2016S2Q1(d)]
The equation of the force-extension curve for sample S2 is given by $T=a x-b x^{2}$ where $T$ is the tension in $\mathrm{N}, x$ the extension in m and $a$ and $b$ are constants. Sample S2 breaks when its extension is 10 mm . How much work does the apparatus do on sample S 2 in breaking it?

## SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

(A) $\frac{a}{2} \times 10^{-4}-\frac{b}{3} \times 10^{-6} \mathrm{Nm}$
(B) $\frac{a}{2} \times 10^{-2}-\frac{b}{3} \times 10^{-3} \mathrm{Nm}$
(C) $\frac{a}{2} \times 10^{2}-\frac{b}{3} \times 10^{3} \mathrm{Nm}$
(D) $\frac{a^{2}}{2} \times 10^{2}-\frac{b^{2}}{3} \times 10^{3} \mathrm{Nm}$
(E) $\frac{a}{2} \times 10^{-2}-\frac{b}{2} \times 10^{-3} \mathrm{Nm}$
[ENGAA, 2016S2Q2]
The figure below shows a network of three non-zero resistances $R_{1}, R_{2}, R_{3}$ connected to a voltage source $V$ with zero internal resistance.

[ENGAA, 2016S2Q2(a)]
Which of the following statements must be correct?
NO WORKING NEEDS TO BE GIVEN FOR THIS PART OF THIS QUESTION.
(A) The currents through resistances $R_{1}$ and $R_{2}$ are the same.
(B) The currents through resistances $R_{1}$ and $R_{3}$ are the same.
(C) The currents through resistances $R_{2}$ and $R_{3}$ are the same.
(D) The voltages across resistances $R_{1}$ and $R_{2}$ are the same.
(E) The voltages across resistances $R_{1}$ and $R_{3}$ are the same.
(F) The voltages across resistances $R_{2}$ and $R_{3}$ are the same.
[ENGAA, 2016S2Q2(b)]
Which of the following expressions gives the current through the voltage source?
SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.
(A) $\frac{V\left(R_{2}+R_{3}\right)}{R_{1} R_{2}+R_{1} R_{3}+R_{2} R_{3}}$
(B) $\frac{V\left(R_{1} R_{2}+R_{1} R_{3}+R_{2} R_{3}\right)}{R_{2}+R_{3}}$
(C) $\frac{V R_{2} R_{3}}{R_{2}+R_{3}+R_{1} R_{2} R_{3}}$
(D) $\frac{V}{R_{1}+R_{2}+R_{3}}$
(E) $\frac{V\left(R_{1} R_{2}+R_{1} R_{3}+R_{2} R_{3}\right)}{R_{1} R_{2} R_{3}}$
[ENGAA, 2016S2Q2(c)]
$R_{1}$ and $R_{2}$ are now fixed such that $R_{1}=R_{2}$. Which of the following expressions gives the power $P$ that is dissipated by resistance $R_{3}$ ?

## SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

(A) $\frac{V^{2} R_{3}^{3}}{\left(R_{1}+2 R_{3}\right)^{2}}$
(B) $\frac{V^{2}}{2 R_{1}+R_{3}}$
(C) $\frac{V^{2}\left(R_{1}+2 R_{3}\right)^{2}}{R_{3}^{3}}$
(D) $\frac{V^{2} R_{3}}{\left(R_{1}+2 R_{3}\right)^{2}}$
(E) $\frac{V^{2}\left(R_{1}+R_{3}\right)^{2}}{\left(R_{1}+2 R_{3}\right)^{2} R_{3}}$
[ENGAA, 2016S2Q2(d)]
For the case where $R_{1}=R_{2}$, which of the following values of $R_{3}$ maximises its power dissipation? You may find it helpful to use the fact that any value of $R_{3}$ that maximises $P$ also minimises $1 / P$.

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.
(A) $R_{3}=\frac{1}{4} R_{1}^{2}$
(B) $R_{3}=\frac{1}{2} R_{1}$
(C) $R_{3}=\exp \left(-\frac{4}{R_{1}^{2}}\right)$
(D) $R_{3}=\frac{1}{\sqrt{2}} R_{1}$
(E) $R_{3}=\frac{1}{4} R_{1}$
(F) Either $R_{3}=\frac{1}{2} R_{1}$ or $R_{3}=-\frac{1}{2} R_{1}$ would result in maximum power dissipation in $R_{3}$.
[ENGAA, 2016S2Q3]
The speed of light in vacuum and air can be taken to be $c=3.0 \times 10^{5} \mathrm{~km} \mathrm{~s}^{-1}$. The refractive index $n$ of a material is the ratio of the speed of light $c$ in vacuum to the speed of light in the material.
[ENGAA, 2016S2Q3(a)]
A lighthouse emits a beam of light. How far does this beam of light travel in 1.0 ns ?

## SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

(A) 0.30 mm
(B) 300 m
(C) 0.30 m
(D) $3.0 \times 10^{-12} \mathrm{~m}$
[ENGAA, 2016S2Q3(b)]
The propagation time $T$ is the time taken for a pulse of light to travel directly along an optical fibre. A straight optical fibre has a length of 9 km . Its refractive index is 1.5 . What is $T$ for this fibre?

## SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

(A) 20 ms
(B) $20 \mu \mathrm{~s}$
(C) 30 ms
(D) $30 \mu \mathrm{~s}$
(E) 45 ms
(F) $45 \mu \mathrm{~s}$
[ENGAA, 2016S2Q3(c)]
An engineer has used a refractive index of $n=1.5$ to estimate the nominal propagation time $T_{\text {nom }}$ for an optical fibre. The actual refractive index of the fibre depends on the wavelength of the light. For red and blue light the refractive indices obey the inequality $n_{\text {red }}<n_{\text {blue }}<1.5$. If $T_{\text {red }}$ and $T_{\text {blue }}$ are the propagation times for red and blue light, respectively, which of the following inequalities is correct?

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) $T_{\text {red }}<T_{\text {blue }}<T_{\text {nom }}$
(B) $T_{\text {blue }}<T_{\text {red }}<T_{\text {nom }}$
(C) $T_{\text {red }}<T_{\text {nom }}<T_{\text {blue }}$
(D) $T_{\text {blue }}<T_{\text {nom }}<T_{\text {red }}$
(E) $T_{\text {nom }}<T_{\text {red }}<T_{\text {blue }}$
(F) $T_{\text {nom }}<T_{\text {blue }}<T_{\text {red }}$
[ENGAA, 2016S2Q4]
A stunt cyclist is preparing a new trick. The track on which he will perform the trick is shown schematically in the figure below. As shown, most of the track is sloped at an angle $\theta$ to the horizontal.


The cyclist starts riding from rest at $A$. In riding down the slope from $A$ to $B$ he transfers an amount of energy $E$ from his muscles to provide kinetic energy to the rider-bicycle system and descends through a vertical distance $h$. The cyclist leaves the track at $B$, travelling horizontally initially. He lands on the track at $C$, a distance $L$ down the slope.
Assume that the rider-bicycle system can be modeled as a point mass of mass $M$, that frictional forces and air resistance can be neglected, and that the gravitational field strength is $g$.
[ENGAA, 2016S2Q4(a)]
What is $V_{a}$, the component of the velocity of the rider-bicycle system along (parallel to) the slope, immediately after the cyclist has left the track at $B$ ?

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.
(A) $\sqrt{\left(\frac{E}{M}+2 g h\right)} \cos \theta$
(B) $\sqrt{\frac{2}{M}(E+M g h)} \sin \theta$
(C) $\sqrt{\frac{2}{M}(E+M g h)} \frac{1}{\cos \theta}$
(D) $\sqrt{\left(\frac{E}{M}+2 g h\right)} \sin \theta$
(E) $\sqrt{2\left(\frac{E}{M}+g h\right)} \cos \theta$
[ENGAA, 2016S2Q4(b)]
The cyclist leaves the track at $B$ at time $t=0$ with an initial speed $V$. By considering motion parallel and/or perpendicular to the slope, or otherwise, find an expression for the time taken to land at $C$.

SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.
(A) $2 \frac{V}{g}$
(B) $2 \frac{V}{g} \tan \theta$
(C) $\frac{V}{g} \sin \theta$
(D) $\frac{V}{g} \tan \theta$
(E) $2 \frac{V}{g} \sin \theta$
[ENGAA, 2016S2Q4(c)]
How far along the slope will the cyclist land, i.e. what is the value of $L$ ?
SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.
(A) $2 \frac{V^{2}}{g} \sin \theta$
(B) $\frac{V^{2}}{2 g}$
(C) $2 \frac{V^{2}}{g} \frac{\sin \theta}{\cos ^{2} \theta}$
(D) $2 \frac{V^{2}}{g} \frac{\cos \theta}{\sin ^{2} \theta}$
(E) $2 \frac{V^{2}}{g}\left(\sin \theta+\tan ^{2} \theta\right)$
[ENGAA, 2016S2Q4(d)]
As part of the trick, the cyclist wants to clear an obstacle placed on the slope between $B$ and $C$. To give the cyclist the greatest chance of clearing the obstacle it should be placed at the point at which the cyclist's perpendicular distance from the track is greatest. At what distance from $B$ should the obstacle be placed?

## SHOW YOUR WORKING IN THE SPACE PROVIDED BELOW.

(A) $\frac{1}{2} L$
(B) $\frac{V^{2}}{2 g} \frac{\sin ^{2} \theta}{\cos \theta}$
(C) $\frac{1}{2} h$
(D) $\frac{V^{2}}{g} \sin \theta\left(1+\frac{1}{2} \tan ^{2} \theta\right)$
(E) $\frac{V^{2}}{2 g}$

## ENGAA 2017 S1



## TIME ALLOWED: 80 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 80 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.
Part A Mathematics and Physics ( 28 questions)
Part B Advanced Mathematics and Advanced Physics (26 questions)
This paper contains 54 multiple choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 54 questions. Each question is worth one mark.

Questions ask you to show your choice between options. Choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators may NOT be used.

## Part A

[ENGAA, 2017S1Q1]
Evaluate

$$
\frac{(\sqrt{12}+\sqrt{3})^{2}}{(\sqrt{12}-\sqrt{3})^{2}}
$$

(A) 1
(B) 3
(C) $\frac{5}{3}$
(D) $\frac{7}{3}$
(E) $3 \sqrt{3}$
(F) 9
[ENGAA, 2017S1Q2]
A car is travelling along a horizontal road in a straight line.
The graph is a velocity-time graph for part of the car's journey.


During this part of the journey, what is the total distance that the car travels while it is decelerating?
(A) 400 m
(B) 500 m
(C) 550 m
(D) 600 m
(E) 750 m
(F) 1400 m
(G) 1800 m
(H) 1900 m
[ENGAA, 2017S1Q3]
Solve fully the inequality

$$
2 x^{2} \geq 15-x
$$

(A) $x \leq-3$
(B) $x \geq 2.5$
(C) $x \leq-1.5, x \geq 5$
(D) $-1.5 \leq x \leq 5$
(E) $x \leq-3, x \geq 2.5$
(F) $-3 \leq x \leq 2.5$
[ENGAA, 2017S1Q4]
When a saucepan of water is heated from below, convection currents form and transfer heat through the liquid.
Here are three statements about the water as it is heated:
1 The mass of a fixed volume of the water increases.
2 The density of a fixed mass of the water decreases.
3 The volume of a fixed mass of the water increases.
Which of these statements help(s) to explain how convection currents are formed?
(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2017S1Q5]
The equation gives $y$ in terms of $x$ :

$$
y=3\left(\frac{x}{2}-1\right)^{2}-5
$$

Which one of the following is a rearrangement for $x$ in terms of $y$ ?
(A) $x=2 \pm 2 \sqrt{\frac{y-5}{3}}$
(B) $x=2 \pm 2 \sqrt{\frac{y+5}{3}}$
(C) $x=2 \pm 3 \sqrt{\frac{y+5}{3}}$
(D) $x=-2 \pm 2 \sqrt{\frac{y+5}{3}}$
(E) $x=-2 \pm 3 \sqrt{\frac{y+5}{2}}$
(F) $x=2+2\left(\frac{y+5}{3}\right)^{2}$
(G) $x=-2+2\left(\frac{y+5}{3}\right)^{2}$

## [ENGAA, 2017S1Q6]

An electric motor is used to pull a broken-down car slowly from the road up a ramp on to the back of a breakdown truck.

The car has a mass of 1200 kg and is lifted through a vertical height of 1.0 m .

In the process of lifting the car, energy is lost to the surroundings from the motor and from other causes.

What is the total energy lost to the surroundings?

(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 7.0 kJ
(B) 9.0 kJ
(C) 12 kJ
(D) 16 kJ
(E) 21 kJ
(F) 33 kJ
[ENGAA, 2017S1Q7]
A fruit stall sells apples costing $£ x$ each, and pears costing $£ y$ each. Sam bought 2 apples and 5 pears, and the total cost of these was $£ P$. Lesley bought 3 apples and 2 pears, and the total cost of these was $£ Q$. Which of the following is an expression for the cost, in pounds ( $£$ ), of a pear?
(A) $\frac{2 Q-3 P}{3}$
(B) $\frac{2 Q-3 P}{11}$
(C) $\frac{Q-P}{3}$
(D) $\frac{Q-P}{11}$
(E) $\frac{P-Q}{3}$
(F) $\frac{3 P-2 Q}{3}$
(G) $\frac{3 P-2 Q}{11}$

## [ENGAA, 2017S1Q8]

In one type of medical scanner a source is placed inside a patient's body. This source causes pairs of gamma-rays to be emitted simultaneously in opposite directions.
Detectors on each side of the patient are used to detect the gamma-rays. The distance between the two detectors is 3.0 m . When the source is at $Q$, half-way between the detectors, the two gamma-rays arrive at the same time.
In a particular scan the gamma-rays arrive at the two detectors with a time difference of $4.0 \times 10^{-10} \mathrm{~s}$.
Assume that, inside the patient, the gamma-rays travel at a speed of $3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$.
How far from $Q$, half-way between the detectors, is the gamma-ray source?
(A) 6.0 mm
(B) 12 mm
(C) 24 mm
(D) 6.0 cm
(E) 12 cm
(F) 24 cm
[ENGAA, 2017S1Q9]
$P$ is directly proportional to $Q$ squared.
When $P$ is $2, Q$ is 4 .
$Q$ is inversely proportional to $R$.
When $Q$ is $2, R$ is 5 .
What is $P$ in terms of $R$ ?
(A) $P=\frac{5}{R}$
(B) $P=\frac{5}{4 R}$
(C) $P=\frac{1}{800 R^{2}}$
(D) $P=\frac{5}{4 R^{2}}$
(E) $P=\frac{25}{2 R^{2}}$
(F) $P=\frac{800}{R^{2}}$
(G) $P=\frac{R^{2}}{50}$
(H) $P=\frac{25 R^{2}}{2}$
[ENGAA, 2017S1Q10]
When a plutonium-239 nucleus absorbs a neutron it undergoes nuclear fission. One particular fission reaction results in the creation of xenon and zirconium as daughter nuclei. The nuclear equation for this reaction is shown but with some non-zero integers replaced by the letters $w$, $x, y$ and $z$.

$$
{ }_{0}^{1} \mathrm{n}+{ }_{94}^{239} \mathrm{Pu} \rightarrow{ }_{54}^{w} \mathrm{Xe}+{ }_{x}^{y} \mathrm{Zr}+z_{0}^{1} \mathrm{n}
$$

Which equation is correct?
(A) $w+y=240$
(B) $z=240-(w+y)$
(C) $x=40-z$
(D) $94=54+x+1$
(E) $240=54+x$
(F) $94=w+y+1$
[ENGAA, 2017S1Q11]
Which one of the following is a simplification of

$$
2-\frac{x^{2}\left(9 x^{2}-4\right)}{x^{3}(2-3 x)}
$$

(A) $-1-\frac{2}{x}$
(B) $-1+\frac{2}{x}$
(C) $5-\frac{2}{x}$
(D) $5+\frac{2}{x}$
(E) $5-\frac{3}{x}$
(F) $5+\frac{3}{x}$

## [ENGAA, 2017S1Q12]

An electric motor is connected to a constant 12 V d.c. supply. The motor is used to lift a mass of 20 kg by means of a rope and pulley. The mass is lifted vertically through a height of 6.0 m in a time of 5.0 s . The complete lifting system (motor, rope and pulley) is $80 \%$ efficient.


What is the current in the electric motor?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 1.6 A
(B) 2.0 A
(C) 2.5 A
(D) 16 A
(E) 20 A
(F) 25 A
[ENGAA, 2017S1Q13]
What is the value of $x$ that makes the following expression correct?

$$
2^{3+2 x} 4^{x} 8^{-x}=4 \sqrt{2}
$$

(A) -2.25
(B) -1.75
(C) -1.5
(D) -0.5
(E) -0.25

## [ENGAA, 2017S1Q14]

The nuclide ${ }_{Q}^{P} X$ decays to the stable nuclide $Y$. During this process four particles are emitted: an $\alpha$-particle and three $\beta^{-}$particles.
Which of the following is not a nuclide that could be formed at any stage during this process?

| nuclide | atomic mass | atomic number |
| :---: | :---: | :---: |
| (A) | $P$ | $Q-1$ |
| (B) | $P$ | $Q+1$ |
| (C) | $P$ | $Q+2$ |
| (D) | $P$ | $Q+3$ |
| (E) | $P-4$ | $Q-2$ |
| (F) | $P-4$ | $Q-1$ |
| (G) | $P-4$ | $Q$ |
| (H) | $P-4$ | $Q+1$ |

[ENGAA, 2017S1Q15]
There are 100 students in Year 10.
Each student studies exactly one of French, German, and Spanish.
$X$ girls study French and there are $3 X$ girls in total.
$2 Y$ boys study German. There are 35 students studying Spanish of which $Y$ are boys.
Which of the following is an expression for the total number of students studying German?
(A) $X+2 Y$
(B) $X+Y+35$
(C) $X+3 Y-35$
(D) $2 X+2 Y$
(E) $2 X+Y-35$
(F) $2 X+3 Y-35$
(G) $2 X+Y+35$
[ENGAA, 2017S1Q16]
The radius of an iron-56 atom is $3.0 \times 10^{4}$ times greater than the radius of an iron- 56 nucleus. What is the value of $\frac{\text { density of an iron atom }}{\text { density of an iron nucleus }}$ ?
(A) $\left(3.0 \times 10^{4}\right)^{-3}$
(B) $\left(3.0 \times 10^{4}\right)^{-2}$
(C) $\left(3.0 \times 10^{4}\right)^{-1}$
(D) $\left(3.0 \times 10^{4}\right)^{1}$
(E) $\left(3.0 \times 10^{4}\right)^{2}$
(F) $\left(3.0 \times 10^{4}\right)^{3}$
[ENGAA, 2017S1Q17]
An exterior angle of a regular polygon with $n$ sides is $4^{\circ}$ larger than an exterior angle of a regular polygon with $(n+3)$ sides.
What is the value of $n$ ?
(A) 10
(B) 12
(C) 15
(D) 18
(E) 21
(F) 24
(G) 27
[ENGAA, 2017S1Q18]
Graph 1 shows how the displacement of one of the particles of a medium varies with time in seconds as a wave travels through the medium.


Graph 2 shows how the displacement varies with distance in metres at one time for the same wave.


Which expression gives the speed in $\mathrm{m} \mathrm{s}^{-1}$ of the wave?
(A) $\frac{4\left(x_{2}-x_{1}\right)}{3\left(t_{2}-t_{1}\right)}$
(B) $\frac{3\left(x_{2}-x_{1}\right)}{2\left(t_{2}-t_{1}\right)}$
(C) $\frac{2\left(x_{2}-x_{1}\right)}{\left(t_{2}-t_{1}\right)}$
(D) $\frac{8\left(x_{2}-x_{1}\right)}{3\left(t_{2}-t_{1}\right)}$
(E) $\frac{3\left(x_{2}-x_{1}\right)}{\left(t_{2}-t_{1}\right)}$
(F) $\frac{6\left(x_{2}-x_{1}\right)}{\left(t_{2}-t_{1}\right)}$

## [ENGAA, 2017S1Q19]

The bearing of a ship $R$ from a lighthouse $L$ is $220^{\circ}$.
A canoe $C$ is due North of $R$.
$C$ is the same distance from the ship and the lighthouse.
What is the bearing of $L$ from $C$ ?
(A) $070^{\circ}$
(B) $080^{\circ}$
(C) $090^{\circ}$
(D) $100^{\circ}$
(E) $140^{\circ}$
[ENGAA, 2017S1Q20]
A kettle is designed to work from a car's power socket. The kettle has a power rating of 150 W when a constant voltage of 12.0 V d.c. is applied across its element.
How much charge passes through the element of this kettle when the voltage of 12.0 V is applied across it for 20 minutes?
(A) 96 C
(B) 250 C
(C) 15000 C
(D) 36000 C
(E) 900000 C
(F) 2160000 C

## [ENGAA, 2017S1Q21]

The hands of a 12-hour analogue clock move continuously. When the time on the clock is 4:00, the angle between the minute hand and the hour hand is $120^{\circ}$.
What is the angle between the two hands at 4:40?
(A) $80^{\circ}$
(B) $100^{\circ}$
(C) $110^{\circ}$
(D) $120^{\circ}$
(E) $140^{\circ}$
[ENGAA, 2017S1Q22]
A freight train travelling on a straight horizontal track at $2.0 \mathrm{~m} \mathrm{~s}^{-1}$ collides with a passenger train travelling at $5.0 \mathrm{~m} \mathrm{~s}^{-1}$ in the opposite direction. Both trains immediately come to a complete stop on the track.
The freight train has three locomotives of 130 tonnes each and seven container wagons of 30 tonnes each. The passenger train has two locomotives of 70 tonnes each and a number of passenger carriages of 10 tonnes each.
How many passenger carriages does the passenger train have?
(A) 7
(B) 9
(C) 10
(D) 24
(E) 46
[ENGAA, 2017S1Q23]
A pet shop has 4 female rabbits and $x$ male rabbits for sale.
A customer buys 2 of the rabbits, chosen at random, and each rabbit is equally likely to be chosen.
The probability that both the chosen rabbits are male is $\frac{1}{3}$.
What is the value of $x$ ?
(A) 2
(B) 4
(C) 6
(D) 8
(E) 9
(F) 11
(G) 12
[ENGAA, 2017S1Q24]
Consider the following three statements about a parachutist of mass 72 kg falling vertically at a constant velocity of $5.0 \mathrm{~m} \mathrm{~s}^{-1}$ after the parachute has opened:

1 The parachutist has a constant kinetic energy of 1800 J .
2 The parachutist is losing gravitational potential energy at a rate of $3600 \mathrm{~J} \mathrm{~s}^{-1}$.
3 Air resistance and the force of gravity acting on the parachutist are a Newton's third law pair of forces.

Which of the statements is/are correct?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2017S1Q25]


The diagram shows a square with side of length $x \mathrm{~cm}$. A circle is drawn with centre $O$ which lies at the mid-point of one of the sides of the square. This side forms part of a diameter of the circle. The circle passes through two corners of the square as shown.
What is the area, in $\mathrm{cm}^{2}$, of the shaded part of the semi-circle?
(A) $(\pi-1) x^{2}$
(B) $\left(\frac{\pi-2}{2}\right) x^{2}$
(C) $\left(\frac{3 \pi-2}{2}\right) x^{2}$
(D) $\left(\frac{3 \pi-4}{4}\right) x^{2}$
(E) $\left(\frac{5 \pi-4}{4}\right) x^{2}$
(F) $\left(\frac{5 \pi-8}{8}\right) x^{2}$
[ENGAA, 2017S1Q26]
Two radioactive sources $X$ and $Y$ have half-lives of 3.0 hours and 2.0 hours respectively. The product of the decay of both of the sources is a stable isotope of the element $Z$.
Six hours ago a mixture contained the same number of atoms of both $X$ and $Y$, and no other atoms.
What fraction of the mixture is now made up of atoms of $Z$ ?
(A) $\frac{10}{16}$
(B) $\frac{11}{16}$
(C) $\frac{12}{16}$
(D) $\frac{13}{16}$
(E) $\frac{14}{16}$
(F) $\frac{15}{16}$
[ENGAA, 2017S1Q27]
A cylindrical hollow metal pipe is 16 cm long.
It has an external diameter of 10 cm and an internal diameter of 8 cm .
The density of the metal from which the pipe is made is 8 grams per $\mathrm{cm}^{3}$.
[diagram not to scale]


What is the mass of the pipe in grams?
(A) $8 \pi$
(B) $16 \pi$
(C) $18 \pi$
(D) $72 \pi$
(E) $128 \pi$
(F) $512 \pi$
(G) $1152 \pi$
(H) $4608 \pi$
[ENGAA, 2017S1Q28]
Car $X$ passes car $Y$ on a motorway.
Car $X$ is travelling at 1.5 times the speed of car $Y$.
The mass of $\operatorname{car} X$ is $\frac{4}{5}$ of the mass of $\operatorname{car} Y$.
How do the kinetic energies of the two cars compare?
(A) kinetic energy of car $X=0.90 \times$ kinetic energy of $\operatorname{car} Y$
(B) kinetic energy of car $X=0.96 \times$ kinetic energy of car $Y$
(C) kinetic energy of car $X=1.20 \times$ kinetic energy of $\operatorname{car} Y$
(D) kinetic energy of $\operatorname{car} X=1.44 \times$ kinetic energy of $\operatorname{car} Y$
(E) kinetic energy of $\operatorname{car} X=1.80 \times$ kinetic energy of $\operatorname{car} Y$

## Part B

## [ENGAA, 2017S1Q29]

Which one of the following is a simplification of

$$
1-\left(\frac{3+\sqrt{3}}{6-2 \sqrt{3}}\right)^{2}
$$

(A) $-\frac{3}{4}$
(B) $\frac{3}{4}$
(C) $-\frac{3}{4}-\frac{\sqrt{3}}{7}$
(D) $\frac{3}{4}-\frac{\sqrt{3}}{7}$
(E) $-\frac{3}{4}-\sqrt{3}$
(F) $\frac{3}{4}-\sqrt{3}$
(G) $-\frac{\sqrt{3}}{2}$
(H) $\frac{\sqrt{3}}{2}$
[ENGAA, 2017S1Q30]
The diagram shows a crane being used on a building site. The crane is perfectly balanced about $P$.


The load is now moved to the left by 5.0 m .
To keep the crane perfectly balanced about $P$, how far does the counterweight have to move, and in which direction?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 1.0 m to the left
(B) 1.0 m to the right
(C) 3.0 m to the left
(D) 3.0 m to the right
(E) 4.0 m to the left
(F) 4.0 m to the right
[ENGAA, 2017S1Q31]
$k$ is the smallest positive value of $x$ which is a solution to both the equations $2 \sin x+1=0$ and $2 \cos 2 x=1$.
How many values of $x$ in the range $0 \leq x \leq k$ are solutions to at least one of these equations?
(A) 0
(B) 2
(C) 3
(D) 4
(E) 8
[ENGAA, 2017S1Q32]
A ball is thrown vertically upwards in still air and is then caught at the same height when it comes back down.
Which velocity-time graph shows this complete motion?
(Take upwards as positive, and ignore air resistance.)

(A)

(D)

(G)

(B)

(E)

(C)
(F)

(H)
[ENGAA, 2017S1Q33]
Which of the following is a solution to the equation $3^{(2 x+1)}-6\left(3^{x}\right)=0$ ?
(A) $\log _{2} 3$
(B) $\log _{3} 2$
(C) 2
(D) $\log _{10} 2$
(E) $\frac{2}{3}$
[ENGAA, 2017S1Q34]
An aircraft is climbing at constant speed in a straight line at an angle of $10^{\circ}$ to the horizontal. Which statement about the resultant force on the aircraft is correct?
(A) It is parallel to its motion.
(B) It is perpendicular to its motion.
(C) It is zero.
(D) It is equal to its weight.
(E) It is equal to the drag acting on the aircraft.
[ENGAA, 2017S1Q35]
The diagram shows the outline of a keyhole consisting of three straight sides and an arc from a circle.
The sides $P Q$ and $R S$ are both 18 mm in length and when extended meet at the centre of the circle $O$ forming an angle of $\frac{\pi}{6}$ radians.
The longer arc from $Q$ to $R$ has length $22 \pi \mathrm{~mm}$.

[diagram not to scale]

What is the area, in $\mathrm{mm}^{2}$, of the keyhole as shaded in the diagram?
(A) $121 \pi+\frac{841}{4}$
(B) $121 \pi+\frac{841 \sqrt{3}}{4}$
(C) $132 \pi+225$
(D) $132 \pi+225 \sqrt{3}$
(E) $144 \pi+225$
(F) $144 \pi+225 \sqrt{3}$
[ENGAA, 2017S1Q36]
A horizontal, uniform bar of mass 60 kg is 4.0 m long and is pivoted at one end. The bar is held in equilibrium by a force $F$ at the other end of the bar, acting at an angle of $60^{\circ}$ to the horizontal.
[diagram not to scale]


Which expression gives the magnitude of $F$ in newtons?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) $\frac{30}{\sin 60^{\circ}}$
(B) $\frac{30}{\cos 60^{\circ}}$
(C) $\frac{60}{\sin 60^{\circ}}$
(D) $\frac{60}{\cos 60^{\circ}}$
(E) $\frac{300}{\sin 60^{\circ}}$
(F) $\frac{300}{\cos 60^{\circ}}$
(G) $\frac{600}{\sin 60^{\circ}}$
(H) $\frac{600}{\cos 60^{\circ}}$

## [ENGAA, 2017S1Q37]

It is given that $y=8^{p}$ and $z=\left(\frac{1}{2}\right)^{2 q}$ where $p$ and $q$ are real numbers.
Which of the following expressions is a simplification of $\log _{2}\left(\frac{y^{3}}{z^{2}}\right)$ ?
(A) $6 p-4 q$
(B) $6 p+4 q$
(C) $6 p-8 q$
(D) $6 p+8 q$
(E) $9 p-4 q$
(F) $9 p+4 q$
(G) $9 p-8 q$
(H) $9 p+8 q$
[ENGAA, 2017S1Q38]
A ball starts at a speed of $40.0 \mathrm{~m} \mathrm{~s}^{-1}$. The ball is subject to a constant deceleration of $14.4 \mathrm{~m} \mathrm{~s}^{-2}$ as it travels a distance of 20.0 m in a straight line. What is the final speed of the ball?
(A) $16.0 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $20.0 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $25.6 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $32.0 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $36.2 \mathrm{~m} \mathrm{~s}^{-1}$
(F) $39.3 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2017S1Q39]
The graph of the function $y=x^{3}+p x^{2}+q x+6$, where $p$ and $q$ are real constants, has a local maximum when $x=2$ and a local minimum when $x=4$. What are the values of $p$ and $q$ ?
(A) $p=-3$ and $q=-8$
(B) $p=-3$ and $q=8$
(C) $p=3$ and $q=-8$
(D) $p=-9$ and $q=24$
(E) $p=9$ and $q=24$
(F) $p=9$ and $q=-24$
[ENGAA, 2017S1Q40]
A block of mass 1.0 kg is at rest on a rough horizontal surface. The block is attached by a light inextensible string to a force meter. The other end of the force meter is attached by another light inextensible string via a frictionless pulley to a load of mass 1.0 kg . The block remains stationary.


What is the reading on the force meter?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 0.0 N
(B) 0.5 N
(C) 1.0 N
(D) 2.0 N
(E) 5.0 N
(F) 10 N
(G) 20 N

## [ENGAA, 2017S1Q41]

In triangle $P Q R, P Q=4 x \mathrm{~cm}, Q R=(8-3 x) \mathrm{cm}, \angle P Q R=60^{\circ}$.
What is the maximum value of the area, in $\mathrm{cm}^{2}$, of triangle $P Q R$ ?
(A) $\frac{8 \sqrt{3}}{3}$
(B) $\frac{16}{3}$
(C) $\frac{69 \sqrt{3}}{16}$
(D) $\frac{16 \sqrt{3}}{3}$
(E) $\frac{32}{3}$
(F) $\frac{32 \sqrt{3}}{3}$
[ENGAA, 2017S1Q42]
An apple of mass 100 g , growing on a tree, falls vertically from a height of 4.0 m above the ground. It hits the ground with a speed of $8.0 \mathrm{~m} \mathrm{~s}^{-1}$.
How much work does the apple do against resistive forces during its descent, before it hits the ground?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 0.80 J
(B) 3.6 J
(C) 4.0 J
(D) 7.2 J
(E) 8.0 J
[ENGAA, 2017S1Q43]
Given that $y=(2+3 x)^{6}$, what is the coefficient of $x^{3}$ in $\frac{\mathrm{d} y}{\mathrm{~d} x}$ ?
(A) 240
(B) 4320
(C) 4860
(D) 12960
(E) 19440
[ENGAA, 2017S1Q44]
A stone is fired vertically upwards at a speed of $13 \mathrm{~m} \mathrm{~s}^{-1}$ on a still day from the top of a 6.0 m high cliff. It then falls down and lands at the bottom of the cliff.


From when the stone passes the top of the cliff on the way down, how long does it take to reach the ground at the bottom of the cliff?
(air resistance can be ignored; gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 0.40 s
(B) $\frac{6.0}{6.5} \mathrm{~s}$
(C) 0.60 s
(D) $\sqrt{1.2} \mathrm{~s}$
(E) 1.3 s
(F) 2.0 s
(G) 2.5 s
(H) 3.0 s

## [ENGAA, 2017S1Q45]

A geometric progression has first term equal to 1 and common ratio $\frac{1}{2} \sin 2 x$.
The sum to infinity of the series is $\frac{4}{3}$.
Find the possible values of $x$ in the range $\pi \leq x \leq 2 \pi$.
(A) $\frac{13}{12} \pi, \frac{17}{12} \pi$
(B) $\frac{7}{6} \pi, \frac{4}{3} \pi$
(C) $\frac{7}{6} \pi, \frac{11}{6} \pi$
(D) $\frac{5}{4} \pi, \frac{7}{4} \pi$
(E) there are no values of $x$ in this range
[ENGAA, 2017S1Q46]
An archer fires an arrow of mass 0.024 kg vertically upwards from a bow.
The graph shows how the force of the bowstring on the arrow varies with distance as the arrow moves upwards.


The work done by the force of the bowstring is given by the area under the force-distance graph. When the arrow leaves the bow, what is the kinetic energy of the arrow, and what is the maximum height that it gains from this point?
(Air resistance can be ignored. The effect of gravity as the arrow is fired is negligible compared to the force of the bowstring. The gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$.)

|  | kinetic energy $/ \mathrm{J}$ | height $/ \mathrm{m}$ |
| :---: | :---: | :---: |
| (A) | 38.4 | 16 |
| (B) | 38.4 | 160 |
| (C) | 38.4 | 1600 |
| (D) | 38.4 | 16000 |
| (E) | 76.8 | 32 |
| (F) | 76.8 | 320 |
| (G) | 76.8 | 3200 |
| (H) | 76.8 | 32000 |

[ENGAA, 2017S1Q47]
The sequence of numbers $u_{1}, u_{2}, u_{3}, \ldots, u_{n}, \ldots$ is given by

$$
\begin{aligned}
u_{1} & =2 \\
u_{n+1} & =p u_{n}+3
\end{aligned}
$$

where $p$ is an integer.
The fourth term, $u_{4}$, is equal to -7 .
What is the value of $u_{1}+u_{2}+u_{3}+u_{4}$ ?
(A) -10
(B) -2
(C) -1
(D) 8
(E) 26
[ENGAA, 2017S1Q48]
A book of mass $m$ rests on a rough horizontal surface. The surface is now tilted as shown:


When the angle of tilt $\theta$ is $20^{\circ}$, the book slides down the slope at a constant speed.
What is the acceleration of the book down the slope when the angle of tilt is $25^{\circ}$ ?
(gravitational field strength $=g$ )
(A) $g\left(\cos 20^{\circ}-\sin 20^{\circ} \tan 5^{\circ}\right)$
(B) $g\left(\cos 20^{\circ}-\sin 20^{\circ} \tan 25^{\circ}\right)$
(C) $g\left(\cos 25^{\circ}-\sin 5^{\circ} \tan 20^{\circ}\right)$
(D) $g\left(\cos 25^{\circ}-\sin 25^{\circ} \tan 20^{\circ}\right)$
(E) $g\left(\sin 20^{\circ}-\cos 20^{\circ} \tan 5^{\circ}\right)$
(F) $g\left(\sin 20^{\circ}-\cos 20^{\circ} \tan 25^{\circ}\right)$
(G) $g\left(\sin 25^{\circ}-\cos 5^{\circ} \tan 20^{\circ}\right)$
(H) $g\left(\sin 25^{\circ}-\cos 25^{\circ} \tan 20^{\circ}\right)$
[ENGAA, 2017S1Q49]
Find the complete set of values of $x$ for which

$$
\frac{x^{3}-6 x^{2}+9 x-4}{x}>0
$$

(A) $x<0, x>4$
(B) $0<x<4$
(C) $0<x<1, x>4$
(D) $x<0,1<x<4$
(E) $x<1, x\rangle 4$
(F) $1<x<4$

## [ENGAA, 2017S1Q50]

A suitcase of mass $m$ is on a conveyor belt which moves upwards at a constant speed at an angle of $\theta$ to the horizontal. The coefficient of friction between the suitcase and the slope is $\mu$. The suitcase does not slip, even if angle $\theta$ is made slightly larger.
Which expression gives the friction force between the suitcase and the belt?
(gravitational field strength $=g$ )
(A) $\mu m g$
(B) $m g \sin \theta$
(C) $m g \cos \theta$
(D) $\mu m g \sin \theta$
(E) $\mu m g \cos \theta$
[ENGAA, 2017S1Q51]
The curve $y=\sin x$ is stretched by a scale factor of $\frac{1}{2}$ parallel to the $x$-axis and then translated by $\frac{\pi}{4}$ in the negative $x$-direction.
What is the equation of the new curve?
(A) $y=\sin \left(\frac{x}{2}-\frac{\pi}{4}\right)$
(B) $y=\sin \left(\frac{x}{2}+\frac{\pi}{4}\right)$
(C) $y=\sin \left(\frac{x}{2}-\frac{\pi}{8}\right)$
(D) $y=\sin \left(\frac{x}{2}+\frac{\pi}{8}\right)$
(E) $y=\sin \left(2 x-\frac{\pi}{4}\right)$
(F) $y=\sin \left(2 x+\frac{\pi}{4}\right)$
(G) $y=\sin \left(2 x-\frac{\pi}{2}\right)$
(H) $y=\sin \left(2 x+\frac{\pi}{2}\right)$
[ENGAA, 2017S1Q52]
The graph shows how the horizontal force on a tennis ball of mass $m$ varies during a shot in a tennis match. The ball is initially travelling horizontally toward the racket with speed $u$ and leaves the racket horizontally travelling in the opposite direction with speed $v$.


Which expression gives the magnitude of the momentum of the ball as it leaves the racket?
(A) $F\left(t_{2}-t_{1}\right)$
(B) $F\left(t_{2}-t_{1}\right)-m u$
(C) $F\left(t_{2}-t_{1}\right)+m u$
(D) $m v-m u$
(E) $F t_{2}-m u$
[ENGAA, 2017S1Q53]
The equations of two straight lines are $y=3+\left(2 p^{2}-p\right) x$ and $y=7+(p-2) x$, where $p$ is a real constant.

For certain values of $p$, the two lines are perpendicular.
Which of the following numbers is closest to the greatest such value of $p$ ?
(A) 2.00
(B) 1.75
(C) 1.50
(D) 1.00
(E) -0.25
(F) -0.50
[ENGAA, 2017S1Q54]
The acceleration versus time graph is for a ball dropped from rest, falling vertically and bouncing on the ground.


The time of contact with the ground can be ignored.
What is the speed of the ball immediately after hitting the ground for the first time, and what is the maximum height reached by the ball after the first bounce?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )

|  | speed $/ \mathrm{m} \mathrm{s}^{-1}$ | height $/ \mathrm{m}$ |
| :---: | :---: | :---: |
| (A) | 4.00 | 0.80 |
| (B) | 4.00 | 1.25 |
| (C) | 5.00 | 0.80 |
| (D) | 5.00 | 1.25 |
| (E) | 8.00 | 3.20 |

## ENGAA 2017 S2



## TIME ALLOWED: 40 MINUTES

This paper is Section 2 of 2 .
This question paper contains 17 multiple choice questions arranged into 4 groups. Some questions are connected to other questions.

Please complete this section in pencil. Your working or reasoning must be written in the spaces provided in this question paper and may be taken into account in the assessment of your work.

Your final choice of answer option must be recorded by shading a circle in the spaces provided on the inside front cover. For each question shade the one option you consider correct. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 17 questions. The number of marks each question is worth is indicated. In total 38 marks are available.

You can use the blank pages for rough working or notes, but no extra paper is allowed. Only answers in the spaces indicated in the paper will be marked.

Calculators may be used in this section. Please record your calculator model in the box below:

## Calculator model

[ENGAA, 2017S2Q1]
A ball of mass $m=0.5 \mathrm{~kg}$ is at rest a distance $d$ above the flat floor of a spacecraft.
Installed in the floor is an artificial gravity generator which produces a field at right angles to the floor, directed towards the floor. There is no air in the spacecraft.
The generator is switched on at time $t=0 \mathrm{~s}$ and produces a field $g$ that increases linearly with time, such that $g=0.4 t \mathrm{~m} \mathrm{~s}^{-2}$. The artificial gravity is the only force experienced by the ball.
[ENGAA, 2017S2Q1(a)]
Assuming that the ball does not hit the floor within the first second of motion, which of these graphs represents the speed of the ball plotted against time?

## SHOW YOUR REASONING IN THE SPACE PROVIDED ON THE NEXT PAGE.



(A)

(D)

(B)

(E)
[ENGAA, 2017S2Q1(b)]
Which of these expressions gives the time taken for the ball to first hit the floor?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) $(15 d)^{\frac{1}{3}}$
(B) $(5 d)^{\frac{1}{3}}$
(C) $(5 d)^{\frac{1}{2}}$
(D) $\left(\frac{15 d}{2}\right)^{\frac{1}{3}}$
(E) $\left(\frac{5 d}{2}\right)^{\frac{1}{3}}$
[ENGAA, 2017S2Q1(c)]
The ball bounces and hits the floor repeatedly. Which of these graphs might represent the position of the ball plotted against time?

## SHOW YOUR REASONING IN THE SPACE PROVIDED ON THE NEXT PAGE.

P

Q


(A) P only
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only
[ENGAA, 2017S2Q1(d)]
Force is usually measured in Newtons (N). Given that $F=m a$, which of the following is an alternative unit for force?

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) $\mathrm{kg} \mathrm{s} \mathrm{m}^{-2}$
(B) $\mathrm{kg}^{-1} \mathrm{~m}^{-1} \mathrm{~s}^{2}$
(C) $\mathrm{kg} \mathrm{m} \mathrm{s}^{-2}$
(D) $\mathrm{N} \mathrm{kg}^{-1} \mathrm{~m}^{-1} \mathrm{~s}^{2}$
(E) $\mathrm{N}^{-1} \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}$
[ENGAA, 2017S2Q1(e)]
Air is now injected into the spacecraft, creating air resistance. The drag force $D$ on the ball is given by

$$
D=\frac{1}{2} X \rho v^{2} A
$$

where $\rho$ is the air density, $v$ is the ball's speed, $A$ is its cross-sectional area and $X$ is an unknown parameter.
What are the units of $X$ ?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) $\mathrm{m} \mathrm{s}^{-2}$
(B) $\mathrm{m} \mathrm{s}^{-1}$
(C) $\mathrm{kg}^{-1} \mathrm{~m}^{-1} \mathrm{~s}^{2}$
(D) $\mathrm{kg} \mathrm{m} \mathrm{s}^{-2}$
(E) $X$ has no units
[ENGAA, 2017S2Q2]
The graph shows the current against voltage characteristics of four different electronic devices $W, X, Y$ and $Z$. One of the devices is an $8 \Omega$ resistor and one is a filament lamp rated 9 W at 6 V . You may assume that the filament lamp does not 'blow' in the context of this question.

[ENGAA, 2017S2Q2(a)]
Which of the devices is the resistor?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) device $W$
(B) device $X$
(C) device $Y$
(D) device $Z$
[ENGAA, 2017S2Q2(b)]
Which of the devices is the filament lamp?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) device $W$
(B) device $X$
(C) device $Y$
(D) device $Z$
[ENGAA, 2017S2Q2(c)]
The filament lamp and the resistor are connected in parallel to a 6.0 V power supply with negligible internal resistance.
Approximately what current is drawn from the supply?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) 0.75 A
(B) 1.5 A
(C) 1.83 A
(D) 2.25 A
(E) 2.42 A
[ENGAA, 2017S2Q2(d)]
The previous circuit is disconnected, and then devices $W$ and $Y$ are connected in series to the same 6.0 V power supply.
Which one of the following statements about the new circuit must be correct?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) Devices $W$ and $Y$ dissipate equal power.
(B) Devices $W$ and $Y$ have equal voltages across them.
(C) Equal currents flow through devices $W$ and $Y$.
(D) The power supply delivers more power than it would if device $W$ or device $Y$ were connected alone.
(E) The power supply delivers more power than it would if devices $W$ and $Y$ were connected in parallel.
[ENGAA, 2017S2Q2(e)]
In the new circuit, approximately what power is dissipated by device $W$ ?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) 0.5 W
(B) 1.0 W
(C) 1.5 W
(D) 2.0 W
(E) 2.5 W
[ENGAA, 2017S2Q3]
Fig. 3(a) shows the results of an experiment in which a 0.5 m length of elastic cord has been extended by a force with a corresponding extension. The cord fails at point $Q$ by fracture.


Fig. 3(a)
[ENGAA, 2017S2Q3(a)]
The elastic behaviour of a material can often be described by Hooke's law, which is given by the equation $F=k x$, where $x$ is extension, $F$ is force and $k$ is an elastic constant which depends on the material studied.
Which of the following statements correctly describes the behaviour of the cord?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) no Hooke's law behaviour and fracture at a strain of 0.05
(B) Hooke's law behaviour up to $P$ and fracture at a strain of 0.05
(C) Hooke's law behaviour up to $Q$ and fracture at a strain of 0.05
(D) Hooke's law behaviour up to $P$ and fracture at a strain of 0.1
(E) Hooke's law behaviour up to $Q$ and fracture at a strain of 0.1
[ENGAA, 2017S2Q3(b)]
What is the work done $U$ in stretching this 0.5 m length of elastic cord by 0.05 m (to 2 significant figures)?

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) 0.15 J
(B) 0.30 J
(C) 0.60 J
(D) 2.0 J
(E) 6.0 J
[ENGAA, 2017S2Q3(c)]
An unstretched 0.25 m length of the same type of cord is used in a catapult to propel a mass $m$, as illustrated in Fig. 3(b).


Fig. 3(b)
What is the maximum speed $V_{\max }$ at which the mass can be propelled (where $U$ is the work done calculated in part b))?

SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) $\sqrt{m U}$
(B) $\sqrt{\frac{U}{m}}$
(C) $\sqrt{\frac{2 U}{m}}$
(D) $\sqrt{2 m U}$
(E) $\sqrt{\frac{U}{2 m}}$

## [ENGAA, 2017S2Q3(d)]

Two parallel 0.25 m lengths of the elastic cord are used in the catapult as shown in Fig. 3(c)


Fig. 3(c)
What is the maximum speed at which the mass can now be propelled?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) $\frac{1}{2} V_{\max }$
(B) $\frac{1}{\sqrt{2}} V_{\max }$
(C) $V_{\max }$
(D) $\sqrt{2} V_{\max }$
(E) $2 V_{\max }$
[ENGAA, 2017S2Q4]
The diagram shows the geometry for two slit diffraction of light, with the slits on the left and the viewing screen on the right; $d=800 \mathrm{~nm}, L=1 \mathrm{~m}$ and the speed of light is $3 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$.

[ENGAA, 2017S2Q4(a)]
The pair of slits is illuminated by laser light of wavelength $\lambda=600 \mathrm{~nm}$.
Which of the following statements are correct (where $n$ is an integer)?
1 Points of maximum brightness on the screen occur where the distances $r_{1}$ and $r_{2}$ differ by $n \lambda$.

2 Points of maximum brightness on the screen occur where the distances $r_{1}$ and $r_{2}$ differ by $\left(n+\frac{1}{2}\right) \lambda$.
3 Points of minimum brightness on the screen occur where the distances $r_{1}$ and $r_{2}$ differ by $\left(n+\frac{1}{2}\right) \lambda$

4 For a diffraction pattern to appear, the light from the two slits must be coherent.
5 The maxima are all of equal brightness.

## SHOW YOUR REASONING IN THE SPACE PROVIDED ON THE NEXT PAGE.

(A) 1 and 4 only
(B) 1, 3 and 4 only
(C) 1, 3 and 5 only
(D) 1, 4 and 5 only
(E) 2 and 4 only
[ENGAA, 2017S2Q4(b)]
A thin piece of transparent material, thickness 300 nm and in which the speed of light is half that in air, is now placed immediately behind one of the two slits.
Which one of the following statements is correct?

## SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.

(A) The diffraction pattern is unchanged.
(B) The diffraction pattern disappears because the light from the two slits is no longer coherent.
(C) The diffraction pattern disappears because the light from the two slits is no longer in phase.
(D) The complete diffraction pattern shifts in the $y$ direction.
(E) Each maximum is replaced by two because the material alters the wavelength of the light coming from it.
[ENGAA, 2017S2Q4(c)]
A radio transmitter transmits a signal at 600 MHz to a receiver 1 km away. In an attempt to double the strength of the signal at the receiver, a second antenna is added at the transmitter, 1 m away alongside the original one, and fed by the same signal. It is suggested that, instead of improving reception, diffraction effects might actually make reception much worse.
Which of the following statements is correct?
SHOW YOUR REASONING IN THE SPACE PROVIDED BELOW.
(A) Diffraction effects would not be a problem because light and radio are different types of wave.
(B) Diffraction effects would not be a problem because the waves are too low frequency to produce diffraction effects.
(C) Diffraction effects would not be a problem as the transmitting antennas are too far apart to produce diffraction effects.
(D) Diffraction effects will occur, but the maxima would be sufficiently close together that this would not be a problem.
(E) Diffraction effects could be a problem because the distance between the transmitting antennas is comparable to the wavelength.

## ENGAA 2018 S1



## TIME ALLOWED: 80 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 80 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.
Part A Mathematics and Physics (28 questions)
Part B Advanced Mathematics and Advanced Physics (26 questions)
This paper contains 54 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 54 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators may NOT be used.

## Part A

[ENGAA, 2018S1Q1]
A group of drivers, consisting of 200 women and 300 men, was asked if they passed their driving test at the first attempt.
Altogether 167 of the group said they passed at the first attempt.
Of the women, 143 said they did not pass at the first attempt.
How many of the men said they passed at the first attempt?
(A) 10
(B) 24
(C) 33
(D) 57
(E) 110
(F) 133
(G) 157
[ENGAA, 2018S1Q2]
An unstable nucleus X becomes a stable nucleus Y after a succession of decays, during which a total of 5 alpha particles and 2 beta ( $\beta^{-}$) particles are emitted.
How many fewer protons does nucleus Y contain than nucleus X ?
(A) 6
(B) 8
(C) 10
(D) 12
(E) 14
(F) 16
(G) 18
(H) 20
[ENGAA, 2018S1Q3]
A cuboid has sides of length $x, \sqrt{2} x$ and $2 x$, measured in cm .
The volume, in $\mathrm{cm}^{3}$, of the cuboid is numerically equal to twice the total surface area, in $\mathrm{cm}^{2}$, of the cuboid.
What is the value of $x$ ?
(A) 10
(B) $6+2 \sqrt{2}$
(C) 5
(D) $3+\sqrt{2}$
(E) $\frac{5}{2}$
(F) $\frac{3}{2}+\frac{1}{2} \sqrt{2}$

## [ENGAA, 2018S1Q4]

The diagram shows three resistors $R_{1}, R_{2}$ and $R_{3}$ connected in series with a battery of constant voltage. The resistance of each resistor and the corresponding current are also shown.


Resistor $R_{3}$ is now removed and the circuit is reconnected.
What is the new current in the circuit?
(A) 0.20 A
(B) 0.22 A
(C) 0.33 A
(D) 0.40 A
(E) 0.50 A
(F) 2.0 A
(G) 6.0 A
[ENGAA, 2018S1Q5]
The line joining the points with coordinates $(p, p-1)$ and $(1-p, 2 p)$ is parallel to the line with equation $2 x+3 y+1=0$.
What is the value of $p$ ?
(A) -1
(B) $-\frac{1}{7}$
(C) $\frac{1}{9}$
(D) $\frac{1}{8}$
(E) 1
(F) $\frac{5}{4}$
(G) 2
(H) 5
[ENGAA, 2018S1Q6]
When travelling in a vacuum, visible light has a wavelength between 400 nm and 700 nm . The speed of light in a vacuum is $3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$.
What can be concluded about ultraviolet radiation from this information?
(A) It has a maximum frequency of $2.7 \times 10^{14} \mathrm{~Hz}$
(B) It has a maximum frequency of $4.3 \times 10^{14} \mathrm{~Hz}$
(C) It has a maximum frequency of $7.5 \times 10^{14} \mathrm{~Hz}$
(D) It has a maximum frequency of $1.0 \times 10^{15} \mathrm{~Hz}$
(E) It has a minimum frequency of $2.7 \times 10^{14} \mathrm{~Hz}$
(F) It has a minimum frequency of $4.3 \times 10^{14} \mathrm{~Hz}$
(G) It has a minimum frequency of $7.5 \times 10^{14} \mathrm{~Hz}$
(H) It has a minimum frequency of $1.0 \times 10^{15} \mathrm{~Hz}$
[ENGAA, 2018S1Q7]
A rectangle $P Q R S$ is drawn inside a circle, with its vertices on the circumference of the circle.

[diagram not to scale]
The ratio of the length of $P Q$ to the length of $Q R$ is 2:1.
The area of the rectangle $P Q R S$ is $96 \mathrm{~cm}^{2}$.
What is the radius, in cm , of the circle?
(A) $\sqrt{6}$
(B) 3
(C) $3 \sqrt{2}$
(D) $2 \sqrt{15}$
(E) $4 \sqrt{6}$
(F) 12
(G) $12 \sqrt{2}$
(H) $8 \sqrt{15}$
[ENGAA, 2018S1Q8]
A filament lamp working at its operating voltage converts electrical energy at a rate of 100 W . The lamp has an efficiency of 5.0\%.

How much energy is wasted by the lamp in 10 minutes?
(A) 50 J
(B) 950 J
(C) 1000 J
(D) 3000 J
(E) 57000 J
(F) 60000 J

$320 \mathrm{~cm}^{3}$

[diagram not to scale]

At a cinema, drinks are sold in regular and large sizes.
The cups for these are mathematically similar.
The ratio of the heights of the cups and the ratio of the depths of the drinks are both $4: 5$.
The volume of drink in a regular size cup is $320 \mathrm{~cm}^{3}$.
What is the volume, in $\mathrm{cm}^{3}$, of drink in a large size cup?
(A) 384
(B) 400
(C) 500
(D) 576
(E) 625
(F) 640
[ENGAA, 2018S1Q10]
The potential difference across the motor in an electric car is 400 V and the current in the motor is 1250 A .
The car accelerates along a horizontal road from rest for 4.0 s .
The efficiency of the overall system is $45 \%$.
What is the kinetic energy of the car at the end of the 4.0 s ?
(Ignore energy losses due to air resistance and due to friction between the tyres and the road.)
(A) 225000 J
(B) 500000 J
(C) 900000 J
(D) 1250000 J
(E) 2000000 J
[ENGAA, 2018S1Q11]
The straight lines

$$
\begin{aligned}
5 x+2 y & =20 \\
y & =3 x-23 \\
x & =0
\end{aligned}
$$

enclose a region with area $K$ square units.
What is the value of $K$ ?
(A) 39
(B) 78
(C) 99
(D) 129
(E) 198
(F) 258
[ENGAA, 2018S1Q12]
The momentum of a small object moving in a straight line is $24 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$ and its kinetic energy is 96 J .
What is the mass of the object?
(A) 3.0 kg
(B) 4.0 kg
(C) 6.0 kg
(D) 8.0 kg
(E) 12 kg
[ENGAA, 2018S1Q13]
A scale model of a cylindrical pillar is to be made.
The full-sized pillar has a volume of $12 \pi \mathrm{~m}^{3}$.
The model will use a length scale of $1: 40$.
The model is to be a solid cylinder made of a plastic which has a density of $\frac{4}{3} \mathrm{~g} \mathrm{~cm}^{-3}$.
What is the mass of the model in grams?
(A) $\frac{9}{640} \pi$
(B) $\frac{1}{40} \pi$
(C) $40 \pi$
(D) $\frac{1125}{8} \pi$
(E) $250 \pi$
(F) $10000 \pi$
(G) $225000 \pi$
(H) $400000 \pi$
[ENGAA, 2018S1Q14]
A radioactive isotope decays in a single step to a stable isotope.
A radiation detector is placed very near to a sample of the radioactive isotope in a laboratory. The count rate on the detector changes as time elapses. The graph shows how the measured count rate changes with time.


What is the background count rate and what is the half-life of the isotope?

|  | background count rate <br> / counts per minute | half-life of isotope <br> / minutes |
| :---: | :---: | :---: |
| (A) | 20 | 40 |
| (B) | 20 | 50 |
| (C) | 20 | 60 |
| (D) | 20 | 65 |
| (E) | 120 | 40 |
| (F) | 120 | 50 |
| (G) | 120 | 60 |
| (H) | 120 | 65 |

[ENGAA, 2018S1Q15]

[diagram not to scale]
$P Q R S T$ is a regular pentagon.
$R S U$ is an equilateral triangle.
What is the size of angle STU?
(A) $48^{\circ}$
(B) $54^{\circ}$
(C) $60^{\circ}$
(D) $66^{\circ}$
(E) $84^{\circ}$
[ENGAA, 2018S1Q16]
A rock falling vertically experiences an air resistance force of 12 N at an instant when its acceleration is $2.0 \mathrm{~m} \mathrm{~s}^{-2}$ downwards.

What is the mass of the rock?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 1.0 kg
(B) 1.2 kg
(C) 1.5 kg
(D) 6.0 kg
(E) 10 kg
(F) 12 kg
(G) 15 kg
(H) 60 kg
[ENGAA, 2018S1Q17]
The original price of an item is $p$.
The price is increased by $125 \%$.
The increased price is then decreased by $40 \%$ to $q$.
The relationship between $p$ and $q$ can be expressed as $m p=q$.
What is the value of $m$ ?
(A) $\frac{7}{20}$
(B) $\frac{17}{20}$
(C) $\frac{27}{20}$
(D) $\frac{33}{20}$
(E) $\frac{37}{20}$
[ENGAA, 2018S1Q18]
A transverse wave with an amplitude of 4.0 cm and a frequency of 10 Hz travels along a rope at a speed of $2.4 \mathrm{~m} \mathrm{~s}^{-1}$.

What is the total distance travelled by a particle in the rope in a time of 20 s ?
(A) 2.4 m
(B) 4.8 m
(C) 8.0 m
(D) 16 m
(E) 32 m
(F) 48 m
[ENGAA, 2018S1Q19]
$Q$ is 5 km away from $P$ on a bearing of $065^{\circ}$.
$R$ is 5 km away from $Q$ on a bearing of $155^{\circ}$.
What is the bearing of $P$ from $R$ ?
(A) $070^{\circ}$
(B) $110^{\circ}$
(C) $225^{\circ}$
(D) $270^{\circ}$
(E) $290^{\circ}$
(F) $315^{\circ}$
(G) $335^{\circ}$
[ENGAA, 2018S1Q20]
A student places a measuring cylinder on a balance. She pours a volume $V$ of water into the measuring cylinder, and finds that the mass of the measuring cylinder and water together is 290 g .
She then empties the measuring cylinder and dries it before putting it back on the balance.
She now pours the same volume $V$ of olive oil into the measuring cylinder, and finds that the mass of the measuring cylinder and olive oil together is 270 g .
What is the mass of the measuring cylinder?
(densities: olive oil $=0.90 \mathrm{~g} \mathrm{~cm}^{-3}$; water $=1.0 \mathrm{~g} \mathrm{~cm}^{-3}$ )
(A) 18 g
(B) 20 g
(C) 90 g
(D) 180 g
(E) 200 g
[ENGAA, 2018S1Q21]

[diagram not to scale]
The line segment $R T$ is a tangent at the point $S$ to a circle with centre $O$. $Q$ and $P$ are points on the circumference of the circle such that $Q S=Q P$.
Angle $P S T=75^{\circ}$.
What is the size of angle $Q S O$ ?
(A) $15^{\circ}$
(B) $30^{\circ}$
(C) $37.5^{\circ}$
(D) $45^{\circ}$
(E) $52.5^{\circ}$
(F) $60^{\circ}$
(G) $67.5^{\circ}$
(H) $75^{\circ}$
[ENGAA, 2018S1Q22]
A skydiver of weight 1000 N falls vertically.
The distance-time graph for the skydiver is shown below.


The air resistance $F$ (in N ) acting on the skydiver travelling at velocity $v$ (in $\mathrm{m} \mathrm{s}^{-1}$ ) is given by the equation

$$
F=k v^{2}
$$

where $k$ (in $\mathrm{N} \mathrm{m}^{-2} \mathrm{~s}^{2}$ ) is a constant.
What is the numerical value of $k$ for the skydiver?
(A) 0.050
(B) 0.40
(C) 0.63
(D) 2.5
(E) 20
[ENGAA, 2018S1Q23]

[diagram not to scale]
The vertical height $h \mathrm{~cm}$ of an isosceles triangle is 3 cm longer than the base length of $b \mathrm{~cm}$.
The sloping side is of length $s \mathrm{~cm}$.
The area of the triangle is $14 \mathrm{~cm}^{2}$.
There is one value of $s$ which satisfies these conditions.
Within which range does this value of $s$ lie?
(A) $5<s<6$
(B) $6<s<7$
(C) $7<s<8$
(D) $8<s<9$
(E) $9<s<10$
(F) $10<s<11$
[ENGAA, 2018S1Q24]
A neutron is absorbed by a uranium-235 ( $\left.{ }_{92}^{235} \mathrm{U}\right)$ nuclide.
The resulting nuclide undergoes fission to produce a bromine- $88\left({ }_{35}^{88} \mathrm{Br}\right)$ nuclide, a lanthanum145 nuclide and some neutrons.
The lanthanum- 145 nuclide is radioactive and emits a beta ( $\beta^{-}$) particle.
How many neutrons are emitted in the fission reaction and how many protons are there in the nuclide formed by the decay of lanthanum-145?

|  | neutrons | protons |
| :---: | :---: | :---: |
| (A) | 2 | 55 |
| (B) | 2 | 56 |
| (C) | 2 | 57 |
| (D) | 2 | 58 |
| (E) | 3 | 55 |
| (F) | 3 | 56 |
| (G) | 3 | 57 |
| (H) | 3 | 58 |

[ENGAA, 2018S1Q25]
The first five terms of a sequence in order are:

$$
\begin{array}{lllll}
2 & 17 & 42 & 77 & 122
\end{array}
$$

The $n$th term of this sequence is $p n^{2}+q$ where $p$ and $q$ are integers.
What is the value of $\frac{p-q}{p+q}$ ?
(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{23}{17}$
(E) $\frac{13}{7}$
(F) 2
(G) 4
(H) 14
[ENGAA, 2018S1Q26]
The diagram shows a circuit containing a battery and three identical resistors $X, Y$ and $Z$.


The total power supplied by the battery is 18 W .
What is the power dissipated as heat in resistor $X$ ?
(A) 1.5 W
(B) 2.0 W
(C) 3.0 W
(D) 4.5 W
(E) 6.0 W
(F) 8.0 W
(G) 12 W
[ENGAA, 2018S1Q27]
A bag contains 6 red and 6 green sweets. The sweets are identical apart from their colour. A child takes a sweet at random from the bag.
If the sweet is red, the child stops taking sweets.
If the sweet is green, it is not replaced and the child takes another sweet.
This continues until a red sweet is taken at which point the child stops taking sweets.
What is the probability that the child takes more green sweets than red sweets?
(A) $\frac{3}{22}$
(B) $\frac{5}{22}$
(C) $\frac{3}{11}$
(D) $\frac{1}{2}$
(E) $\frac{8}{11}$
(F) $\frac{17}{22}$
[ENGAA, 2018S1Q28]
Three detectors $X, Y$ and $Z$ are separated by large distances.
Each of the detectors records a seismic wave from the same earthquake whose epicentre (source) is very close to the surface of the Earth.
The wave travels out from the epicentre at $4.0 \mathrm{~km} \mathrm{~s}^{-1}$.
Detectors $X$ and $Y$ start to detect the wave at the same time, but detector $Z$ starts to detect it one minute later.
Which of the following statements must be correct?
1 The epicentre is at the midpoint of the line $X Y$.
$2 \quad Z$ is equidistant from $X$ and $Y$.
$3 \quad Z$ is no more than 240 km away from $X$ and from $Y$.
(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3

## Part B

[ENGAA, 2018S1Q29]
Curve $C$ has equation $y=9-x^{2}$.
Line $L$ has equation $y=5$.
What is the area enclosed between $C$ and $L$ ?
(A) $\frac{32}{3}$
(B) $\frac{62}{3}$
(C) $\frac{92}{3}$
(D) $\frac{122}{3}$
(E) $\frac{152}{3}$
[ENGAA, 2018S1Q30]
An aircraft moves from rest with uniform acceleration along a horizontal runway. After travelling 1600 m it reaches a speed of $80 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the acceleration of the aircraft?
(A) $0.025 \mathrm{~m} \mathrm{~s}^{-2}$
(B) $0.050 \mathrm{~m} \mathrm{~s}^{-2}$
(C) $0.10 \mathrm{~m} \mathrm{~s}^{-2}$
(D) $0.50 \mathrm{~m} \mathrm{~s}^{-2}$
(E) $2.0 \mathrm{~m} \mathrm{~s}^{-2}$
(F) $4.0 \mathrm{~m} \mathrm{~s}^{-2}$
(G) $10 \mathrm{~m} \mathrm{~s}^{-2}$
(H) $20 \mathrm{~m} \mathrm{~s}^{-2}$

## [ENGAA, 2018S1Q31]

How many solutions of the equation $2 \sin ^{3} \theta=\sin \theta$ lie in the interval $-\frac{\pi}{2} \leq \theta \leq \pi$ ?
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6
(F) 7
[ENGAA, 2018S1Q32]
The diagram represents a mass that is moving in a straight line at constant speed up a slope of constant gradient.


Which statement about the forces acting on the mass must be correct?
(A) All the forces acting on the mass are equal in magnitude.
(B) Only three forces act on the mass.
(C) The force of friction on the mass is equal to the driving force.
(D) The weight of the mass acts in the opposite direction to the contact force.
(E) There is no air resistance acting on the mass.
(F) There is no resultant force acting on the mass.

## [ENGAA, 2018S1Q33]

The line $y=x+k$, where $k$ is a constant, is a tangent to the curve $y=3 x^{2}-2 x+1$.
What is the value of $k$ ?
(A) -2
(B) -1
(C) $\frac{1}{4}$
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$
(F) $\frac{3}{4}$
(G) 1
(H) 2
[ENGAA, 2018S1Q34]
The diagram shows four objects $W, X, Y$ and $Z$, of masses $3.0 \mathrm{~kg}, 4.0 \mathrm{~kg}, 6.0 \mathrm{~kg}$ and 2.0 kg respectively, connected by light, inextensible rods.
The objects are pulled along a smooth, horizontal surface by a constant force of 30 N in the direction indicated.


What is the tension in the rod connecting $X$ and $Y$ ?
(A) 8.0 N
(B) 10 N
(C) 12 N
(D) 14 N
(E) 16 N

## [ENGAA, 2018S1Q35]

A sector $S$ of a circle has area $10 \pi \mathrm{~cm}^{2}$. The angle of sector $S$ is increased by $\frac{\pi}{20}$ radians to form sector $T$. The total area of sector $T$ is $\frac{25}{2} \pi \mathrm{~cm}^{2}$.
What is the total arc length, in cm , of sector $T$ ?
(A) $\frac{9 \sqrt{5}}{10} \pi$
(B) $\frac{5 \sqrt{2}}{4} \pi$
(C) $2 \pi$
(D) $\frac{5}{2} \pi$
[ENGAA, 2018S1Q36]
An object of mass 40 kg is placed on a uniform, horizontal plank of mass 10 kg between two supports $X$ and $Y$ as shown in the diagram.


What is the contact force at $X$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 15 N
(B) 35 N
(C) 150 N
(D) 250 N
(E) 300 N
(F) 350 N
(G) 375 N
[ENGAA, 2018S1Q37]
In a particular arithmetic progression:
1 the 13th term is six times the 1st term
2 the 11th term is 1 less than twice the 5th term
What is the 3rd term of the progression?
(A) -14.5
(B) -11
(C) $\frac{29}{19}$
(D) 3.5
(E) 11
(F) 14.5
[ENGAA, 2018S1Q38]
A block of mass $m$ slides a distance $l$ down a slope that is inclined at angle $\theta$ to the horizontal, as shown:


The block experiences a friction force of $k W \sin \theta$, where $W$ is the weight of the block and $k$ is a constant.
The block starts from rest at the top of the slope and slides down a distance $l$ to the bottom, where its potential energy is zero.
What fraction of the initial potential energy at the top has become kinetic energy as the block reaches the bottom?
(A) $k$
(B) $1-k$
(C) $k \sin \theta$
(D) $1-k \sin \theta$
(E) $k \tan \theta$
(F) $1-k \tan \theta$
[ENGAA, 2018S1Q39]
The first three terms of a geometric progression, whose terms are all greater than zero, are $(p-2),(2 p+2)$ and $(5 p+14)$.
What is the fifth term of the progression?
(A) 324
(B) 486
(C) 1250
(D) 1458
(E) 3888
[ENGAA, 2018S1Q40]
An object $X$ of mass 2.0 kg is initially moving at a speed of $4.5 \mathrm{~m} \mathrm{~s}^{-1}$ on a smooth, horizontal surface.
A 5.0 N force is applied to $X$ in the direction of its motion for 3.0 seconds.
A short time later it collides head on with, and sticks to, a stationary object $Y$ of mass 3.0 kg . What is the speed of $X$ and $Y$ as they move off together after the collision?
(A) $1.8 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $3.0 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $3.6 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $4.8 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $5.4 \mathrm{~m} \mathrm{~s}^{-1}$

## [ENGAA, 2018S1Q41]

Evaluate

$$
\log _{2}\left(\frac{5}{4}\right)+\log _{2}\left(\frac{6}{5}\right)+\log _{2}\left(\frac{7}{6}\right)+\cdots+\log _{2}\left(\frac{64}{63}\right)
$$

(A) -2
(B) 3
(C) 4
(D) 6
(E) $\log _{2}(3!)$
(F) $\log _{2} 60$
[ENGAA, 2018S1Q42]
A ball of mass 0.20 kg is thrown vertically downwards at an initial speed of $4.0 \mathrm{~m} \mathrm{~s}^{-1}$ and travels a distance of 0.45 m to the ground.
The ball hits the ground, and rebounds with an initial speed of $2.0 \mathrm{~m} \mathrm{~s}^{-1}$.
How much energy does the ball lose in the bounce?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; air resistance can be ignored)
(A) 0.10 J
(B) 0.40 J
(C) 0.50 J
(D) 0.90 J
(E) 1.2 J
(F) 1.6 J
(G) 2.1 J
[ENGAA, 2018S1Q43]
Circle $C$ has equation $(x+3)^{2}+(y-2)^{2}=5$.
The length of the tangent from the circle $C$ to the point $P$ is $5 \sqrt{3}$.
What is the shortest distance from $P$ to $C$ ?
(A) $5 \sqrt{3}$
(B) $5 \sqrt{3}+\sqrt{5}$
(C) $3 \sqrt{5}$
(D) 5
(E) 10
[ENGAA, 2018S1Q44]
Two solid spheres $X$ and $Y$ have masses $m$ and $2 m$ respectively. They travel in opposite directions towards each other along the same line with speeds $v$ and $2 v$ respectively and collide head on.
The graph shows the variation of velocity with time for sphere $X$ before, during, and after the collision.


Which sketch shows the variation of velocity with time for sphere $Y$ ?

(A)

(D)

(B)

(E)

(C)

(F)
[ENGAA, 2018S1Q45]
The points $A(-3,2), B(1,3)$ and $C(-1, u)$ are such that the distances $A C$ and $A B$ are related by:

$$
A C=2 A B
$$

What are the possible values of $u$ ?
(A) 2 and -6
(B) -2 and 6
(C) 6 and -10
(D) -6 and 10
(E) $2+2 \sqrt{13}$ and $2-2 \sqrt{13}$
(F) $-3+2 \sqrt{13}$ and $-3-2 \sqrt{13}$
[ENGAA, 2018S1Q46]
A metal ball suspended from a steel cable is held at rest by a horizontal force $P$. The cable makes an angle of $30^{\circ}$ to the vertical as shown in the diagram. The cable exerts a force $T$ on the ball.

What is the magnitude of $P$ ?
(A) $\frac{T}{2}$
(B) $T$
(C) $2 T$
(D) $\frac{T}{\sqrt{2}}$
(E) $\frac{T}{\sqrt{3}}$
(F) $\frac{2 T}{\sqrt{3}}$
(G) $\frac{\sqrt{3} T}{2}$
[ENGAA, 2018S1Q47]
What is the coefficient of $x^{3}$ in the expansion of $(1-2 x)^{5}(1+2 x)^{5}$ ?
(A) -6400
(B) -640
(C) -80
(D) 0
(E) 80
(F) 800
(G) 960
[ENGAA, 2018S1Q48]
A pendulum bob of mass 10 g is suspended by a light, inextensible string of length 50 cm . The bob is released from rest at position $X$.


What is the speed of the bob as it passes through position $Y$ ?
(gravitational field strength $g=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that resistive forces are negligible)
(A) $\sqrt{2} \mathrm{~m} \mathrm{~s}^{-1}$
(B) $\sqrt{4} \mathrm{~m} \mathrm{~s}^{-1}$
(C) $\sqrt{6} \mathrm{~m} \mathrm{~s}^{-1}$
(D) $\sqrt{8} \mathrm{~m} \mathrm{~s}^{-1}$
(E) $\sqrt{10} \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2018S1Q49]
Given that

$$
\int_{0}^{2} x^{m} \mathrm{~d} x=\frac{16 \sqrt{2}}{7}
$$

And

$$
\int_{0}^{2} x^{m+1} \mathrm{~d} x=\frac{32 \sqrt{2}}{9}
$$

what is the value of $m$ ?
(A) $-\frac{11}{2}$
(B) $-\frac{9}{2}$
(C) $-\frac{22}{29}$
(D) $\frac{7}{22}$
(E) $\frac{5}{2}$
(F) $\frac{7}{2}$
[ENGAA, 2018S1Q50]
An object of mass $m$ is initially moving at constant speed $u$ to the right. It collides with a stationary object of greater mass $M$ and bounces back in the opposite direction at speed $v$. What is the speed of the greater mass immediately after the collision?
(A) $\frac{m u}{M}$
(B) $\frac{M u}{m}$
(C) $\frac{m(v-u)}{M}$
(D) $\frac{M(v-u)}{m}$
(E) $\frac{m(v+u)}{M}$
(F) $\frac{M(v+u)}{m}$
[ENGAA, 2018S1Q51]
The two functions $f$ and $g$ satisfy

$$
f^{\prime}(x)=a x+g(x)
$$

where $a$ is a constant.
Given that

$$
\int_{2}^{4} g(x) \mathrm{d} x=12
$$

and

$$
f(4)=18+f(2)
$$

what is the value of $a$ ?
(A) 1
(B) 3
(C) 5
(D) 6
(E) 15
[ENGAA, 2018S1Q52]
A load drops from rest through a vertical height $h$ to the ground.
A light cable attached to the load passes over a friction pulley that provides a braking force during the fall. As the load falls through height $h, 50 \%$ of the gravitational potential energy lost is transferred into thermal energy. The load reaches a final speed of $10 \mathrm{~m} \mathrm{~s}^{-1}$.


What is the vertical height $h$ ?
(gravitational field strength $g=10 \mathrm{~N} \mathrm{~kg}^{-1}$; air resistance can be ignored)
(A) 0.50 m
(B) 1.0 m
(C) 2.0 m
(D) 2.5 m
(E) 5.0 m
(F) 10 m
(G) 20 m
[ENGAA, 2018S1Q53]
The dimensions of a solid cuboid, in cm , are $x, 2 x$ and $y$.
The volume of the cuboid is $576 \mathrm{~cm}^{3}$.
At this volume, the surface area of the cuboid has its maximum value.
What is the area, in $\mathrm{cm}^{2}$, of the face that has the largest area?
(A) $2(288)^{\frac{2}{3}}$
(B) 72
(C) 96
(D) 432
(E) $4(144)^{\frac{2}{3}}$
[ENGAA, 2018S1Q54]
An object is thrown vertically upwards from ground level with an initial velocity of $40 \mathrm{~m} \mathrm{~s}^{-1}$.
2.0 seconds later another object is released from a height above the ground and falls vertically from rest.

Both of the objects hit the ground at the same time.
From what height above the ground was the second object released?
(gravitational field strength $g=10 \mathrm{~N} \mathrm{~kg}^{-1}$; air resistance can be ignored)
(A) 80 m
(B) 180 m
(C) 320 m
(D) 500 m
(E) 900 m

## ENGAA 2018 S2



## TIME ALLOWED: 40 MINUTES

This paper is Section 2 of 2 .
A separate answer sheet is provided for this paper. Please check that you have one. You also require a soft pencil and an eraser.

Please complete both the answer sheet and this question paper with your candidate number, centre number, date of birth, and name.

This question paper contains 18 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 18 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
There is space in the question paper for your working or notes, but no extra paper is allowed.

Calculators may NOT be used in this section.
[ENGAA, 2018S2Q1]
A man is cycling along a straight horizontal road at a constant speed of $9.00 \mathrm{~m} \mathrm{~s}^{-1}$.
He passes a boy who is cycling at $5.00 \mathrm{~m} \mathrm{~s}^{-1}$ in the same direction.
When the man is level with the boy, the boy begins to accelerate at a constant rate of 0.800 m $\mathrm{s}^{-2}$.
The boy maintains this constant acceleration and the man continues at constant speed until the boy passes the man.
What is the time interval between the two instances when the man and the boy are level?
(A) 5.00 s
(B) 10.0 s
(C) 22.5 s
(D) 35.0 s
(E) 90.0 s
[ENGAA, 2018S2Q2]
Two liquids $P$ and $Q$ can be mixed together in any proportion.
The density of liquid $P$ is $\rho_{P}$ and the density of liquid $Q$ is $\rho_{Q}$.
A volume $V_{P}$ of liquid $P$ and a volume $V_{Q}$ of liquid $Q$ are mixed together to produce a volume that is equal to $V_{P}+V_{Q}$.
What is the density of the mixture?
(A) $\frac{\rho_{P}+\rho_{Q}}{2}$
(B) $\frac{\rho_{P} V_{P}+\rho_{Q} V_{Q}}{V_{P}+V_{Q}}$
(C) $\left(\frac{\rho_{P}}{V_{P}}+\frac{\rho_{Q}}{V_{Q}}\right)\left(V_{P}+V_{Q}\right)$
(D) $\frac{\rho_{P} V_{Q}+\rho_{Q} V_{P}}{V_{P}+V_{Q}}$
(E) $\frac{\left(\frac{\rho_{P}}{V_{P}}+\frac{\rho_{Q}}{V_{Q}}\right)}{\left(V_{P}+V_{Q}\right)}$
(F) $\left(\frac{\rho_{P}}{V_{Q}}+\frac{\rho_{Q}}{V_{P}}\right)\left(V_{P}+V_{Q}\right)$
[ENGAA, 2018S2Q3]
A circuit contains two fixed resistors, $X$ and $Y$, and a variable resistor $W$. The power supply has no internal resistance.


The resistance of $W$ increases.
What happens to the power dissipated in $X$ and in $Y$ ?

|  | power dissipated in $X$ | power dissipated in $Y$ |
| :---: | :---: | :---: |
| (A) | decreases | decreases |
| (B) | decreases | stays constant |
| (C) | decreases | increases |
| (D) | increases | decreases |
| (E) | increases | stays constant |
| (F) | increases | increases |

[ENGAA, 2018S2Q4]
The diagram shows two bar magnets, $X$ and $Y$, held at rest a short distance apart from one another on a smooth horizontal surface. They are aligned as shown in the diagram and both are released at the same time. They move towards each other, collide and coalesce. Just before they collide $X$ has a velocity $v$ to the right.


The mass of $X$ is double the mass of $Y$.
Which row in the table gives the magnitude and direction of the velocity of the two magnets after the collision?

|  | magnitude of velocity | direction of velocity |
| :---: | :---: | :---: |
| (A) | 0 | not applicable |
| (B) | $\frac{1}{3} v$ | to the left |
| (C) | $\frac{2}{3} v$ | to the left |
| (D) | $\frac{4}{3} v$ | to the left |
| (E) | $\frac{1}{3} v$ | to the right |
| (F) | $\frac{2}{3} v$ | to the right |
| (G) | $\frac{4}{3} v$ | to the right |

[ENGAA, 2018S2Q5]
A stone is suspended from a newtonmeter and the meter reads 3 N. A beaker of water is placed on a top pan balance and the top pan balance reads 400 g .
The stone is lowered into the water so that it is at rest and fully submerged, but not touching the bottom of the beaker. The reading on the newtonmeter is now 2 N and the top pan balance reads $X$ g.
The stone is detached from the newtonmeter and allowed to rest under water on the base of the beaker. The top pan balance now reads $Y$ g.


What are the values of $X$ and $Y$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )

|  | $X$ | $Y$ |
| :---: | :---: | :---: |
| (A) | 400 | 500 |
| (B) | 400 | 600 |
| (C) | 400 | 700 |
| (D) | 500 | 500 |
| (E) | 500 | 600 |
| (F) | 500 | 700 |


[ENGAA, 2018S2Q6]
Diagram 1 represents a stationary wave produced by sound in an open-ended tube of length 0.50 m containing a liquid. The speed of the wave in the liquid is $1000 \mathrm{~m} \mathrm{~s}^{-1}$.

diagram 1
Diagram 2 is a displacement-time graph representing a progressive sound wave with the same frequency in the same liquid.

diagram 2
What is the value of the time at point $X$ on the graph?
(End effects of the stationary wave can be ignored.)
(A) $2.5 \times 10^{-4} \mathrm{~s}$
(B) $3.75 \times 10^{-4} \mathrm{~s}$
(C) $5.0 \times 10^{-4} \mathrm{~s}$
(D) $7.5 \times 10^{-4} \mathrm{~s}$
(E) $1.0 \times 10^{-3} \mathrm{~s}$
(F) $1.5 \times 10^{-3} \mathrm{~s}$
[ENGAA, 2018S2Q7]
A tennis ball travelling at a speed of $30 \mathrm{~m} \mathrm{~s}^{-1}$ hits a racket elastically with a kinetic energy of 27 J.

The racket applies a variable force to the tennis ball for a time of 4.0 ms as shown.


The ball moves away in the opposite direction to its initial motion.
With what speed does the ball leave the racket?
(Assume that resistive forces are negligible.)
(A) $20 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $28 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $32 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $50 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $70 \mathrm{~m} \mathrm{~s}^{-1}$
(F) $80 \mathrm{~m} \mathrm{~s}^{-1}$
(G) $100 \mathrm{~m} \mathrm{~s}^{-1}$
(H) $130 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2018S2Q8]
A painter of mass 74 kg stands on a uniform wooden plank of length 2.5 m and of mass 24 kg . The painter stands at the middle of the plank. The plank rests on two supports.
Support $P$ is 0.25 m from one end of the plank and support $Q$ is 0.75 m from the other end. A pot of paint of mass 5.0 kg is 0.80 m from the centre of mass of the painter.


What is the force exerted by support $Q$ on the plank?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 50 N
(B) 51.5 N
(C) 66 N
(D) 177 N
(E) 500 N
(F) 515 N
(G) 660 N
(H) 1770 N
[ENGAA, 2018S2Q9]
The circuit shown in the diagram contains six resistors and an ideal digital voltmeter.


What is the reading on the voltmeter?
(A) 0 V
(B) 2 V
(C) 4 V
(D) 6 V
(E) 8 V
(F) 10 V
(G) 12 V
[ENGAA, 2018S2Q10]
Four identical light springs are connected together using a light rod. A load of mass $m$ is suspended from the system so that the rod is horizontal, as shown in the diagram.


The spring constant of each spring is $k$ and all four springs obey Hooke's law throughout. Which of the following expressions gives the elastic potential energy stored in the system? (gravitational field strength $=g$ )
(A) $\frac{(m g)^{2}}{6 k}$
(B) $\frac{(m g)^{2}}{2 k}$
(C) $\frac{5(m g)^{2}}{9 k}$
(D) $\frac{2(m g)^{2}}{3 k}$
(E) $\frac{(m g)^{2}}{k}$
(F) $\frac{2(m g)^{2}}{k}$
[ENGAA, 2018S2Q11]
A block of mass 3.60 kg is held stationary on a rough slope inclined at $30.0^{\circ}$ to the horizontal. The edge $X$ of the block is 1.50 m from the bottom of the slope.


The block is released and it accelerates uniformly down the slope. When $X$ reaches the bottom of the slope, the speed of the block is $2.00 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the average rate at which work is done against resistive forces? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 4.8 W
(B) 13.2 W
(C) 15.6 W
(D) 18.0 W
(E) 26.4 W
[ENGAA, 2018S2Q12]
A circuit contains a battery with internal resistance and two resistors, connected as shown in the diagram.
The emf of the battery is 6.00 V . The pd across the $20.0 \Omega$ resistor is 4.80 V .


What is the internal resistance of the battery?
(A) $0.0800 \Omega$
(B) $0.333 \Omega$
(C) $0.480 \Omega$
(D) $3.00 \Omega$
(E) $12.0 \Omega$
(F) $12.5 \Omega$
(G) $15.0 \Omega$
[ENGAA, 2018S2Q13]
A stick at position $X$ dips into water every 0.80 s , creating a circular wave which travels at a constant speed.
The diagrams show the wave crests at a time $t$ and 1.0 s later. One of the wave crests, labelled $Q$, appears in both diagrams.
In each diagram, the distance from $X$ to a wave crest is labelled.


What is the wavelength of the wave?
(A) 1.2 cm
(B) 1.5 cm
(C) 2.0 cm
(D) 3.5 cm
(E) 4.4 cm
(F) 6.0 cm
(G) 7.5 cm
(H) 14 cm
[ENGAA, 2018S2Q14]
A block $P$ has a smaller block $Q$ resting on its top surface.
$Q$ is connected to a hanging block, $R$, by a light, inextensible string. The string passes over a smooth pulley which is connected to block $P$, as shown in the diagram.


The masses of blocks $P, Q$ and $R$ are $m_{P}, m_{Q}$ and $m_{R}$ respectively.
The surfaces of the three blocks are smooth.
$P$ is accelerated horizontally to the right by an external force. While this is happening, $Q$ and $R$ do not move relative to $P$.
What is the acceleration of $P$ ?
(gravitational field strength $=g$ )
(A) $\frac{m_{Q} g}{m_{R}}$
(B) $\frac{m_{R} g}{m_{Q}}$
(C) $\frac{m_{R} g}{m_{R}+m_{Q}}$
(D) $\frac{m_{Q} g}{\left(m_{P}+m_{Q}+m_{R}\right)}$
(E) $\frac{m_{R} g}{\left(m_{P}+m_{Q}+m_{R}\right)}$
(F) $\frac{\left(m_{Q}+m_{R}\right) g}{\left(m_{P}+m_{Q}+m_{R}\right)}$
[ENGAA, 2018S2Q15]
A solid cube with a total surface area of $96 \mathrm{~cm}^{2}$ is suspended from a spring of spring constant $2.0 \times 10^{4} \mathrm{~N} \mathrm{~m}^{-1}$ and causes the spring to extend by $1.6 \times 10^{-4} \mathrm{~m}$.
The cube is removed from the spring and placed on a horizontal surface where it rests with one face on the surface.
What is the pressure exerted by the cube on the surface and what is the density of the material from which the cube is made?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )

|  | pressure $/ \mathrm{N} \mathrm{m}^{-2}$ | density $/ \mathrm{kg} \mathrm{m}^{-3}$ |
| :---: | :---: | :---: |
| (A) | $2.0 \times 10^{1}$ | $5.0 \times 10^{-2}$ |
| (B) | $2.0 \times 10^{1}$ | $5.0 \times 10^{4}$ |
| (C) | $3.3 \times 10^{2}$ | $5.0 \times 10^{-2}$ |
| (D) | $3.3 \times 10^{2}$ | $5.0 \times 10^{3}$ |
| (E) | $3.3 \times 10^{2}$ | $5.0 \times 10^{4}$ |
| (F) | $2.0 \times 10^{3}$ | $5.0 \times 10^{-2}$ |
| (G) | $2.0 \times 10^{3}$ | $5.0 \times 10^{3}$ |
| (H) | $2.0 \times 10^{3}$ | $5.0 \times 10^{4}$ |

[ENGAA, 2018S2Q16]
A power cable consists of a cylindrical copper (Cu) wire surrounded by six cylindrical aluminium (Al) wires. All the wires are of the same cross-sectional area as shown:


The table gives the densities and resistivities of aluminium and copper.

| material | density | resistivity |
| :---: | :---: | :---: |
| aluminium | $d$ | $3 \rho$ |
| copper | $3 d$ | $2 \rho$ |

The cable has mass $M$ and length $L$.
Which expression gives the resistance between the two ends of the cable?
(A) $\frac{18 \rho d L^{2}}{5 M}$
(B) $\frac{21 \rho d L^{2}}{M}$
(C) $\frac{81 \rho d L^{2}}{5 M}$
(D) $\frac{180 \rho d L^{2}}{M}$
(E) $\frac{12 \rho d L^{2}}{5 M}$
(F) $\frac{28 \rho d L^{2}}{3 M}$
(G) $\frac{36 \rho d L^{2}}{5 M}$
(H) $\frac{80 \rho d L^{2}}{M}$
[ENGAA, 2018S2Q17]
Oil of density $800 \mathrm{~kg} \mathrm{~m}^{-3}$ is pumped through a pipe of circular cross-sectional area $0.60 \mathrm{~m}^{2}$ at a speed of $5.0 \mathrm{~m} \mathrm{~s}^{-1}$. Between $X$ and $Y$, the cross-sectional area of the pipe decreases to $0.25 \mathrm{~m}^{2}$.


What is the resultant force exerted on the oil as it passes from $X$ to $Y$ ?
(A) 0 N
(B) 7000 N
(C) 12000 N
(D) 16800 N
(E) 19000 N
(F) 24000 N
(G) 40800 N
(H) 143000 N
[ENGAA, 2018S2Q18]
A small steel ball of mass $m$ is released from the top of a semi-circular ramp of radius $r$ as shown in the diagram:


After being released, the ball moves around the semi-circle to the lowest point at position $P$ and then rises to a maximum height on the other side at position $Q$ before falling down again. Assume that the friction force acting on the ball has a constant magnitude whilst the ball is moving.
What is the kinetic energy of the ball as it first passes position $P$ ?
(gravitational field strength $=g$ )
(A) $\operatorname{mgr}(\sqrt{2}-1)$
(B) $\operatorname{mgr}\left(1-\frac{\sqrt{2}}{3}\right)$
(C) $\operatorname{mgr}\left(1-\frac{\sqrt{2}}{4}\right)$
(D) $\frac{2 m g r}{3}$
(E) $\frac{3 m g r}{4}$
(F) $\operatorname{mgr}\left(\frac{1+\sqrt{2}}{3}\right)$
(G) $m g r$
(H) $\operatorname{mgr}\left(2-\frac{\sqrt{2}}{2}\right)$

## ENGAA 2019 S1



## TIME ALLOWED: 60 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2 .

This paper contains two parts, $\mathbf{A}$ and $\mathbf{B}$, and you should attempt both parts.
Part A Mathematics and Physics (20 questions)
Part B Advanced Mathematics and Advanced Physics (20 questions)
This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 40 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

## Part A

[ENGAA, 2019S1Q1]
Evaluate

$$
(\sqrt{7}+\sqrt{3})^{2}-(\sqrt{7}-\sqrt{3})^{2}
$$

(A) 0
(B) $2 \sqrt{7}$
(C) $4 \sqrt{7}$
(D) $2 \sqrt{21}$
(E) 10
(F) $4 \sqrt{21}$
(G) 20

## [ENGAA, 2019S1Q2]

The current-voltage graph for a diode is shown.


The diode is connected in series with a resistor and a 6.0 V battery. The current in the circuit is 8.0 mA .
What is the resistance of the resistor?
(Assume that the battery has negligible resistance.)
(A) $0.15 \Omega$
(B) $0.60 \Omega$
(C) $0.75 \Omega$
(D) $4.8 \Omega$
(E) $150 \Omega$
(F) $600 \Omega$
(G) $750 \Omega$
[ENGAA, 2019S1Q3]
The equation gives $y$ in terms of $x$ :

$$
y=3-4\left(1-\frac{x}{2}\right)^{2}
$$

Which one of the following is a rearrangement for $x$ in terms of $y$ ?
(A) $x=-2 \pm 2 \sqrt{\frac{3-y}{4}}$
(B) $x=-2 \pm 2 \sqrt{\frac{4-y}{3}}$
(C) $x=1 \pm \sqrt{\frac{3-y}{4}}$
(D) $x=1 \pm 2 \sqrt{\frac{3-y}{4}}$
(E) $x=2 \pm 2 \sqrt{\frac{3-y}{4}}$
(F) $x=2 \pm 2 \sqrt{\frac{4-y}{3}}$
(G) $x=2 \pm 2 \sqrt{\frac{3+y}{4}}$
[ENGAA, 2019S1Q4]
Two electromagnetic waves $P$ and $Q$ travel in a vacuum and the ratio of their wavelengths is:

$$
\frac{\text { wavelength of } P}{\text { wavelength of } Q}=1.0 \times 10^{8}
$$

Which row in the table shows the ratio of their speeds, the ratio of their frequencies, and identifies the possible natures of $P$ and $Q$ ?

|  | speed of $P$ <br> speed of $Q$ | frequency of $P$ <br> frequency of $Q$ | nature ofP | nature of $Q$ |
| :---: | :---: | :---: | :---: | :---: |
| (A) | 1.0 | $1.0 \times 10^{-8}$ | microwave | X-ray |
| (B) | 1.0 | $1.0 \times 10^{-8}$ | microwave | radio wave |
| (C) | 1.0 | $1.0 \times 10^{8}$ | infrared | ultraviolet |
| (D) | 1.0 | $1.0 \times 10^{8}$ | visible light | infrared |
| (E) | $1.0 \times 10^{8}$ | 1.0 | gamma | X-ray |
| (F) | $1.0 \times 10^{8}$ | 1.0 | gamma | infrared |
| (G) | $1.0 \times 10^{8}$ | $1.0 \times 10^{16}$ | infrared | radio wave |
| (H) | $1.0 \times 10^{8}$ | $1.0 \times 10^{16}$ | visible light | ultraviolet |

[ENGAA, 2019S1Q5]
The resistance to the motion of a car is directly proportional to the square of the speed of the car.
The car increases its speed by $20 \%$.
What is the percentage increase in the resistance to the motion of the car?
(A) $20 \%$
(B) $24 \%$
(C) $44 \%$
(D) $120 \%$
(E) $224 \%$
(F) $240 \%$
(G) $400 \%$
[ENGAA, 2019S1Q6]
A water-tight cylinder with a thin, freely moving piston contains $2.0 \times 10^{-3} \mathrm{~m}^{3}$ of trapped air at atmospheric pressure of $1.0 \times 10^{5} \mathrm{~Pa}$.
When the cylinder is submerged in water of constant density $1000 \mathrm{~kg} \mathrm{~m}^{-3}$, the volume of air in the cylinder decreases to $4.0 \times 10^{-4} \mathrm{~m}^{3}$.
The piston is at a depth $h$ below the surface of the water and the water surface is open to the atmosphere.
What is the depth $h$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that the temperature of the air remains constant and that air is an ideal gas)
(A) 40 m
(B) 50 m
(C) 60 m
(D) 400 m
(E) 500 m
(F) 600 m

## [ENGAA, 2019S1Q7]

The equation of a curve is $y=p x^{2}+q x$ where $p$ and $q$ are constants.
The curve passes through the points $(2,6)$ and $(4,-4)$.
What is the value of $q-p$ ?
(A) 1
(B) 2
(C) 5
(D) 6
(E) 9
(F) 16

## [ENGAA, 2019S1Q8]

The secondary coil of an ideal, $100 \%$ efficient transformer is connected to a resistor by cables of total resistance $1500 \Omega$. The current in the primary coil is 4.0 A . There are 240 turns in the primary coil and 4800 turns in the secondary coil.
What is the power produced as heat in the cables?
(A) 60 W
(B) 300 W
(C) 6000 W
(D) 24000 W
(E) 120000 W
(F) 9600000 W
[ENGAA, 2019S1Q9]
Which of the following is a simplification of

$$
4-\frac{x(3 x+1)}{x^{2}\left(3 x^{2}-2 x-1\right)}
$$

(A) $\frac{12 x^{3}-8 x^{2}-7 x-1}{x(3 x-1)(x-1)}$
(B) $\frac{4 x^{2}+4 x-1}{x(x+1)}$
(C) $\frac{4 x^{2}+4 x+1}{x(x+1)}$
(D) $\frac{4 x^{2}-4 x-1}{x(x-1)}$
(E) $\frac{4 x^{2}-4 x+1}{x(x-1)}$
(F) $\frac{12 x^{3}-8 x^{2}-x+1}{x(3 x-1)(x-1)}$
[ENGAA, 2019S1Q10]
Two tanks of water are connected by a solid cylindrical copper bar of length $l$ and diameter $d$. The bar is insulated.
One tank contains water at $90^{\circ} \mathrm{C}$ and the other tank contains water at temperature $\theta$.


For which of the following conditions is thermal energy conducted along the bar at the lowest rate?

|  | $l / \mathrm{m}$ | $d / \mathrm{cm}$ | $\theta /{ }^{\circ} \mathrm{C}$ |
| :---: | :---: | :---: | :---: |
| (A) | 0.40 | 4.0 | 20 |
| (B) | 0.40 | 4.0 | 40 |
| (C) | 0.40 | 8.0 | 20 |
| (D) | 0.40 | 8.0 | 40 |
| (E) | 0.80 | 4.0 | 20 |
| (F) | 0.80 | 4.0 | 40 |
| (G) | 0.80 | 8.0 | 20 |
| (H) | 0.80 | 8.0 | 40 |

[ENGAA, 2019S1Q11]
The ball for a garden game is a solid sphere of volume $192 \mathrm{~cm}^{3}$.
For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by $25 \%$.
What is the volume, in $\mathrm{cm}^{3}$, of the children's ball?
(A) 48
(B) 81
(C) 96
(D) 108
(E) 144
[ENGAA, 2019S1Q12]
The radioactive isotope $X$ becomes the stable isotope $Y$ after a succession of decays involving only the emission of alpha and beta ( $\beta^{-}$) particles.
During the decay of one nucleus from $X$ to $Y$, a total of seven particles are emitted. It is known that more of these particles are alpha particles than beta particles.
The atomic number of $X$ is $Z$ and the mass number of $X$ is $A$.
Which row in the table could give the atomic number and the mass number of $Y$ ?

|  | atomic number of $Y$ | mass number of $Y$ |
| :---: | :---: | :---: |
| (A) | $Z-2$ | $A-12$ |
| (B) | $Z-5$ | $A-8$ |
| (C) | $Z-8$ | $A-20$ |
| (D) | $Z-10$ | $A-24$ |
| (E) | $Z-11$ | $A-16$ |

## [ENGAA, 2019S1Q13]

The diagram shows a right-angled triangle, with sides of length $x+4,2 x+2$ and $3 x$, all in cm .

$2 x+2$
What is the area, in $\mathrm{cm}^{2}$, of the triangle?
(A) 10
(B) 12
(C) 28
(D) 36
(E) 40
(F) 54
(G) 70

## [ENGAA, 2019S1Q14]

The kinetic energy of an object of mass 4.0 kg , travelling in a straight line, increases from 32 J to 200 J in 3.0 seconds due to a constant resultant force.
What is the value of this resultant force?
(A) 2.0 N
(B) 4.0 N
(C) 8.0 N
(D) 24 N
(E) 28 N
(F) 56 N

## [ENGAA, 2019S1Q15]

$P R$ and $Q S$ are the diagonals of a rhombus $P Q R S$.
$P R=(3 x+2) \mathrm{cm}$
$Q S=(8-2 x) \mathrm{cm}$
The area of $P Q R S$ is $11 \mathrm{~cm}^{2}$.
What is the difference, in cm , between the two possible lengths of $P R$ ?
(A) $2 \frac{2}{3}$
(B) $4 \frac{1}{2}$
(C) $5 \frac{1}{3}$
(D) 8
(E) 14
[ENGAA, 2019S1Q16]
In the following circuit, all five resistors have the same resistance.


The reading on the voltmeter is 1.0 V .
What is the voltage across the battery?
(A) 4.0 V
(B) 5.0 V
(C) 6.0 V
(D) 7.0 V
(E) 8.0 V
(F) 9.0 V
(G) 10 V
[ENGAA, 2019S1Q17]


The diagram shows two congruent right-angled triangles $P Q R$ and $T S R$ with right angles at $Q$ and $S$, respectively.
$P Q=T S=3 \mathrm{~cm}$
$Q R=S R=4 \mathrm{~cm}$
$P R T$ is a straight line.
What is the length, in cm , of $Q S$ ?
(A) 4
(B) $3 \sqrt{2}$
(C) 5.2
(D) $4 \sqrt{2}$
(E) 6.4
(F) 8.2
(G) 10
[ENGAA, 2019S1Q18]
A block is designed with a cylindrical channel to accommodate a hot-water pipe. The block is 100 cm long and it has a square cross-section of side 22.0 cm with a cylindrical hole in the middle, as shown in the diagram:


The diameter of the cylindrical hole is 14.0 cm and the density of the material from which the block is made is $0.100 \mathrm{~g} \mathrm{~cm}^{-3}$.
What is the mass of the block?
(take $\pi$ to be $\frac{22}{7}$ )
(A) 1.32 kg
(B) 3.30 kg
(C) 13.2 kg
(D) 33.0 kg
(E) 132 kg
(F) 330 kg
(G) 1320 kg
(H) 3300 kg
[ENGAA, 2019S1Q19]


A solid pyramid has a square base of side length 12 cm and a vertical height of $h \mathrm{~cm}$.
The volume of the pyramid, in $\mathrm{cm}^{3}$, is equal to the total surface area of the pyramid, in $\mathrm{cm}^{2}$.
What is the value of $h$ ?
(volume of pyramid $=\frac{1}{3} \times$ area of base $\times$ vertical height)
(A) $\frac{72}{35}$
(B) $2 \sqrt{3}$
(C) 6
(D) $\frac{144}{23}$
(E) 8
(F) $2 \sqrt{21}$
[ENGAA, 2019S1Q20]
A sample initially contains equal numbers of atoms of a radioactive isotope $X$ and a stable isotope $Y$.
Isotope $X$ has a half-life of 3 years and decays in a single stage to the stable isotope $Y$.
What is the ratio

$$
\text { number of atoms of } X \text { : number of atoms of } Y
$$

in the sample 6 years later?
(A) The sample contains only isotope $Y$.
(B) $1: 7$
(C) $1: 4$
(D) $1: 3$
(E) $7: 4$

## Part B

[ENGAA, 2019S1Q21]
Find the area of the shape bounded by the four lines:

$$
\begin{aligned}
2 y+x & =4 \\
x & =-6 \\
x & =0 \\
y & =0
\end{aligned}
$$

(A) 4
(B) 12
(C) 21
(D) 25
(E) 27
(F) 30
[ENGAA, 2019S1Q22]
A hydroelectric power station uses the water in a reservoir to power the generators. The water falls through a vertical height of 150 m to the turbines which power the generators.
The efficiency of the power station is $90 \%$ and the output power of the power station is 1800 MW.
The gravitational field strength is $10 \mathrm{~N} \mathrm{~kg}^{-1}$ and the density of water is $1000 \mathrm{~kg} \mathrm{~m}^{-3}$.
What volume of water passes through the turbines in one minute?
(A) $6.48 \times 10^{4} \mathrm{~m}^{3}$
(B) $7.20 \times 10^{4} \mathrm{~m}^{3}$
(C) $8.00 \times 10^{4} \mathrm{~m}^{3}$
(D) $6.48 \times 10^{7} \mathrm{~m}^{3}$
(E) $7.20 \times 10^{7} \mathrm{~m}^{3}$
(F) $8.00 \times 10^{7} \mathrm{~m}^{3}$
[ENGAA, 2019S1Q23]
The curve

$$
y=x^{3}+p x^{2}+q x+r
$$

has a local maximum when $x=-1$ and a local minimum when $x=3$
What is the value of $p$ ?
(A) -9
(B) -3
(C) -1
(D) 1
(E) 3
(F) 9

## [ENGAA, 2019S1Q24]

A car $P$ of mass 1000 kg is travelling north at $30 \mathrm{~m} \mathrm{~s}^{-1}$ along a straight, horizontal road when it hits another car $Q$ which is directly ahead of $P$ and travelling in the same direction. Car $Q$ has a mass of 500 kg and is travelling at $20 \mathrm{~m} \mathrm{~s}^{-1}$.
The collision lasts for 0.20 s and immediately after the collision car $Q$ is moving north at 30 m $\mathrm{s}^{-1}$.
What is the speed of $P$ immediately after the collision and what is the size of the average resultant force that acts on $Q$ during the collision?
(Assume that no external forces act on the cars during the collision.)

|  | speed ofP $/ \mathrm{m} \mathrm{s}^{-1}$ | average force on $Q / \mathrm{N}$ |
| :---: | :---: | :---: |
| (A) | 20 | 25000 |
| (B) | 20 | 50000 |
| (C) | 20 | 100000 |
| (D) | 20 | 125000 |
| (E) | 25 | 25000 |
| (F) | 25 | 50000 |
| (G) | 25 | 100000 |
| (H) | 25 | 125000 |

[ENGAA, 2019S1Q25]
When simplified, $\frac{1}{(1-\sqrt{2})^{3}}$ is written in the form $a+b \sqrt{2}$ where $a$ and $b$ are integers.
What is the value of $b$ ?
(A) -7
(B) -5
(C) -1
(D) 1
(E) 5
(F) 7
[ENGAA, 2019S1Q26]
A metal wire of length 0.50 m has a uniform cross-sectional area of $4.0 \times 10^{-7} \mathrm{~m}^{2}$. There is a current of 4.0 A in the wire.

What is the potential difference across the ends of the wire?
(resistivity of the metal $1.6 \times 10^{-7} \Omega \mathrm{~m}$ )
(A) 0.05 V
(B) 0.20 V
(C) 0.80 V
(D) 3.2 V
(E) 5.0 V
(F) 20 V
[ENGAA, 2019S1Q27]
It is given that

$$
7 \cos x+\tan x \sin x=5
$$

where $0^{\circ}<x<90^{\circ}$
What are the possible values of $\tan x$ ?
(A) $\frac{1}{2}$ or $\frac{1}{3}$
(B) $\frac{1}{\sqrt{3}}$ or $\frac{1}{2 \sqrt{2}}$
(C) $\frac{\sqrt{3}}{2}$ or $\frac{2 \sqrt{2}}{3}$
(D) $\sqrt{3}$ or $2 \sqrt{2}$
(E) 3 or 2
[ENGAA, 2019S1Q28]
A uniform square trap door of side 0.80 m and mass 14 kg has a smooth hinge at one edge and is held open at an angle of $30^{\circ}$ to the horizontal. It is supported by a single rigid rod placed so that it meets the surface of the trap door at $90^{\circ}$ at a distance 0.10 m from the top edge of the trap door, as shown.


What is the normal contact force exerted on the trap door by the rod?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 40 N
(B) $35 \sqrt{3} \mathrm{~N}$
(C) $40 \sqrt{3} \mathrm{~N}$
(D) 80 N
(E) $80 \sqrt{3} \mathrm{~N}$
(F) $280 \frac{\sqrt{3}}{3} \mathrm{~N}$
[ENGAA, 2019S1Q29]
An equilateral triangle of side 8 cm is drawn so that its vertices lie on the circumference of a circle, as shown in the diagram.


What is the total of the three areas shaded in the diagram, $\mathrm{in}^{\mathrm{cm}^{2}}$ ?
(A) $8(2 \pi-3)$
(B) $24(\pi-\sqrt{3})$
(C) $48(4 \pi-\sqrt{3})$
(D) $\frac{16}{3}(4 \pi-6-3 \sqrt{3})$
(E) $\frac{16}{3}(4 \pi-3 \sqrt{3})$
[ENGAA, 2019S1Q30]
Two small loudspeakers are placed side by side 30 cm apart.
They are connected to the same signal generator so that they emit sound of frequency 400 Hz in phase with one another.
The sounds both reach a microphone placed 40 cm directly in front of one of the two loudspeakers as shown.


What is the phase difference between waves from the loudspeakers as they arrive at the microphone?
(speed of sound $=320 \mathrm{~m} \mathrm{~s}^{-1}$ )
(A) $30^{\circ}$
(B) $36^{\circ}$
(C) $45^{\circ}$
(D) $60^{\circ}$
(E) $72^{\circ}$
(F) $90^{\circ}$
(G) $120^{\circ}$
[ENGAA, 2019S1Q31]
Which one of the following is the real solution of the equation

$$
3 \times 5^{2 x+1}-5^{x}-2=0
$$

(A) $x=\log _{5}\left(\frac{1}{3}\right)$
(B) $x=\log _{5}\left(\frac{2}{5}\right)$
(C) $x=\log _{5}\left(\frac{3}{5}\right)$
(D) $x=\log _{5}\left(\frac{2}{3}\right)$
(E) $x=\log _{5}\left(\frac{5}{3}\right)$
(F) $x=\log _{5}\left(\frac{5}{2}\right)$
[ENGAA, 2019S1Q32]
An astronaut on the Moon throws a ball vertically upwards. The ball has a mass of 2.0 g and is thrown upwards at $80 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the maximum height gained by the ball?
(gravitational field strength close to the Moon's surface $=1.6 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 25 m
(B) 50 m
(C) 320 m
(D) 2000 m
(E) 3200 m
(F) 4000 m
[ENGAA, 2019S1Q33]
For a particular function $f(x)$, it is given that:

$$
\int_{-2}^{2} 2 f(x) \mathrm{d} x+\int_{2}^{4} f(x) \mathrm{d} x=4
$$

and also:

$$
\int_{-2}^{2} 5 f(x) \mathrm{d} x-\int_{-2}^{4} f(x) \mathrm{d} x=7
$$

Find the value of $\int_{2}^{4} f(x) \mathrm{d} x$.
(A) $\frac{1}{3}$
(B) $\frac{11}{7}$
(C) $\frac{11}{6}$
(D) $\frac{13}{6}$
(E) $\frac{13}{3}$
[ENGAA, 2019S1Q34]
A student has one $300 \Omega$ resistor and another resistor of resistance $R$.
The student plots a graph of current $I$ against potential difference $V$ for the $300 \Omega$ resistor and then for both resistors connected in parallel.


What is the resistance $R$ ?
(A) $3.3 \Omega$
(B) $5.0 \Omega$
(C) $10 \Omega$
(D) $100 \Omega$
(E) $200 \Omega$
(F) $600 \Omega$
(G) $1000 \Omega$
[ENGAA, 2019S1Q35]
Given that

$$
f(x)=\int_{0}^{x}(3+2 t)^{7} \mathrm{~d} t
$$

what is the coefficient of $x^{4}$ in the expansion of $f(x)$ in powers of $x$ ?
(A) 70
(B) 162
(C) $\frac{2835}{4}$
(D) 3024
(E) 5670
(F) 15120
(G) 22680
[ENGAA, 2019S1Q36]
A light, vertical, copper wire of length 2.4 m and uniform cross-sectional area $2.0 \times 10^{-6} \mathrm{~m}^{2}$ supports a load of mass 4.0 kg .
The Young modulus of copper is $1.2 \times 10^{11} \mathrm{~Pa}$.
What is the strain energy in the wire?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that the wire obeys Hooke's law and that the cross-sectional area remains constant)
(A) $8.0 \times 10^{-5} \mathrm{~J}$
(B) $1.7 \times 10^{-4} \mathrm{~J}$
(C) $4.0 \times 10^{-4} \mathrm{~J}$
(D) $8.0 \times 10^{-3} \mathrm{~J}$
(E) $4.0 \times 10^{-2} \mathrm{~J}$
(F) $1.6 \times 10^{-2} \mathrm{~J}$
[ENGAA, 2019S1Q37]
The three internal angles in a triangle are $\alpha, \beta$ and $\theta$, and

$$
\begin{aligned}
& 3 \tan \alpha-2 \sin \beta=2 \\
& 5 \tan \alpha+6 \sin \beta=8
\end{aligned}
$$

What is the value of $\theta$ in degrees?
(A) 15
(B) 45
(C) 75
(D) 105
(E) 135
[ENGAA, 2019S1Q38]
Light travelling in a transparent liquid strikes the surface from below. The angle between the surface of the liquid and the direction of travel of the light is $\alpha$.
The light then travels along the surface between the liquid and the air as shown in the diagram.


Now, light travelling in air strikes the surface from above so that the angle between the surface and the direction of travel of this light is also $\alpha$.
After the light strikes the surface from above, the angle between the surface and the direction of travel of the refracted light is $\beta$.
Which expression gives $\beta$ ?
(all angles are in degrees)
(A) $\cos \beta=\cos ^{2} \alpha$
(B) $\cos \beta=\cos \alpha \sin ^{-1}\left(\frac{1}{\alpha}\right)$
(C) $\sin \beta=\sin ^{2} \alpha$
(D) $\beta=90-(90-\alpha)^{2}$
(E) $\beta=0$
(F) $\beta=\alpha$

## [ENGAA, 2019S1Q39]

Find the complete set of values of $x$ for which

$$
x^{3}-2 x^{2}-7 x-4>0
$$

(A) $x<-1$
(B) $x>-1$
(C) $-1<x<4$
(D) $x<-1$ or $x>4$
(E) $x<4$
(F) $x>4$
[ENGAA, 2019S1Q40]
The velocity-time graph is for an 80 kg person in a lift that is moving vertically upwards.


What is the magnitude of the contact force between the person and the lift floor at the time corresponding to $X$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 640 N
(B) 768 N
(C) 800 N
(D) 832 N
(E) 960 N

## ENGAA 2019 S2



## TIME ALLOWED: 60 MINUTES

This paper is Section 2 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each uestion is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.
[ENGAA, 2019S2Q1]
The ray diagram shows light passing from a vacuum into a medium.


Two angles, $x$ and $y$, are shown on the diagram.
When $x$ is $60^{\circ}, y$ is $45^{\circ}$.
When $x$ is $45^{\circ}$, what is the value of $\sin y$ ?
(A) $\frac{1}{\sqrt{3}}$
(B) $\frac{2}{\sqrt{3}}$
(C) 1
(D) $\frac{\sqrt{3}}{2}$
(E) $\sqrt{3}$
[ENGAA, 2019S2Q2]
Identical resistors are used to produce three different arrangements $X, Y$ and $Z$.


Each arrangement is connected, in turn, across the same battery which has a negligible internal resistance.
The total power developed in each of the arrangements is determined.
What is the order of the arrangements when placed in order of increasing power?
(A) $X, Y, Z$
(B) $X, Z, Y$
(C) $Y, X, Z$
(D) $Y, Z, X$
(E) $Z, X, Y$
(F) $Z, Y, X$
[ENGAA, 2019S2Q3]
A block of mass 2.0 kg is on a plane which is inclined to the horizontal at an angle of $30^{\circ}$.
The block is attached to a load of mass $M$ by a light, inextensible string which passes over a smooth pulley.


The load moves downwards at a constant speed.
A constant friction force of 5.0 N acts on the block while it moves.
What is the value of $M$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that air resistance is negligible)
(A) 0.50 kg
(B) 1.0 kg
(C) 1.5 kg
(D) 2.5 kg
(E) 4.0 kg
(F) 6.0 kg
[ENGAA, 2019S2Q4]
The battery in the circuit shown has an emf of 16 V and an internal resistance of $1.0 \Omega$.


Which line in the table gives the voltmeter readings when switch $S$ is in its open and closed states?

|  | voltmeter reading / V |  |
| :---: | :---: | :---: |
|  | whenS is open | whenS is closed |
| (A) | $\frac{4.0}{3.0}$ | 2.0 |
| (B) | 4.0 | 6.0 |
| (C) | 4.0 | 2.4 |
| (D) | 6.0 | 2.4 |
| (E) | 6.0 | 4.0 |
| (F) | $\frac{48}{11}$ | $\frac{48}{19}$ |
| (G) | $\frac{48}{11}$ | $\frac{48}{7.0}$ |
| (H) | $\frac{128}{11}$ | $\frac{64}{7.0}$ |

[ENGAA, 2019S2Q5]
A stationary wave is set up in a medium in which the speed of the wave is $3.2 \mathrm{~m} \mathrm{~s}^{-1}$.
The stationary wave is formed by the superposition of two longitudinal waves, each of amplitude 1.5 cm , travelling in opposite directions.
The distance between adjacent nodes in the stationary wave is 4.0 cm .
What is the total distance travelled by a particle at an antinode during a time interval of 1.0 minute?
(A) 0 m
(B) 72 m
(C) 144 m
(D) 192 m
(E) 288 m
(F) 576 m
[ENGAA, 2019S2Q6]
A ray of light of single frequency $f$ is travelling in a block of transparent material.
The ray strikes the boundary between the block and air at an angle $\theta$ to the boundary. When $\theta=65^{\circ}$ the ray is just at the threshold of being totally internally reflected.
Which of the following is an expression for the wavelength of the light in the material?
(The speed of light in air is $v_{\text {air }}$.)
(A) $\frac{v_{\text {air }} \times \cos 65^{\circ}}{f}$
(B) $\frac{v_{\text {air }} \times \sin 65^{\circ}}{f}$
(C) $\frac{f}{v_{\text {air }} \times \cos 65^{\circ}}$
(D) $\frac{f}{v_{\text {air }} \times \sin 65^{\circ}}$
(E) $\frac{v_{\text {air }}}{f \times \cos 65^{\circ}}$
(F) $\frac{v_{\text {air }}}{f \times \sin 65^{\circ}}$
(G) $\frac{f \times \cos 65^{\circ}}{v_{\text {air }}}$
(H) $\frac{f \times \sin 65^{\circ}}{v_{\text {air }}}$
[ENGAA, 2019S2Q7]
A solid pyramid of height 140 m has a square base.
The density of the stone from which the pyramid is made is $2100 \mathrm{~kg} \mathrm{~m}^{-3}$.
Atmospheric pressure is 100 kPa .
What is the average pressure on the ground under the pyramid?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; volume of a pyramid $=\frac{1}{3} \times$ base area $\times$ vertical height)
(A) 98 kPa
(B) 108 kPa
(C) 198 kPa
(D) 980 kPa
(E) 1080 kPa
(F) 2940 kPa
(G) 3040 kPa
[ENGAA, 2019S2Q8]
The pressure exerted by a gas at constant temperature is directly proportional to its density. A spherical bubble of gas forms at the bottom of a glass containing a fizzy drink. The radius of the bubble at the point of formation, at the bottom of the drink, is $R$. The depth of the liquid in the glass is $h$, and the density of the liquid of the drink is $\rho$. Atmospheric pressure is $P$.
As the bubble rises, its radius changes.
Which expression gives the radius of the bubble when it is at a distance $x$ below the surface of the drink?
(gravitational field strength $g$; volume of sphere $=\frac{4}{3} \pi r^{3}$ where $r$ is the radius; the mass and the temperature of the gas in the bubble remain constant)
(A) $R\left(\frac{h \rho g-P}{x \rho g-P}\right)^{\frac{1}{3}}$
(B) $R\left(\frac{h}{x}\right)^{\frac{1}{3}}$
(C) $R\left(\frac{h \rho g+P}{x \rho g+P}\right)^{\frac{1}{3}}$
(D) $R\left(\frac{x \rho g-P}{h \rho g-P}\right)^{\frac{1}{3}}$
(E) $R\left(\frac{x}{h}\right)^{\frac{1}{3}}$
(F) $R\left(\frac{x \rho g+P}{h \rho g+P}\right)^{\frac{1}{3}}$
[ENGAA, 2019S2Q9]
A block of mass 2.0 kg slides directly down a smooth slope.
The slope is at an angle of $30^{\circ}$ to the horizontal.
The block reaches a speed of $8.0 \mathrm{~m} \mathrm{~s}^{-1}$, at which point the slope becomes rough and the block begins to decelerate.
After travelling a distance of 4.0 m down the rough slope the block comes to rest.
What is the magnitude of the average friction force between the block and the rough slope?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that air resistance is negligible)
(A) 2.0 N
(B) 6.0 N
(C) 10 N
(D) 12 N
(E) 16 N
(F) $10 \sqrt{3} \mathrm{~N}$
(G) 26 N
(H) $(16+10 \sqrt{3}) \mathrm{N}$
[ENGAA, 2019S2Q10]
A non-uniform beam $P Q$ of length 5.0 m and weight $X$ rests on a pivot placed 3.0 m from end $P$. It is kept in equilibrium in a horizontal position by an upward force of magnitude 0.60 X acting at end $P$.
The normal contact force at the pivot is 800 N .
What is the weight of the beam and how far is the centre of gravity of the beam from the pivot?

|  | weight of beam $/ \mathrm{N}$ | distance from pivot/ m |
| :---: | :---: | :---: |
| (A) | 500 | 0.50 |
| (B) | 500 | 1.8 |
| (C) | 500 | 3.0 |
| (D) | 2000 | 0.50 |
| (E) | 2000 | 1.8 |
| (F) | 2000 | 3.0 |

[ENGAA, 2019S2Q11]
A car is travelling along a straight road with constant acceleration. It passes a road sign.
It travels 12.2 m in the 3 rd second and 14.4 m in the 4th second after passing the road sign.
What was the speed of the car as it passed the road sign?
(A) $2.20 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $4.50 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $6.70 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $7.80 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $13.3 \mathrm{~m} \mathrm{~s}^{-1}$
(F) $37.2 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2019S2Q12]
A light spring has unstretched length 0.40 m and spring constant $50 \mathrm{~N} \mathrm{~m}^{-1}$.
The spring is stretched by a varying tension force that starts at a value of zero and increases at a constant rate of $0.20 \mathrm{~N} \mathrm{~s}^{-1}$ up to a maximum value.
When the force reaches its maximum value, the strain energy of the spring is 0.25 J .
What is the average power used to stretch the spring?
(Assume that the spring obeys Hooke's law.)
(A) 0.010 W
(B) 0.020 W
(C) 0.040 W
(D) 0.080 W
(E) 1.0 W
(F) 2.0 W
(G) 4.0 W
(H) 8.0 W
[ENGAA, 2019S2Q13]
The circuit below contains three identical resistors, and two identical cells. When the switch is open, the power dissipated by resistor $X$ is $P$.


What is the power dissipated by resistor $X$ after the switch is closed?
(A) $\frac{P}{4}$
(B) $\frac{9 P}{16}$
(C) $\frac{3 P}{4}$
(D) $P$
(E) $\frac{16 P}{9}$
(F) $\frac{9 P}{4}$
[ENGAA, 2019S2Q14]
A car of mass $m$ is pulling a caravan of mass $M$.
The caravan is connected to the car by a metal bar of length $l$ and cross-sectional area $A$.
The Young modulus of the metal from which the bar is made is $E$.
The car and caravan have a constant forward acceleration $a$ and there are total resistive forces $D_{1}$ acting on the car and $D_{2}$ acting on the caravan.
What is the extension of the bar?
(Assume that the bar obeys Hooke's law and that the cross-sectional area of the bar remains unchanged.)
(A) $\frac{M a l}{E A}$
(B) $\frac{M a}{E A}$
(C) $\frac{\left(M a+D_{2}\right) l}{E A}$
(D) $\frac{M a+D_{2}}{E A}$
(E) $\frac{\left(M a+m a+D_{1}+D_{2}\right) l}{E A}$
(F) $\frac{M a+m a+D_{1}+D_{2}}{E A}$
[ENGAA, 2019S2Q15]
Four resistors, $P, Q, R$ and $S$, are connected to a battery with negligible internal resistance, as shown in the diagram.
$P$ and $S$ each have resistance $x . Q$ and $R$ each have resistance $2 x$.


A fifth resistor, $T$, which has resistance $x$, is to be added to the circuit in one of the following listed positions, as shown in the diagram:

1 in parallel with $P$
2 in series with $Q$
3 in parallel with $R$


Which of the positions for resistor $T$ causes an increase in the magnitude of the voltmeter reading?
(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2019S2Q16]
A ball is thrown vertically upwards in air. The ball travels upwards to reach its highest point and then falls back down to its initial starting position. The velocity-time graph for the ball is shown.


Which of the following statements is/are correct?
1 The magnitude of the acceleration of the ball is only equal to the magnitude of the acceleration of free fall when it is at its highest point.

2 The time taken for the upward journey of the ball is equal to the time taken for the journey back down to its starting position.

3 The maximum increase in the gravitational potential energy of the ball is less than its initial kinetic energy and greater than its kinetic energy when it returns to its starting position.
(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2019S2Q17]
A stone is projected from level ground at an angle of $30^{\circ}$ to the horizontal.
After 1.0 s the stone lands on a ledge at height $h$ above the level ground.
During this journey the vertical component of velocity of the stone is upwards for the first 0.60 s and downwards for the remaining 0.40 s .
What is the value of $h$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that air resistance is negligible)
(A) 1.0 m
(B) 1.6 m
(C) 2.0 m
(D) 3.0 m
(E) 3.2 m
(F) 6.0 m
(G) 7.0 m
(H) 11 m
[ENGAA, 2019S2Q18]
A drawbridge system consists of a uniform ramp, of weight $W$, that is smoothly hinged at its lower end. The upper end is connected by a light, inextensible cable to a winch that is fixed to the wall in the position shown in the diagram.


The ramp is lowered slowly, at constant speed, from its closed (vertical) position $\left(\theta=0^{\circ}\right)$ to its open (horizontal) position $\left(\theta=90^{\circ}\right.$ ).
What is the maximum tension in the cable during this process?
(double-angle identities: $\sin 2 \theta=2 \sin \theta \cos \theta ; \cos 2 \theta=\cos ^{2} \theta-\sin ^{2} \theta$ )
(A) $\frac{W}{2}$
(B) $\frac{W}{\sqrt{2}}$
(C) $\frac{\sqrt{3} W}{2}$
(D) $W$
(E) $\frac{2 W}{\sqrt{3}}$
(F) $\sqrt{2} W$
(G) 2 W
[ENGAA, 2019S2Q19]
A particle of mass $m$ has kinetic energy $E$ when it collides with a stationary particle of mass $M$. The two particles coalesce. Which of the following expressions gives the total kinetic energy transferred to other forms of energy in the collision?
(A) 0
(B) $\frac{M E}{(M+m)}$
(C) $\frac{m E}{(M+m)}$
(D) $\frac{(M+m) E}{m}$
(E) $\frac{(M+m) E}{M}$
(F) $\frac{m M E}{(M+m)^{2}}$
(G) $E$
[ENGAA, 2019S2Q20]
The critical angle for light incident on a boundary from medium $X$ to air is $45^{\circ}$.
The critical angle for light of the same frequency incident on a boundary from medium $Y$ to air is $60^{\circ}$.
There is a boundary between medium $X$ and medium $Y$. Light of the same frequency travelling in one of these mediums is incident on this boundary.
In which direction of incidence is there a critical angle at this boundary, and within what range is this critical angle?

|  | direction of incidence | critical angle |
| :---: | :---: | :---: |
| (A) | $X$ to $Y$ | between $0^{\circ}$ and $30^{\circ}$ |
| (B) | $X$ to $Y$ | between $30^{\circ}$ and $45^{\circ}$ |
| (C) | $X$ to $Y$ | between $45^{\circ}$ and $60^{\circ}$ |
| (D) | $X$ to $Y$ | between $60^{\circ}$ and $90^{\circ}$ |
| (E) | $Y$ to $X$ | between $0^{\circ}$ and $30^{\circ}$ |
| (F) | $Y$ to $X$ | between $30^{\circ}$ and $45^{\circ}$ |
| (G) | $Y$ to $X$ | between $45^{\circ}$ and $60^{\circ}$ |
| (H) | $Y$ to $X$ | between $60^{\circ}$ and $90^{\circ}$ |

## ENGAA 2020 S1



## TIME ALLOWED: 60 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.

## Part A Mathematics and Physics (20 questions)

Part B Advanced Mathematics and Advanced Physics (20 questions)
You are strongly advised to divide your time equally between the two parts: 30 minutes on Part A and 30 minutes on Part B. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 40 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

## Part A

[ENGAA, 2020S1Q1]
A soldering iron has a copper tip of mass 2.0 g .
The tip is heated with 30 W of thermal power. In 50 s , the temperature of the tip increases by $200^{\circ} \mathrm{C}$.
How much energy is transferred from the tip to the surroundings in this time?
(specific heat capacity of copper $=400 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{C}^{-1}$ )
(A) 160 J
(B) 500 J
(C) 1340 J
(D) 1500 J
(E) 1660 J
(F) 1840 J
(G) 2500 J
[ENGAA, 2020S1Q2]
The admission charge to a cinema is different for adults and children.
Admission for 2 adults and 3 children costs $£ 20$.
Admission for 4 adults and 4 children costs $£ 34$.
What does admission cost for 6 adults and 2 children?
(A) $£ 27$
(B) $£ 29$
(C) $£ 33$
(D) $£ 39$
(E) $£ 44$
(F) $£ 48$
(G) $£ 72$
[ENGAA, 2020S1Q3]
Uranium-238 ( ${ }_{92}^{238} \mathrm{U}$ ) decays by a series of alpha and beta ( $\beta^{-}$) emissions to become the stable isotope lead-206 ( $\left.{ }_{82}^{206} \mathrm{~Pb}\right)$.
How many beta ( $\beta^{-}$) particles are emitted in the decay of one uranium- 238 nucleus to lead206?
(A) 6
(B) 8
(C) 10
(D) 12
(E) 14
(F) 16
[ENGAA, 2020S1Q4]
A fair spinner has eight equal sections.
Each section has one number written on it, as shown.


The spinner is spun twice, and the two numbers scored are added
What is the probability that the sum of the two numbers is 5 ?
(A) $\frac{1}{8}$
(B) $\frac{5}{8}$
(C) $\frac{1}{16}$
(D) $\frac{3}{16}$
(E) $\frac{25}{64}$
(F) $\frac{55}{64}$
[ENGAA, 2020S1Q5]
A dc electricity transmission system uses an undersea cable to send electricity from one country to another. On a particular day, the first country supplies electricity at a voltage of 400 kV and 2000 A to the transmission system. The second country receives electricity from the transmission system at 160 kV and 4000 A .
What is the percentage efficiency of the system and how much energy is wasted every minute?

|  | efficiency\% | energy wasted every minute $/ \mathrm{J}$ |
| :---: | :---: | :---: |
| (A) | 20 | $9.6 \times 10^{9}$ |
| (B) | 20 | $3.84 \times 10^{10}$ |
| (C) | 20 | $4.8 \times 10^{10}$ |
| (D) | 80 | $9.6 \times 10^{9}$ |
| (E) | 80 | $3.84 \times 10^{10}$ |
| (F) | 80 | $4.8 \times 10^{10}$ |

[ENGAA, 2020S1Q6]
Consider the four lines with the following equations.
$12 x+6 y=3$
$29 y=3 x-4$
$32 y=6 x+3$
$4 \quad 4 x+6 y-9=0$
Which two lines are perpendicular?
(A) 1 and 2
(B) 1 and 3
(C) 1 and 4
(D) 2 and 3
(E) 2 and 4
(F) 3 and 4
[ENGAA, 2020S1Q7]
Two fixed horizontal metal rails are side by side and 12 cm apart. The rails are connected to a d.c. power supply by a switch that is initially open.

A freely moveable metal rod of length 20 cm is placed on the rails as shown in the diagram.
The diagram shows the arrangement seen from above.
The angle between the rod and the rails is $90^{\circ}$.


The whole arrangement is placed in a uniform magnetic field of magnitude 0.50 T that is directed perpendicularly into the page.
The moveable rod has a weight of 0.40 N .
The switch is now closed. As a result, there is a current of 2.4 A in the circuit and the rod moves. What is the initial magnitude of the acceleration of the rod and what is its direction?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )

|  | acceleration $/ \mathrm{m} \mathrm{s}^{-2}$ | direction |
| :---: | :---: | :---: |
| (A) | 0.36 | to the left |
| (B) | 0.36 | to the right |
| (C) | 0.60 | to the left |
| (D) | 0.60 | to the right |
| (E) | 3.6 | to the left |
| (F) | 3.6 | to the right |
| (G) | 6.0 | to the left |
| (H) | 6.0 | to the right |

[ENGAA, 2020S1Q8]
Find the sum of the solutions of

$$
2\left(\frac{x}{4}+3\right)^{2}-\left(\frac{x}{4}+3\right)-36=0
$$

(A) 2
(B) $\frac{3}{2}$
(C) $\frac{1}{2}$
(D) -4
(E) -13
(F) -22
(G) -26
(H) -34
[ENGAA, 2020S1Q9]
Two trolleys are moving towards each other along a straight horizontal track.
One trolley has mass 8.0 kg and is travelling to the right at $4.0 \mathrm{~m} \mathrm{~s}^{-1}$.
The other trolley has mass 2.0 kg and is travelling to the left at $1.0 \mathrm{~m} \mathrm{~s}^{-1}$.
When the trolleys collide they stick together.
How much kinetic energy is transferred to other forms of energy in the collision?
(A) 2.0 J
(B) 18 J
(C) 20 J
(D) 28 J
(E) 35 J
(F) 40 J
(G) 45 J
(H) 65 J
[ENGAA, 2020S1Q10]
When the expression

$$
(2 x+3)^{2}-(x-3)^{2}
$$

is written in the form $p(x+q)^{2}+r$, where $p, q$ and $r$ are constants, what is the value of $r$ ?
(A) -27
(B) -9
(C) 0
(D) 3
(E) 15
[ENGAA, 2020S1Q11]
A car of mass 800 kg travels in a straight line along a horizontal road.
The car accelerates non-uniformly from rest for 5.0 seconds and then moves at constant speed, as shown in the distance-time graph:


What is the average resultant force acting on the car over the time for which it is accelerating?
(A) 320 N
(B) 480 N
(C) 640 N
(D) 960 N
(E) 1600 N
(F) 3200 N
(G) 4800 N
[ENGAA, 2020S1Q12]
The number of pairs of winter boots sold on a day is inversely proportional to the cube of the outside temperature on that day, measured in ${ }^{\circ} \mathrm{C}$.
On a day when the outside temperature is $8^{\circ} \mathrm{C}, 250$ pairs of boots are sold.
The next day, when the outside temperature is $x^{\circ} \mathrm{C}$, the number of pairs of boots sold is $700 \%$ more than on the previous day.
What is the value of $x$ ?
(A) 2
(B) 4
(C) $\frac{8}{\sqrt[3]{7}}$
(D) $8 \sqrt[3]{7}$
(E) 16
[ENGAA, 2020S1Q13]
$P$ and $Q$ are two fixed points on the surface of the ocean which are 6.0 m apart.
An ocean wave travels in the direction $P$ to $Q$.
The wave has a frequency of 0.50 Hz and travels at a constant speed.
A wave peak passes $Q$ at time $t=0 \mathrm{~s}$.
The next wave peak travelling towards $Q$ passes $P$ at time $t=0.80 \mathrm{~s}$.
What is the speed of the wave?
(A) $2.1 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $3.4 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $5.0 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $7.5 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $20 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2020S1Q14]
In a sale, all prices are reduced by $25 \%$.
A customer calculates the pre-sale price of a bicycle incorrectly by increasing the marked sale price by $25 \%$.
The customer's calculated pre-sale price is incorrect by $£ 15$.
What is the correct pre-sale price of the bicycle?
(A) $£ 180$
(B) $£ 195$
(C) $£ 210$
(D) $£ 225$
(E) $£ 240$

## [ENGAA, 2020S1Q15]

A parachutist of mass 80.0 kg drops from a plane travelling at $40.0 \mathrm{~m} \mathrm{~s}^{-1}, 2000 \mathrm{~m}$ above the Earth's surface.

The parachutist hits the ground at a speed of $5.00 \mathrm{~m} \mathrm{~s}^{-1}$.
How much work is done by the parachutist against drag forces during the fall?
(Take the Earth's gravitational field strength to be $10.0 \mathrm{~N} \mathrm{~kg}^{-1}$.)
(A) 1535000 J
(B) 1624000 J
(C) 1649000 J
(D) 1663000 J
(E) 1726000 J
[ENGAA, 2020S1Q16]


In the diagram, $Q S$ is perpendicular to $P R$.
$P S=x \mathrm{~cm}$
$P Q=y \mathrm{~cm}$
$Q R=z \mathrm{~cm}$
angle $Q R S=61^{\circ}$
$P S R$ is a straight line.
Which one of the following is an expression for the length $z$, in cm ?
(A) $\sqrt{y^{2}+x^{2}} \sin 61^{\circ}$
(B) $\sqrt{y^{2}-x^{2}} \sin 61^{\circ}$
(C) $\sqrt{y^{2}+x^{2}} \cos 61^{\circ}$
(D) $\sqrt{y^{2}-x^{2}} \cos 61^{\circ}$
(E) $\frac{\sqrt{y^{2}+x^{2}}}{\sin 61^{\circ}}$
(F) $\frac{\sqrt{y^{2}-x^{2}}}{\sin 61^{\circ}}$
(G) $\frac{\sqrt{y^{2}+x^{2}}}{\cos 61^{\circ}}$
(H) $\frac{\sqrt{y^{2}-x^{2}}}{\cos 61^{\circ}}$
[ENGAA, 2020S1Q17]
A light spring of unstretched length 0.10 m has a spring constant of $20 \mathrm{~N} \mathrm{~m}^{-1}$. The spring is suspended so that it is vertical and a load of mass 0.050 kg is attached to the end of the spring. The load is pulled vertically downwards until the length of the spring is 0.30 m . The load is then released.
What is the speed of the load at the instant that the spring returns to its unstretched length? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that resistive forces are negligible)
(A) $0 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $4.0 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $6.0 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $12 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $16 \mathrm{~m} \mathrm{~s}^{-1}$
(F) $\sqrt{6} \mathrm{~m} \mathrm{~s}^{-1}$
(G) $\sqrt{12} \mathrm{~m} \mathrm{~s}^{-1}$
(H) $\sqrt{30} \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2020S1Q18]
Two vertices of a square are at $(1,1)$ and $(3,5)$.
What is the difference between the perimeters of the largest and smallest possible squares that can be drawn with these points as two of their vertices?
(A) 0
(B) $4 \sqrt{3}(2-\sqrt{2})$
(C) $4 \sqrt{3}(\sqrt{2}-1)$
(D) $4 \sqrt{5}(2-\sqrt{2})$
(E) $4 \sqrt{5}(\sqrt{2}-1)$
(F) $4 \sqrt{13}(2-\sqrt{2})$
(G) $4 \sqrt{13}(\sqrt{2}-1)$
(H) $4 \sqrt{3} \sqrt{5}(2-\sqrt{2})$
[ENGAA, 2020S1Q19]
A rocket travelling in space is burning its fuel at a constant rate. By expelling the burnt fuel through a nozzle, the engine is applying a constant force to the rocket.
What is happening to the magnitude of the acceleration of the rocket?
(A) It is increasing at an increasing rate.
(B) It is increasing at a constant rate.
(C) It is increasing at a decreasing rate.
(D) It is not changing.
(E) It is decreasing at an increasing rate.
(F) It is decreasing at a constant rate.
(G) It is decreasing at a decreasing rate.
[ENGAA, 2020S1Q20]
The quadratic equation $2 x^{2}-p x-4=0$, where $p$ is a positive constant, has two solutions that differ by 6 .
What is the value of $p$ ?
(A) 2
(B) $4 \sqrt{7}$
(C) 12
(D) $4 \sqrt{11}$
(E) $4 \sqrt{34}$
(F) $6 \sqrt{30}$

## Part B

[ENGAA, 2020S1Q21]
A block of mass $m$ slides down a rough slope.
At position 1 the velocity of the block is $v_{1}$.
At position 2 , which is a vertical distance $h$ below position 1 , the velocity of the block is $v_{2}$.


Which expression gives the work done against friction by the block as it slides from position 1 to position 2?
(gravitational field strength $=g$; assume that air resistance is negligible)
(A) $m g h+\frac{1}{2} m\left(v_{2}{ }^{2}-v_{1}{ }^{2}\right)$
(B) $m g h-\frac{1}{2} m\left(v_{2}{ }^{2}-v_{1}{ }^{2}\right)$
(C) $m g h+\frac{1}{2} m\left(v_{2}-v_{1}\right)^{2}$
(D) $m g h-\frac{1}{2} m\left(v_{2}-v_{1}\right)^{2}$
(E) $\frac{1}{2} m\left(v_{2}{ }^{2}-v_{1}{ }^{2}\right)-m g h$
(F) $\frac{1}{2} m\left(v_{1}{ }^{2}-v_{2}{ }^{2}\right)-m g h$
(G) $\frac{1}{2} m\left(v_{2}-v_{1}\right)^{2}-m g h$
(H) $\frac{1}{2} m\left(v_{1}-v_{2}\right)^{2}-m g h$

## [ENGAA, 2020S1Q22]

$(x-1)$ and $(x-2)$ are both factors of $x^{4}+a x^{3}+b x^{2}-12 x+4$.
What are the values of $a$ and $b$ ?
(A) $a=-6$ and $b=-23$
(B) $a=-6$ and $b=13$
(C) $a=6$ and $b=-11$
(D) $a=6$ and $b=1$
[ENGAA, 2020S1Q23]
The braking system of a car includes two cylinders containing an incompressible oil, linked by a flexible tube that also contains oil. There is a freely moving piston in each cylinder. One piston is labelled $X$, and the other is labelled $Y$ in the diagram.


When the driver presses on the brake pedal, a force is exerted on piston $X$. The pressure produced in the oil by this force is transmitted through the oil so that it causes a force to act on piston $Y$. This presses the brakes against the moving parts.
The diameter of piston $X$ is 4.0 cm . The diameter of piston $Y$ is 12.0 cm .
The driver exerts a force of 36.0 N on piston $X$ and it moves a distance of 5.4 cm to the right.
What is the resultant force on piston $Y$ and how far does it move along the cylinder?

|  | force on piston $Y / \mathrm{N}$ | distance moved by piston $Y / \mathrm{cm}$ |
| :--- | :--- | :--- |
| (A) | 4.0 | 0.60 |
| (B) | 4.0 | 48.6 |
| (C) | 12.0 | 1.80 |
| (D) | 12.0 | 16.2 |
| (E) | 108 | 1.80 |
| (F) | 108 | 16.2 |
| (G) | 324 | 0.60 |
| (H) | 324 | 48.6 |

[ENGAA, 2020S1Q24]
What is the area of the region enclosed between the curve $y=\frac{1}{2} x^{2}$, the line $y=-x$, and the lines $x=1$ and $x=3$ ?
(A) $\frac{1}{3}$
(B) 2
(C) 4
(D) 6
(E) $\frac{25}{3}$
(F) $\frac{28}{3}$
[ENGAA, 2020S1Q25]
The graph shows how the displacement due to a wave in air varies with time.


The speed of the wave in air is $300 \mathrm{~m} \mathrm{~s}^{-1}$.
The wave now travels into water.
$\frac{\text { wave speed in air }}{\text { wave speed in water }}=0.2$
What is the wavelength of the wave in water?
(A) $\frac{1}{6} \mathrm{~m}$
(B) $\frac{2}{9} \mathrm{~m}$
(C) $\frac{5}{6} \mathrm{~m}$
(D) $\frac{9}{10} \mathrm{~m}$
(E) $\frac{10}{9} \mathrm{~m}$
(F) $\frac{6}{5} \mathrm{~m}$
(G) $\frac{9}{2} \mathrm{~m}$
(H) 6 m
[ENGAA, 2020S1Q26]
A line with non-zero gradient $m$ is reflected in the line $y=x$.
What is the gradient of the reflected line?
(A) $m$
(B) $-m$
(C) $\frac{1}{m}$
(D) $-\frac{1}{m}$
[ENGAA, 2020S1Q27]
A flat rectangular coil of wire with sides of length 30 cm and 20 cm is freely pivoted about an axle. The axle passes through the middle of the sides of length 30 cm .
Part of the coil is between the poles of a U-shaped magnet as shown in the diagram. The poles are 4.0 cm long. The magnetic field can be considered uniform between the poles, and zero elsewhere.

The coil is connected to a power supply so that there is a current in it.


The current is 0.60 A and the magnetic flux density is 0.050 T . There are 50 turns of wire in the coil.
What is the moment about the axle, in Ncm , produced by the magnetic force acting on the coil?
(A) 0.018 N cm
(B) 0.036 N cm
(C) 0.045 N cm
(D) 0.90 N cm
(E) 1.8 N cm
(F) 2.25 N cm
(G) 4.5 N cm
[ENGAA, 2020S1Q28]
The sum of the first 20 terms of an arithmetic progression is 50.
The sum of the next 20 terms of the arithmetic progression is -50 .
What is the sum of the first 100 terms of the arithmetic progression?
(A) -750
(B) -350
(C) -50
(D) $-\frac{159}{8}$
(E) $\frac{159}{8}$
(F) 50
(G) 350
(H) 750
[ENGAA, 2020S1Q29]
A box of mass 3.0 kg is pulled a distance 5.0 m directly up a smooth slope by a constant applied force of 30 N acting parallel to the slope.
The initial speed of the box is $3.0 \mathrm{~m} \mathrm{~s}^{-1}$ and the final speed is $7.0 \mathrm{~m} \mathrm{~s}^{-1}$. Its acceleration is constant.


What is the component of the weight acting down the slope?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; air resistance is negligible)?
(A) 12 N
(B) 18 N
(C) 22 N
(D) 28 N
(E) 29 N
(F) 42 N
(G) 90 N
[ENGAA, 2020S1Q30]
The line $L$ with equation $y=m x+c$, where $m>0$ and $c \geq 0$, passes through the point $(2,4)$. A line is drawn through the point $(2,4)$ perpendicular to $L$.
The triangle enclosed between the two lines and the $y$-axis has area 5 square units.
What is the larger of the two possible values of $m$ ?
(A) -0.5
(B) 0.5
(C) 1.25
(D) 2
(E) 5
[ENGAA, 2020S1Q31]
Electrical power is supplied through a d.c. transmission line that consists of two metal wires. Each wire is 8.0 km long and has a cross-sectional area of $1.0 \mathrm{~cm}^{2}$.


The resistivity of the metal from which the wires are made is $2.5 \times 10^{-7} \Omega \mathrm{~m}$.
Electrical power is transmitted to the transmission line at a potential difference of 24000 V . At what rate is energy being wasted as heat in the wires when the power supplied to the transmission line is 120 kW ?
(A) 0.40 W
(B) 0.80 W
(C) 1.6 W
(D) 250 W
(E) 500 W
(F) 1000 W
(G) $1.44 \times 10^{7} \mathrm{~W}$
(H) $5.76 \times 10^{7} \mathrm{~W}$
[ENGAA, 2020S1Q32]
$P$ and $Q$ are two different geometric progressions.
The 3rd term of each geometric progression is 4.
The 5th term of each geometric progression is 2 .
What is the modulus of the difference between the sums to infinity of $P$ and $Q$ ?
(A) 0
(B) 8
(C) $8 \sqrt{2}$
(D) 16
(E) $16 \sqrt{2}$
(F) 32
(G) $32 \sqrt{2}$
[ENGAA, 2020S1Q33]
A tennis ball travelling at $24.0 \mathrm{~m} \mathrm{~s}^{-1}$ is hit by a racket. As a result of the impact, the ball returns back along its original path having undergone a change in velocity of $48.0 \mathrm{~m} \mathrm{~s}^{-1}$. The acceleration of the ball whilst in contact with the racket is constant with magnitude $6000 \mathrm{~m} \mathrm{~s}^{-}$ ${ }^{2}$.
What is the total distance travelled by the ball whilst in contact with the racket?
(A) 0.00 cm
(B) 4.80 cm
(C) 9.60 cm
(D) 14.4 cm
(E) 19.2 cm

## [ENGAA, 2020S1Q34]

The curve

$$
y=x^{3}+3 \sqrt{5} p x^{2}+3 p x+13
$$

has two distinct turning points.
What are all the possible values of $p$ ?
(A) $p<0, p\rangle 0.2$
(B) $p \leq 0, p \geq 0.2$
(C) $0<p<0.2$
(D) $0 \leq p \leq 0.2$
(E) $p<0, p\rangle 1.2$
(F) $p \leq 0, p \geq 1.2$
(G) $0<p<1.2$
(H) $0 \leq p \leq 1.2$
[ENGAA, 2020S1Q35]
The diagram shows the relative positions of two identical light springs, both in equilibrium. The springs are supporting loads of mass 0.20 kg and 1.0 kg as shown.


The same two springs are now connected in parallel, supporting a 2.0 kg mass as shown.


In this parallel arrangement, what is the total strain energy in the springs?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that the springs obey Hooke's law)
(A) 0.25 J
(B) 0.40 J
(C) 0.50 J
(D) 1.0 J
(E) 25 J
(F) 40 J
(G) 50 J
(H) 100 J
[ENGAA, 2020S1Q36]
Find the number of solutions of the equation

$$
14 \cos ^{3} x+10 \sin ^{2} x \cos x=13 \cos x
$$

in the range $-2 \pi \leq x \leq 2 \pi$.
(A) 4
(B) 6
(C) 8
(D) 10
(E) 12
(F) 14
[ENGAA, 2020S1Q37]
An object of mass 2.5 kg is at rest at time $t=0 \mathrm{~s}$. A resultant force acts on the object in a constant direction.
The magnitude of the resultant force acting on the object varies with time as shown by the graph.


What is the kinetic energy of the object at time $t=0.20 \mathrm{~s}$ ?
(A) 0 J
(B) 0.80 J
(C) 1.0 J
(D) 1.6 J
(E) 2.0 J
(F) 3.2 J
(G) 6.4 J
[ENGAA, 2020S1Q38]
Find the product of the real roots of the equation

$$
\left(\log _{10} x^{2}\right)^{2}+\log _{10} x=3
$$

(A) $10^{-\frac{3}{2}}$
(B) $10^{-1}$
(C) $10^{-\frac{1}{2}}$
(D) $10^{-\frac{1}{4}}$
(E) $10^{\frac{3}{5}}$
(F) $10^{1}$

## [ENGAA, 2020S1Q39]

In the following circuit, the ammeter records a current of zero.


What is the resistance of resistor $R$ ?
(A) $0 \Omega$
(B) $200 \Omega$
(C) $300 \Omega$
(D) $400 \Omega$
(E) $600 \Omega$
(F) $1200 \Omega$
(G) $1800 \Omega$
(H) $2400 \Omega$
[ENGAA, 2020S1Q40]
Find the maximum value of the gradient of the curve with equation

$$
y=2-4 x+4 x^{\frac{3}{2}}-x^{2}
$$

where $x>0$.
(A) -4
(B) $-\frac{8}{9}$
(C) $\frac{1}{2}$
(D) 2
(E) 4

## ENGAA 2020 S2



## TIME ALLOWED: 60 MINUTES

This paper is Section 2 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, datecof birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.
[ENGAA, 2020S2Q1]
Spring $P$ has spring constant $1.0 \mathrm{~N} \mathrm{~cm}^{-1}$ and spring $Q$ has spring constant $3.0 \mathrm{~N} \mathrm{~cm}^{-1}$.
The two springs are connected in series.
The springs are stretched by 6.0 cm in total.
What is the extension of spring $P$ ?
(The springs have negligible mass and obey Hooke's law.)
(A) 1.5 cm
(B) 2.0 cm
(C) 3.0 cm
(D) 4.0 cm
(E) 4.5 cm
[ENGAA, 2020S2Q2]
A single strand of wire has a radius of $2.0 \times 10^{-4} \mathrm{~m}$ and length 15 m . The resistivity of the material from which the wire is made is $4.8 \times 10^{-7} \Omega \mathrm{~m}$.
Twelve strands of this wire are connected in parallel to make a cable.
What is the resistance of the cable?
(A) $\frac{\pi}{2160} \Omega$
(B) $\frac{\pi}{180} \Omega$
(C) $\frac{\pi}{15} \Omega$
(D) $\frac{15}{\pi} \Omega$
(E) $\frac{180}{\pi} \Omega$
(F) $\frac{2160}{\pi} \Omega$
[ENGAA, 2020S2Q3]
A ray of light is directed into a semicircular transparent block, entering at $P$. The direction of the ray is adjusted until it strikes the centre of the flat face $X Y$ of the block at the critical angle and reflects to $Q$ as shown.


The length of $X Y$ is $L$.
The speed of light in air is $c$.
What is the time taken by the light to travel from $P$ to $Q$ in the block?
(A) $\frac{L \sqrt{3}}{2 c}$
(B) $\frac{L}{c}$
(C) $\frac{2 L}{c \sqrt{3}}$
(D) $\frac{L \sqrt{3}}{c}$
(E) $\frac{2 L}{c}$
(F) $\frac{4 L}{c \sqrt{3}}$
[ENGAA, 2020S2Q4]
A solid cube with sides of length 20 cm is made from material with density $2000 \mathrm{~kg} \mathrm{~m}^{-3}$. The cube is suspended, in equilibrium, from an initially unstretched spring, and this results in the spring gaining strain energy of 3.2 J .
What is the spring constant of the spring?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; the spring obeys Hooke's law)
(A) $40 \mathrm{~N} \mathrm{~m}^{-1}$
(B) $80 \mathrm{~N} \mathrm{~m}^{-1}$
(C) $400 \mathrm{~N} \mathrm{~m}^{-1}$
(D) $800 \mathrm{~N} \mathrm{~m}^{-1}$
(E) $4000 \mathrm{~N} \mathrm{~m}^{-1}$
(F) $8000 \mathrm{~N} \mathrm{~m}^{-1}$
[ENGAA, 2020S2Q5]
A projectile is fired upwards from the ground at an angle of $60^{\circ}$ to the vertical at a speed of 20 $\mathrm{m} \mathrm{s}^{-1}$.
It travels a horizontal distance $d$ and lands with a downwards vertical component of velocity of $4.0 \mathrm{~m} \mathrm{~s}^{-1}$ on ground that is height $h$ above the starting point of the projectile.
What are $d$ and $h$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that air resistance is negligible)

|  | $d / \mathrm{m}$ | $h / \mathrm{m}$ |
| :---: | :---: | :---: |
| (A) | $6.0 \sqrt{3}$ | 4.2 |
| (B) | $6.0 \sqrt{3}$ | 5.8 |
| (C) | $10 \sqrt{3}-4.0$ | 4.2 |
| (D) | $10 \sqrt{3}-4.0$ | 14.2 |
| (E) | $10 \sqrt{3}+4.0$ | 5.8 |
| (F) | $10 \sqrt{3}+4.0$ | 14.2 |
| (G) | $14 \sqrt{3}$ | 4.2 |
| (H) | $14 \sqrt{3}$ | 5.8 |

[ENGAA, 2020S2Q6]
Diagram 1 shows the positions of nine equally spaced particles in a medium.


Diagram 1
Diagram 2 shows the positions of the same nine particles, at a particular time, while a longitudinal wave is travelling through the medium.


## Diagram 2

What is the amplitude of the wave?
(A) 0.4 m
(B) 0.5 m
(C) 0.6 m
(D) 0.7 m
(E) 2.0 m
(F) 4.0 m
(G) 6.0 m
(H) 8.0 m
[ENGAA, 2020S2Q7]
A spaceship with mass $8.0 \times 10^{4} \mathrm{~kg}$ travels at constant velocity and has $1.0 \times 10^{12} \mathrm{~J}$ of kinetic energy.
An external impulse of $8.0 \times 10^{7} \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$, lasting for 2.0 s , is applied to the spaceship acting in the opposite direction to the motion of the spaceship.
What is the average rate of loss of kinetic energy of the spaceship during the application of the impulse?
(A) $9.5 \times 10^{10} \mathrm{~W}$
(B) $1.8 \times 10^{11} \mathrm{~W}$
(C) $2.2 \times 10^{11} \mathrm{~W}$
(D) $3.2 \times 10^{11} \mathrm{~W}$
(E) $3.6 \times 10^{11} \mathrm{~W}$
(F) $7.2 \times 10^{11} \mathrm{~W}$
[ENGAA, 2020S2Q8]
The diagram shows a solid triangular prism.


The sides of the triangular cross section of the prism are of length $x$.
The height of the prism is $3 x$.
The uniform density of the prism is $\rho$.
The gravitational field strength is $g$.
What is the minimum pressure the prism can exert when it rests on level ground?
(A) $3 \rho g$
(B) $3 \rho g x$
(C) $\frac{\rho g}{4}$
(D) $\frac{\rho g x}{4}$
(E) $\frac{\sqrt{3} \rho g}{4}$
(F) $\frac{\sqrt{3} \rho g x}{4}$
[ENGAA, 2020S2Q9]
An apple of mass $m_{\mathrm{a}}$ is placed on a uniform metre rule with the centre of gravity of the apple at the 10 cm mark. The rule is balanced on a pivot placed at the 35 cm mark.
The apple is replaced with an orange of mass $m_{0}$. The rule now balances with the pivot at the 40 cm mark.
What is the ratio $\frac{m_{\mathrm{a}}}{m_{\mathrm{o}}}$ ?
(A) $\frac{5}{9}$
(B) $\frac{4}{5}$
(C) $\frac{5}{6}$
(D) $\frac{6}{5}$
(E) $\frac{5}{4}$
(F) $\frac{9}{5}$

## [ENGAA, 2020S2Q10]

A cyclist travels at a constant speed of $12 \mathrm{~m} \mathrm{~s}^{-1}$ on level ground. During this time the power needed to maintain a constant speed is 900 W . The total weight of the cyclist and bicycle is 850 N .
The cyclist now cycles up a slope at the same constant speed. The slope is at an angle of $30^{\circ}$ to the horizontal.
What is the driving force on the bicycle as it travels up the slope?
(Assume that the magnitude of the resistive forces is constant.)
(A) 75 N
(B) 350 N
(C) 500 N
(D) $(425 \sqrt{3}-75) \mathrm{N}$
(E) 775 N
(F) $(425 \sqrt{3}+75) \mathrm{N}$
(G) 925 N
[ENGAA, 2020S2Q11]
Three identical resistors can be combined in four different arrangements.
One of the arrangements has a resistance of $18 \Omega$.
(A) different arrangement has a resistance of $8.0 \Omega$.

What are the resistances of the other two arrangements?
(All three resistors contribute to the total resistance in all arrangements.)?
(A) $2.0 \Omega$ and $4.0 \Omega$
(B) $2.0 \Omega$ and $9.0 \Omega$
(C) $4.0 \Omega$ and $12 \Omega$
(D) $4.0 \Omega$ and $36 \Omega$
(E) $36 \Omega$ and $162 \Omega$
(F) $81 \Omega$ and $162 \Omega$
[ENGAA, 2020S2Q12]
A $4.0 \mathrm{k} \Omega$ fixed resistor is connected in series with a light dependent resistor (LDR) across a 100 V d.c. power supply.
The current in the LDR is 5.0 mA .
The intensity of light falling on the LDR now decreases and the voltage across the fixed resistor changes by $50 \%$.
What is the change in the resistance of the LDR as a result of the change in intensity?
(A) $8.0 \mathrm{k} \Omega$
(B) $12 \mathrm{k} \Omega$
(C) $16 \mathrm{k} \Omega$
(D) $20 \mathrm{k} \Omega$
(E) $32 \mathrm{k} \Omega$
(F) $36 \mathrm{k} \Omega$
[ENGAA, 2020S2Q13]
An elastic cord with spring constant $k$ is fixed to two points $P$ and $Q$ on the diameter of a ring so that the cord is taut but unstretched. The radius of the ring is $r$.


The midpoint of the cord is then pulled and fixed to a point on the ring halfway between $P$ and $Q$.
What is the energy stored in the elastic cord?
(A) $\frac{1}{2} k r^{2}$
(B) $2 k r^{2}$
(C) $\frac{1}{2}(\sqrt{2}-1) k r^{2}$
(D) $2(\sqrt{2}-1) k r^{2}$
(E) $\frac{1}{2}(3-2 \sqrt{2}) k r^{2}$
(F) $2(3-2 \sqrt{2}) k r^{2}$
[ENGAA, 2020S2Q14]
An object of mass $M$ experiences a resultant force of magnitude $F$. The force acts in a single horizontal direction with a magnitude that varies with time $t$ according to

$$
F=X+Y \sqrt{t}
$$

where $X$ and $Y$ are constants.
The object is at rest at $t=0$.
What is the magnitude of the momentum of the object at time $t=T$ ?
(A) $T\left(X+\frac{2}{3} Y \sqrt{T}\right)$
(B) $T(X+Y \sqrt{T})$
(C) $\frac{T}{M}\left(X+\frac{2}{3} Y \sqrt{T}\right)$
(D) $\frac{T}{M}(X+Y \sqrt{T})$
(E) $\frac{Y}{2 \sqrt{T}}$
(F) $\frac{Y}{2 M \sqrt{T}}$
[ENGAA, 2020S2Q15]
A trolley of mass 3.0 kg is moving horizontally along a smooth track. Its displacement $x$ from a point at time $t$ is given by the equation:

$$
x=8+4 t+2 t^{2}
$$

where $x$ is in metres and $t$ is in seconds.
How much work is done on the trolley between times $t=0$ and $t=5.0 \mathrm{~s}$ ?
(A) 12 J
(B) 24 J
(C) 78 J
(D) 270 J
(E) 840 J
(F) 864 J
(G) 936 J
[ENGAA, 2020S2Q16]
The diagram shows a ray of light passing through three mediums, $P, Q$ and $R$. The boundaries between the three mediums are parallel.


The ratio of the speed of light in medium $P$ to the speed of light in medium $Q$ is $2: \sqrt{5}$. The ratio of the speed of light in medium $Q$ to the speed of light in medium $R$ is $3: \sqrt{6}$. What is the value of $\sin \theta$ ?
(A) $\frac{\sqrt{2}}{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{\sqrt{3}}{6}$
(D) $\frac{\sqrt{5}}{5}$
(E) $\frac{\sqrt{15}}{5}$
(F) $\frac{\sqrt{15}}{6}$
[ENGAA, 2020S2Q17]
Water in a wide river flows at a constant speed of $0.50 \mathrm{~m} \mathrm{~s}^{-1}$. A swimmer swims around a square path of side 30 m marked out by 4 posts $R, S, T$ and $U$ which are fixed to the river bed, as shown.
The swimmer has a constant speed of $1.0 \mathrm{~m} \mathrm{~s}^{-1}$ relative to the water.


How long does it take for the swimmer to swim around the square path once?
(A) $(60+24 \sqrt{5}) \mathrm{s}$
(B) $(60+40 \sqrt{3}) \mathrm{s}$
(C) $(80+24 \sqrt{5}) \mathrm{s}$
(D) $(80+40 \sqrt{3}) \mathrm{s}$
(E) 120 s
(F) 140 s
[ENGAA, 2020S2Q18]
The stress in a steel cable increases with time and is then maintained at a constant value, as shown. The wire does not reach its limit of proportionality.


The table shows properties of the steel used in the cable and the dimensions of the cable.

| length $/ \mathrm{m}$ | cross-sectional area $/ \mathrm{m}^{2}$ | Young modulus $/ \mathrm{Pa}$ |
| :---: | :---: | :---: |
| 4.0 | $2.0 \times 10^{-4}$ | $2.0 \times 10^{11}$ |

How much work was done to stretch the cable?
(A) 320 J
(B) 1.28 kJ
(C) 2.56 kJ
(D) 320 kJ
(E) 640 kJ
(F) 1.60 MJ
(G) 6.40 MJ
[ENGAA, 2020S2Q19]
The following graph shows how the displacement of an object travelling along a straight, horizontal track varies with time.


Which graph shows the velocity of this object against displacement?

(A)

(B)

(D)

(G)

(C)

(F)
[ENGAA, 2020S2Q20]
A cell has emf $E$ and internal resistance $r$ that varies with current $I$ according to:

$$
r=k I^{2}
$$

where $k$ is a constant.
A variable resistor is connected to the terminals of the cell. The resistance of the variable resistor is adjusted.

Which expression gives the resistance of the variable resistor, in terms of $k$ and $E$, that causes maximum power dissipation in it?
(A) $3\left(\frac{k E^{2}}{2}\right)^{\frac{1}{3}}$
(B) $3\left(\frac{k E^{2}}{4}\right)^{\frac{1}{3}}$
(C) $3\left(\frac{k E^{2}}{9}\right)^{\frac{1}{3}}$
(D) $3\left(\frac{k E^{2}}{16}\right)^{\frac{1}{3}}$
(E) $\left(2 k E^{2}\right)^{\frac{1}{3}}$
(F) $\left(4 k E^{2}\right)^{\frac{1}{3}}$
(G) $\left(9 k E^{2}\right)^{\frac{1}{3}}$
(H) $\left(16 k E^{2}\right)^{\frac{1}{3}}$

## ENGAA 2021 S1



## TIME ALLOWED: 60 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.

## Part A Mathematics and Physics (20 questions)

Part B Advanced Mathematics and Advanced Physics (20 questions)
You are strongly advised to divide your time equally between the two parts: 30 minutes on Part A and 30 minutes on Part B. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 40 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

## Part A

[ENGAA, 2021S1Q1]
Simplify fully

$$
5 x y^{2} \times\left(5 x^{2} y\right)^{-3} \times 5 x^{2} y
$$

where $x$ and $y$ are positive.
(A) $\frac{1}{125 x^{7} y^{2}}$
(B) $\frac{1}{125 x^{6} y^{2}}$
(C) $\frac{1}{25 x^{6} y}$
(D) $\frac{1}{25 x^{4} y}$
(E) $\frac{1}{5 x^{3}}$
(F) $\frac{1}{5 x^{2}}$
(G) $\frac{y}{x^{2}}$
(H) $5 x y^{2}$
[ENGAA, 2021S1Q2]
Air is trapped in a cylinder by a piston. The density of the air in the cylinder is $\rho$.


The piston is moved so that the pressure of the trapped air increases by $20 \%$. The temperature of the trapped air does not change.
What is the new density of the trapped air?
(Assume that air is an ideal gas.)
(A) $0.69 \rho$
(B) $0.80 \rho$
(C) $0.83 \rho$
(D) $1.00 \rho$
(E) $1.20 \rho$
(F) $1.44 \rho$
[ENGAA, 2021S1Q3]
Which of the following is a rearrangement of

$$
\frac{p}{2}+\frac{3}{q}=\frac{4}{r}
$$

so that $q$ is the subject?
(A) $q=\frac{2 r}{24-3 p r}$
(B) $q=\frac{3 r}{2 r-p}$
(C) $q=\frac{6 r}{4-p}$
(D) $q=\frac{6 r}{8-p r}$
(E) $q=\frac{r-2}{12 p}$
(F) $q=\frac{3 r-6}{4 p}$
(G) $q=\frac{p r-8}{12 p}$
(H) $q=\frac{3 p r-24}{4 p}$
[ENGAA, 2021S1Q4]
A non-ideal transformer has 100 turns on the primary coil and 25 turns on the secondary coil. It is provided with 3.0 kW of electrical power at a current of 12.5 A .
The voltage output is the same as for an ideal transformer, but the current in the output coil is 40 A .
What is the efficiency of the transformer?
(A) $20 \%$
(B) $25 \%$
(C) $31 \%$
(D) $69 \%$
(E) $75 \%$
(F) $80 \%$
(G) $91 \%$
(H) $100 \%$
[ENGAA, 2021S1Q5]
Two solid cylinders, P and Q , are shown, where $x>y$.


Cylinder P has diameter $x$ and height $y$.
Cylinder Q has diameter $y$ and height $x$.
What is the positive difference between the total surface areas of P and Q ?
(A) 0
(B) $\frac{\pi}{4}\left(x^{2}-y^{2}\right)$
(C) $\frac{\pi}{2}\left(x^{2}-y^{2}\right)$
(D) $\pi\left(x^{2}-y^{2}\right)$
(E) $2 \pi\left(x^{2}-y^{2}\right)$
(F) $\frac{\pi}{4} x y(x-y)$
(G) $\pi x y(x-y)$

## [ENGAA, 2021S1Q6]

A light spring has an uncompressed length of 0.10 m . When an object of mass 0.5 kg rests in equilibrium on top of the spring, the length of the spring reduces to 0.08 m as shown.


What is the energy stored in the spring due to the compression?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; the spring obeys Hooke's law)
(A) 0.005 J
(B) 0.02 J
(C) 0.05 J
(D) 0.1 J
(E) 0.2 J
(F) 0.4 J
[ENGAA, 2021S1Q7]
The price of item P is reduced by $10 \%$. The next day, the new price is increased by $10 \%$.
The price of item Q is increased by $10 \%$. The next day, the new price is reduced by $10 \%$.
How does the final price of each item compare to the original price of that item?

|  | item P final price | item Q final price |
| :---: | :---: | :---: |
| (A) | lower than original | lower than original |
| (B) | lower than original | higher than original |
| (C) | higher than original | lower than original |
| (D) | higher than original | higher than original |
| (E) | the same as original | the same as original |

[ENGAA, 2021S1Q8]
A set of decorative lights consists of 20 lamps connected in series to a dc supply of constant voltage.
The total power transferred by all the lamps is $P$.
The set is designed so that if one of the lamps fails, that lamp becomes short-circuited and it then has zero resistance. The remaining lamps are still lit.
If this happens, with the set connected to the same supply, what is the new total power transferred by the remaining 19 lamps?
(Assume that the resistance of each functioning lamp remains constant.)
(A) $\left(\frac{19}{20}\right)^{2} P$
(B) $\left(\frac{19}{20}\right) P$
(C) $P$
(D) $\left(\frac{20}{19}\right) P$
(E) $\left(\frac{20}{19}\right)^{2} P$
[ENGAA, 2021S1Q9]
$S Q T$ is a right-angled triangle with the right angle at $Q$.
The point $R$ is on $S Q$ such that $S R: R Q=1: 3$
$Q R P$ is a right-angled triangle with the right angle at $Q$.
$P R=S T=8 \mathrm{~cm}$
$Q T=4 \mathrm{~cm}$
What is the length of $P Q$, in cm ?

[diagram not to scale]
(A) $2 \sqrt{3}$
(B) $4 \sqrt{3}$
(C) $\sqrt{19}$
(D) $\sqrt{37}$
(E) $\sqrt{55}$
(F) $\sqrt{61}$
[ENGAA, 2021S1Q10]
A train accelerates from rest along a straight, horizontal section of track.
The force exerted on the train due to its motors is constant and there is a constant friction force of $1.8 \times 10^{7} \mathrm{~N}$.
The graph shows how the momentum of the train changes with time.


What is the force exerted on the train due to its motors?
(A) $3.0 \times 10^{6} \mathrm{~N}$
(B) $6.0 \times 10^{6} \mathrm{~N}$
(C) $1.2 \times 10^{7} \mathrm{~N}$
(D) $1.5 \times 10^{7} \mathrm{~N}$
(E) $2.1 \times 10^{7} \mathrm{~N}$
(F) $2.4 \times 10^{7} \mathrm{~N}$
(G) $3.0 \times 10^{7} \mathrm{~N}$
(H) $4.2 \times 10^{7} \mathrm{~N}$
[ENGAA, 2021S1Q11]
The curve with equation $y=x^{2}-4 x+5$ meets the straight line with equation $y=2 x+c$ at two points, which have $x$-coordinates $p$ and $q$, where $q>p$.
Given that $q-p=8$, what is the value of the constant $c$ ?
(A) -43
(B) -12
(C) -2
(D) 0
(E) 2
(F) 12
(G) 43
[ENGAA, 2021S1Q12]
A ship travels into a wave that is travelling in the opposite direction to the ship. The ship has a horizontal speed of $8.0 \mathrm{~m} \mathrm{~s}^{-1}$. The speed of the wave is $3.0 \mathrm{~m} \mathrm{~s}^{-1}$. The front of the ship rises and falls with a time period of 8.0 s .
What is the wavelength of the wave?
(A) $\frac{3}{8} \mathrm{~m}$
(B) $\frac{5}{8} \mathrm{~m}$
(C) 1.0 m
(D) $\frac{11}{8} \mathrm{~m}$
(E) 24 m
(F) 40 m
(G) 64 m
(H) 88 m
[ENGAA, 2021S1Q13]
Given that

$$
y=\frac{\sin 60^{\circ}-1}{\cos 60^{\circ}}
$$

what is the value of $y^{3}$ ?
(A) $-\frac{\sqrt{3}}{9}$
(B) $-5 \sqrt{2}+10$
(C) $3 \sqrt{3}-8$
(D) $6 \sqrt{3}-10$
(E) $14 \sqrt{2}-20$
(F) $15 \sqrt{3}-26$
(G) $21 \sqrt{3}-38$
[ENGAA, 2021S1Q14]
A 6.0 V battery is connected to an $8.0 \Omega$ resistor and a filament lamp as shown in the circuit diagram.


The reading on the ammeter is 0.25 A .
Which graph is a possible $V-I$ graph for the filament lamp?

[ENGAA, 2021S1Q15]
Charlie has a bowl containing red sweets and green sweets only. The sweets are identical in all respects except colour.
There are nine sweets in total in the bowl.
Charlie eats two sweets from the bowl at random.
The probability of Charlie not eating any green sweets is $\frac{5}{12}$.
What is the probability that Charlie eats two green sweets?
(A) $\frac{2}{27}$
(B) $\frac{1}{12}$
(C) $\frac{1}{9}$
(D) $\frac{4}{27}$
(E) $\frac{1}{6}$
(F) $\frac{1}{4}$
(G) $\frac{7}{12}$
[ENGAA, 2021S1Q16]
A radioactive nuclide $X$ decays in a single stage to a stable nuclide $R$.
A radioactive nuclide $Y$ decays in a single stage to a stable nuclide $S$.
When a rock formed it contained equal numbers of atoms of all four nuclides $X, Y, R$ and $S$.
The half-life of X is $T$ years and the half-life of Y is $2 T$ years.
What is the value of $\frac{\text { number of atoms of } \mathrm{R}}{\text { number of atoms of } \mathrm{S}}$ a a time $4 T$ years after the rock has formed?
(Assume that no other processes add or remove $\mathrm{X}, \mathrm{Y}, \mathrm{R}$ or S from the rock during this time.)
(A) $\frac{1}{4}$
(B) $\frac{17}{20}$
(C) $\frac{31}{28}$
(D) $\frac{6}{5}$
(E) $\frac{5}{4}$
(F) 2
[ENGAA, 2021S1Q17]
The greatest diagonal distance between the two vertices of a cuboid, as shown in the diagram, is $\sqrt{77} \mathrm{~cm}$.


A similar cuboid has all its lengths exactly half the lengths of the original cuboid.
The sides of this smaller cuboid are $2 \mathrm{~cm}, 3 \mathrm{~cm}$ and $x \mathrm{~cm}$.
What is the value of $x$, in cm ?
(A) $\frac{5}{2}$
(B) 5
(C) $\frac{5 \sqrt{2}}{2}$
(D) $5 \sqrt{2}$
(E) $\frac{\sqrt{102}}{2}$
(F) $\sqrt{102}$

## [ENGAA, 2021S1Q18]

A beaker containing 180 g of water at $25^{\circ} \mathrm{C}$ has a 20 g ice cube at $0^{\circ} \mathrm{C}$ added to it.
No heat is transferred between the water and the surroundings (including the beaker).
What is the final temperature of all the water in the beaker after all the ice has melted?
(Take the specific heat capacity of water to be $4 \mathrm{~J} \mathrm{~g}^{-1}{ }^{\circ} \mathrm{C}^{-1}$ and the specific latent heat of fusion of water to be $300 \mathrm{~J} \mathrm{~g}^{-1}$.)
(A) $2.5^{\circ} \mathrm{C}$
(B) $8.3^{\circ} \mathrm{C}$
(C) $10.0^{\circ} \mathrm{C}$
(D) $15.0^{\circ} \mathrm{C}$
(E) $16.7^{\circ} \mathrm{C}$
(F) $22.5^{\circ} \mathrm{C}$
[ENGAA, 2021S1Q19]
A car journey is $m$ miles long.
One kilometre is equivalent to $x$ miles.
The car uses one litre of fuel to travel a distance of $f$ kilometres.
Fuel for the car costs $p$ pence per litre.
Which of the following expressions gives the cost of fuel for this journey, in pounds?
(There are 100 pence in one pound.)
(A) 100 fmpx
(B) $\frac{100 \mathrm{fmp}}{x}$
(C) $\frac{100 \mathrm{mpx}}{f}$
(D) $\frac{100 m p x}{f x}$
(E) $\frac{f m p x}{100}$
(F) $\frac{f m p}{100 x}$
(G) $\frac{m p x}{100 f}$
(H) $\frac{m p}{100 f x}$
[ENGAA, 2021S1Q20]
A pulse of ultrasound travels from one end of a solid uniform rod of length $L$, starting at time $t=0$.
The pulse is partially reflected by a crack in the rod and partially by the far end of the rod.
These two reflected pulses travel back along the rod, arriving at the end from which they started at times $t_{1}$ and $t_{2}$, where $t_{2}>t_{1}$.
What is the distance between the crack and the far end of the rod?
(A) $\frac{t_{1}}{t_{2}} L$
(B) $\frac{t_{2}}{t_{1}} L$
(C) $\frac{t_{1}}{2 t_{2}} L$
(D) $\frac{t_{2}}{2 t_{1}} L$
(E) $\frac{\left(t_{2}-t_{1}\right)}{t_{2}} L$
(F) $\frac{\left(t_{2}-t_{1}\right)}{2 t_{2}} L$

## Part B

[ENGAA, 2021S1Q21]
Given that

$$
y=\left(2 \sqrt{x}-\frac{1}{2 \sqrt{x}}\right)^{2}
$$

find the value of $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $x=\frac{1}{2}$
(A) -12
(B) $-\frac{1}{4}$
(C) 3
(D) $\frac{63}{16}$
(E) 5

## [ENGAA, 2021S1Q22]

Object P of mass 2.4 kg is on a smooth plane inclined at an angle of $60^{\circ}$ to the horizontal. A constant force of magnitude $2 F$ parallel to the plane is applied to $P$. As a result $P$ moves directly up the plane with constant velocity.
Object Q of mass 0.75 kg is on a smooth, horizontal plane. A constant force of magnitude $F$ parallel to the plane is applied to Q . As a result Q moves along the plane with constant acceleration.

What is the acceleration of Q ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) $4.5 \mathrm{~m} \mathrm{~s}^{-2}$
(B) $6.0 \mathrm{~m} \mathrm{~s}^{-2}$
(C) $8.0 \mathrm{~m} \mathrm{~s}^{-2}$
(D) $16 \mathrm{~m} \mathrm{~s}^{-2}$
(E) $4.5 \sqrt{3} \mathrm{~m} \mathrm{~s}^{-2}$
(F) $6.0 \sqrt{3} \mathrm{~m} \mathrm{~s}^{-2}$
(G) $8.0 \sqrt{3} \mathrm{~m} \mathrm{~s}^{-2}$
(H) $16 \sqrt{3} \mathrm{~m} \mathrm{~s}^{-2}$
[ENGAA, 2021S1Q23]
A particular arithmetic series has first term $a$ and common difference $d$.
The sum of the first $k$ terms of this series is denoted by $S_{k}$
Which of the following is a simplification of $S_{n+1}-S_{n-1}$ ?
(A) $d$
(B) $2 d$
(C) $2 a+d$
(D) $2 a+2 d$
(E) $2 a+n d$
(F) $2 a+2 n d$
(G) $2 a+(2 n-1) d$
(H) $2 a+(4 n-2) d$

## [ENGAA, 2021S1Q24]

A sound wave is travelling from left to right in air. The diagram represents the wave at a particular instant, and a distance of 33 cm is labelled.


The speed of sound in air is $330 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the frequency of the sound and in which direction has the air at P been displaced from its mean position?

|  | frequency of sound <br> $/ H z$ | displacement of air at $P$ |
| :---: | :---: | :---: |
| (A) | 1000 | to the left |
| (B) | 2500 | to the left |
| (C) | 5000 | to the left |
| (D) | 1000 | to the right |
| (E) | 2500 | to the right |
| (F) | 5000 | to the right |

[ENGAA, 2021S1Q25]
Find how many distinct real solutions there are to the equation

$$
\left(x^{2}-4 x+3\right)^{2}=1
$$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
[ENGAA, 2021S1Q26]
A resistor $R$ is connected between terminals $X$ and $Y$ in the circuit shown.
30 V dc supply


The power transferred in the $4.0 \Omega$ heater is 9.0 W .
What is the resistance of $R$ ?
(A) $1.6 \Omega$
(B) $2.0 \Omega$
(C) $2.67 \Omega$
(D) $4.0 \Omega$
(E) $8.0 \Omega$
[ENGAA, 2021S1Q27]
The line $x=1$ divides the circle $x^{2}+y^{2}=4$ into two segments.
What is the area of the smaller segment?
(A) $\frac{2 \pi}{3}-\frac{\sqrt{3}}{2}$
(B) $\frac{2 \pi}{3}-\sqrt{3}$
(C) $\frac{\pi}{2}-\frac{1}{2}$
(D) $\frac{\pi}{2}-1$
(E) $\pi-\frac{1}{2}$
(F) $\pi-1$
(G) $\frac{4 \pi}{3}-\frac{\sqrt{3}}{2}$
(H) $\frac{4 \pi}{3}-\sqrt{3}$
[ENGAA, 2021S1Q28]
A uniform plank of length 5.0 m rests horizontally as shown.

[diagram not to scale]
There is a pivot 1.0 m from one end of the plank.
A cable at an angle of $60^{\circ}$ to the horizontal supports the plank at the other end so that it is in equilibrium.
The tension in the cable is 75 N .
What is the weight of the plank?
(A) 60 N
(B) $60 \sqrt{3} \mathrm{~N}$
(C) 100 N
(D) $100 \sqrt{3} \mathrm{~N}$
(E) 125 N
(F) $125 \sqrt{3} \mathrm{~N}$
[ENGAA, 2021S1Q29]
What is the mean of $\log _{10} 27, \log _{10} 64$, and $\log _{10} 216$ ?
(A) $\frac{\log _{10} 307}{3}$
(B) $\frac{\log _{10} 81}{3}$
(C) $\frac{\log _{10} 6^{12}}{3}$
(D) $\log _{10} 64$
(E) $\log _{10} 72$
(F) $\log _{10} 108$
[ENGAA, 2021S1Q30]
A lorry accelerates along a straight, horizontal road with uniform acceleration.
Oil droplets from the lorry fall a small distance onto the road at a constant rate. The time interval between successive drips is $t$.
The diagram shows four successive oil droplets on the road after the lorry has passed.


The distance between the first two of these droplets is $x$ and the distance between the final two is $y$.
Which expression gives the acceleration of the lorry?
(A) $\frac{y-x}{3 t^{2}}$
(B) $\frac{y-x}{2 t^{2}}$
(C) $\frac{y-x}{t^{2}}$
(D) $\frac{2(y-x)}{t^{2}}$
(E) $\frac{y+x}{t^{2}}$
(F) $\frac{y+x}{3 t^{2}}$
[ENGAA, 2021S1Q31]
Which of the following is the largest in value?
(All angles are in radians.)
(A) $\cos 0.5$
(B) $\cos 0.75$
(C) $\cos 1$
(D) $\sin 0.5$
(E) $\sin 0.75$
(F) $\sin 1$

## [ENGAA, 2021S1Q32]

A light, metal wire of length 2.5 m and cross-sectional area $1.8 \times 10^{-6} \mathrm{~m}^{2}$ is suspended vertically. A mass of 7.2 kg is attached to the lower end of the wire. The wire extends by 0.50 mm .
What is the Young modulus of the metal and how much energy is stored in the extended wire? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that the wire obeys Hooke's law and that changes in the cross-sectional area are negligible)

|  | Young modulus $/ \mathrm{Pa}$ | energystored/J |
| :---: | :---: | :---: |
| (A) | $5.0 \times 10^{-12}$ | 0.018 |
| (B) | $5.0 \times 10^{-12}$ | 0.036 |
| (C) | $2.0 \times 10^{11}$ | 0.018 |
| (D) | $2.0 \times 10^{11}$ | 0.036 |
| (E) | $2.0 \times 10^{14}$ | 18 |
| (F) | $2.0 \times 10^{14}$ | 36 |

## [ENGAA, 2021S1Q33]

A geometric progression has first term $u_{1}=a$ and common ratio $r$.
The sum to infinity of the geometric progression is $\frac{8}{5}$.
The sum to infinity of the even-numbered terms $\left(u_{2}+u_{4}+u_{6}+\cdots\right)$ is $\frac{3}{5}$.
What is the value of $a+r$ ?
(A) $\frac{3}{5}$
(B) $\frac{31}{25}$
(C) $\frac{23}{5}$
(D) $\frac{28}{5}$
(E) $\frac{67}{8}$
[ENGAA, 2021S1Q34]
A child of mass 30 kg is on a sledge of mass 10 kg which is moving down a smooth slope at an instantaneous speed of $4.0 \mathrm{~m} \mathrm{~s}^{-1}$.
At this instant, the child jumps backwards off the sledge and lands stationary on the slope.
What is the speed of the sledge immediately after the child jumps off?
(A) $4.0 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $8.0 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $12 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $16 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $20 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2021S1Q35]
At how many distinct points do the following two curves meet?

$$
\begin{aligned}
& y=(x-4)\left(x^{2}-2 x-8\right) \\
& y=-x^{2}+8 x-16
\end{aligned}
$$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
(F) 5
[ENGAA, 2021S1Q36]
A piece of electrically conducting putty is formed into the shape of a uniform cylinder. The resistance between the ends of the cylinder is $R$.
The same piece of putty is now formed into a new uniform cylinder with half the diameter of the first cylinder.
What is the resistance between the ends of the new cylinder?
(A) $\sqrt{2} R$
(B) $2 \sqrt{2} R$
(C) $4 \sqrt{2} R$
(D) $2 R$
(E) $4 R$
(F) $8 R$
(G) $16 R$
[ENGAA, 2021S1Q37]
Evaluate

$$
\frac{3}{\sqrt{27}+\sqrt{21}}+\frac{3}{\sqrt{24}+\sqrt{18}}+\frac{3}{\sqrt{21}+\sqrt{15}}+\cdots+\frac{3}{\sqrt{9}+\sqrt{3}}
$$

(A) $\frac{3 \sqrt{2}}{2}$
(B) $3 \sqrt{2}$
(C) $3 \sqrt{3}$
(D) $\sqrt{3}$
(E) $1+\sqrt{2}$
(F) $3(1+\sqrt{2})$
(G) $\frac{\sqrt{3}}{3}\left(1+\frac{\sqrt{2}}{2}\right)$
(H) $\sqrt{3}\left(1+\frac{\sqrt{2}}{2}\right)$
[ENGAA, 2021S1Q38]
A car accelerates from rest in a straight line. During the first 10 s , its acceleration, $a$, in $\mathrm{m} \mathrm{s}^{-2}$ is given by the equation

$$
a=4.0-0.36 t
$$

where $t$ is the time in seconds.
What is its displacement from its original position after 10 s ?
(A) 22 m
(B) 110 m
(C) 136 m
(D) 140 m
(E) 220 m
(F) 1100 m
(G) 1360 m
(H) 1400 m
[ENGAA, 2021S1Q39]
$P Q R S$ is a rectangle.
$P$ and $Q$ lie on the $x$-axis.
$Q$ and $R$ lie on the line $x=15$
$S$ lies on the curve $y=\sqrt{x}$
What is the maximum possible area of the rectangle?

(A) $5 \sqrt{5}$
(B) $10 \sqrt{5}$
(C) 50
(D) $25 \sqrt{5}$
(E) 100
(F) 125
[ENGAA, 2021S1Q40]
Two trolleys are free to move on a smooth one-dimensional track. A light spring is compressed between the two stationary trolleys, the trolleys are released and then separate.
The trolleys have masses $m$ and $4 m$ and the work done by the spring as it expands is $W$. Assume that no work is done against frictional forces.
What is the difference in kinetic energy between the two trolleys when the spring has expanded?
(A) 0
(B) $\frac{W}{5}$
(C) $\frac{W}{4}$
(D) $\frac{W}{2}$
(E) $\frac{3 W}{5}$
(F) $\frac{3 W}{4}$
(G) $\frac{4 W}{5}$
(H) $W$

## ENGAA 2021 S2



## TIME ALLOWED: 60 MINUTES

This paper is Section 2 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.
[ENGAA, 2021S2Q1]
Two loudspeakers are positioned 8.0 m apart as shown.


The loudspeakers emit sound waves of the same single frequency. The wave emitted by one loudspeaker is $180^{\circ}$ out of phase with the wave emitted by the other loudspeaker.
A point $P$ is in front of the loudspeakers. $P$ is 18.0 m from one loudspeaker and 24.0 m from the other loudspeaker. As a result of superposition of the two waves arriving at $P$, the amplitude of the sound at position P is a minimum.
The speed of the sound is $336 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the lowest possible frequency of the sound?
(A) 21 Hz
(B) 28 Hz
(C) 42 Hz
(D) 56 Hz
(E) 63 Hz
(F) 84 Hz
[ENGAA, 2021S2Q2]
A block is at rest on a rough inclined plane.
The acute angle between the plane and the horizontal is greater than $45^{\circ}$.
The forces acting on the block are: friction $(F)$, weight $(W)$ and normal contact force ( $N$ ).
How do the magnitudes of the three forces compare?
(A) $F<N<W$
(B) $F<W<N$
(C) $N<F<W$
(D) $N<W<F$
(E) $W<F<N$
(F) $W<N<F$

A dc power supply, a resistor of constant resistance $50 \Omega$ and a piece of resistance wire are connected in series.

The length of the resistance wire is 20 m and its cross-sectional area is $0.10 \mathrm{~mm}^{2}$. The wire is made from a material with resistivity $1.0 \times 10^{-7} \Omega \mathrm{~m}$ and the current in it is 200 mA .
What is the voltage across the terminals of the power supply?
(A) 4.0 V
(B) 6.0 V
(C) 9.9 V
(D) 10.0 V
(E) 10.1 V
(F) 12.0 V
(G) 14.0 V
[ENGAA, 2021S2Q4]
Two objects of mass $M$ and $m$ are connected by a rope over a pulley on an inclined plane as shown.

[diagram not to scale]
There is no friction between the plane and the object. The pulley is smooth, and the rope has negligible mass.
The angle $\theta$ of the plane to the horizontal is such that $\sin \theta=0.80$ and $\cos \theta=0.60$.
The object with mass $M$ accelerates down the slope.
Which expression describes the full range of possible values of $M$ compared with $m$ ?
(A) $M>\frac{3}{5} m$
(B) $M>\frac{4}{5} m$
(C) $M>m$
(D) $M>\frac{5}{4} m$
(E) $M>\frac{5}{3} m$
[ENGAA, 2021S2Q5]
An object P falls vertically from rest through air and reaches terminal velocity.
An identical object Q is projected vertically upwards from the ground.
When $Q$ reaches its maximum height, $P$ collides with it. The two objects join together in such a way that there is no change to the area of cross section passing through the air.
The two combined objects then fall through the air as one object.
Which sketch graph shows the variation of velocity with time for object $P$ before and after the collision?

(A)

(C)

(E)

(B)

(D)

(F)
[ENGAA, 2021S2Q6]
A lorry of mass $m$ has an engine that develops a constant mechanical output power $P$.
The lorry is accelerated from rest by the engine in a horizontal straight line. The lorry experiences a total resistive force that is always proportional to the square of its speed.
The process is repeated for different values of $P$, and the maximum speed of the lorry is found to be proportional to $P^{n}$, where $n$ is a constant.
What is the value of $n$ ?
(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) 3
[ENGAA, 2021S2Q7]
A battery pack consists of 6 cells, each with an emf of 1.50 V and each with an internal resistance of $0.20 \Omega$.
The cells are arranged in two rows connected in parallel. Each row contains 3 cells connected in series.
The battery pack is connected to an external resistor of resistance $1.20 \Omega$.
What is the electrical power transferred in the external resistor?
(A) 2.7 W
(B) 3.6 W
(C) 7.5 W
(D) 10.8 W
(E) 13.5 W
(F) 43.2 W
[ENGAA, 2021S2Q8]
A light spring is used to support a uniform rod horizontally against a wall as shown. The angle between the spring and the rod is $\theta$.


The spring constant of the spring is $20 \mathrm{~N} \mathrm{~m}^{-1}$ and the weight of the rod is 16 N .
The angle $\theta$ is such that $\cos \theta=\frac{3}{5}$ and $\sin \theta=\frac{4}{5}$.
How much energy is stored in the spring?
(A) 1.6 J
(B) 2.5 J
(C) 3.2 J
(D) 4.4 J
(E) 5.0 J
(F) 6.4 J
(G) 10 J
(H) 40 J
[ENGAA, 2021S2Q9]
An object of mass 2.0 kg moves in a straight line under the action of a resultant force. The displacement $x$ of the object from its position at time $t=0$ is given by

$$
x=4.0 t^{3}
$$

where $x$ is in metres and $t$ is in seconds.
At $t=5.0 \mathrm{~s}$, what is the rate of change of momentum of the object?
(A) $6.7 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}$
(B) $66.7 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}$
(C) $120 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}$
(D) $240 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}$
(E) $600 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-2}$
[ENGAA, 2021S2Q10]
In an industrial process to test the purity of a metal, a narrow beam of ultrasound passes into a block of the metal. The ultrasound generator $U$ is immersed in a gel that is in contact with the metal. The ultrasound passes from the gel into the metal.
The arcs of circles shown in the gel are lines that represent the positions of the compressions (known as wavefronts) of the ultrasound wave that comes from $U$.


Ultrasound travels faster in the metal than in the gel.
The wavefronts in the metal are circular arcs with their centre at a point $X$ that is on the dashed line.
Where on the dashed line is X ?
(A) above $U$
(B) at U
(C) in the gel below $U$
(D) on the boundary between the gel and the metal
(E) in the metal
[ENGAA, 2021S2Q11]
The diagram shows a circuit containing two power supplies with negligible internal resistance and two resistors with resistances $R$ and $5 R$.

The emfs of the power supplies and the magnitude and direction of the current in one part of the circuit are shown.
One point in the circuit is labelled $P$.


What is the magnitude of the current at P ?
(A) 3.0 mA
(B) 7.0 mA
(C) 8.5 mA
(D) 11.5 mA
(E) 13 mA
(F) 25 mA
[ENGAA, 2021S2Q12]
A selection of five wires made from the same metal have different unstretched lengths but equal masses. The wires are all subjected to the same small tension force and each wire extends within its limit of proportionality.
Which graph shows the relationship between the extension of the wires and the unstretched length of the wires?

(A)

(C)

(E)

(B)

(D)

(F)
[ENGAA, 2021S2Q13]
Water enters a horizontal pipe of cross-sectional area $0.0040 \mathrm{~m}^{2}$ at constant speed $0.50 \mathrm{~m} \mathrm{~s}^{-1}$. At the end of the pipe the cross-sectional area reduces to $0.0020 \mathrm{~m}^{2}$ and the water leaves the pipe as shown. The density of water is $1000 \mathrm{~kg} \mathrm{~m}^{-3}$.


How much power must be supplied to the water to maintain the flow in this section of the pipe? (Assume that the water is incompressible and that frictional forces can be neglected.)
(A) 0.25 W
(B) 0.50 W
(C) 0.75 W
(D) 1.0 W
(E) 1.25 W
(F) 1.5 W
(G) 3.75 W
[ENGAA, 2021S2Q14]
Two light wires P and Q support a load of weight $W$ in equilibrium as shown. Wire P is horizontal and wire Q is at an angle of $60^{\circ}$ to the vertical. The wires are made from the same material.


The radius of wire $Q$ is twice the radius of wire $P$.
What is the ratio

$$
\frac{\text { strain in wire } P}{\text { strain in wire } Q} ?
$$

(The wires do not exceed their limits of proportionality.)
(A) $\frac{\sqrt{3}}{8}$
(B) $\frac{\sqrt{3}}{4}$
(C) $\frac{\sqrt{3}}{2}$
(D) $\sqrt{3}$
(E) $2 \sqrt{3}$
(F) $\frac{4}{\sqrt{3}}$
(G) $\frac{8}{\sqrt{3}}$
[ENGAA, 2021S2Q15]
The speed of light in a block of glass is $2.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$. The block of glass is immersed in a liquid of refractive index 1.2.
The diagram shows a ray of light travelling in the glass block striking the side of the block at the point labelled X . The acute angle between the ray and the side of the block is $\theta$.


What is the full range of values of the acute angle $\theta$ for which light is refracted at X ? (The speed of light in a vacuum is $3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$.)
(A) $0^{\circ}<\theta<\cos ^{-1}\left(\frac{2}{3}\right)$
(B) $0^{\circ}<\theta<\cos ^{-1}\left(\frac{\sqrt{5}}{3}\right)$
(C) $0^{\circ}<\theta<\cos ^{-1}\left(\frac{3}{5}\right)$
(D) $0^{\circ}<\theta<\cos ^{-1}\left(\frac{4}{5}\right)$
(E) $\cos ^{-1}\left(\frac{2}{3}\right)<\theta<90^{\circ}$
(F) $\cos ^{-1}\left(\frac{\sqrt{5}}{3}\right)<\theta<90^{\circ}$
(G) $\cos ^{-1}\left(\frac{3}{5}\right)<\theta<90^{\circ}$
(H) $\cos ^{-1}\left(\frac{4}{5}\right)<\theta<90^{\circ}$
[ENGAA, 2021S2Q16]
A car is at rest on a straight horizontal road. At time $t=0 \mathrm{~s}$ the car starts to move along the road. The graph shows how its acceleration varies from $t=0 \mathrm{~s}$ to $t=20 \mathrm{~s}$.


What is the displacement of the car from its starting position when $t=20 \mathrm{~s}$ ?
(A) 5.0 m
(B) 25 m
(C) 35 m
(D) 175 m
(E) 225 m
(F) 375 m
[ENGAA, 2021S2Q17]
An empty measuring cylinder is placed on a balance, and the balance reading is then set to zero. A mass of 8.7 g of a powder is poured into the measuring cylinder as shown in the diagram.


Liquid is poured into the cylinder to cover the powder completely. The powder does not dissolve. The reading on the measuring cylinder and the reading on the balance are recorded. More liquid is added and a second pair of readings is recorded.
The table shows the two pairs of readings.

| reading on measuring cylinder $/ \mathrm{cm}^{3}$ | reading on balance $/ \mathrm{g}$ |
| :---: | :---: |
| 10.0 | 15.0 |
| 25.0 | 27.6 |

What is the density of the material from which the powder is made?
(A) $0.414 \mathrm{~g} \mathrm{~cm}^{-3}$
(B) $1.16 \mathrm{~g} \mathrm{~cm}^{-3}$
(C) $1.31 \mathrm{~g} \mathrm{~cm}^{-3}$
(D) $1.45 \mathrm{~g} \mathrm{~cm}^{-3}$
(E) $2.00 \mathrm{~g} \mathrm{~cm}^{-3}$
(F) $2.50 \mathrm{~g} \mathrm{~cm}^{-3}$
(G) $3.48 \mathrm{~g} \mathrm{~cm}^{-3}$
(H) $6.00 \mathrm{~g} \mathrm{~cm}^{-3}$
[ENGAA, 2021S2Q18]
A stone of mass 100 g is fired horizontally from an 80 m high vertical cliff. The ground below the cliff is horizontal.

The kinetic energy of the stone when it hits the ground is 125 J .
What is the distance from the bottom of the cliff to the point where the stone hits the ground? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; ignore air resistance and any effect of wind)
(A) 60 m
(B) 80 m
(C) 120 m
(D) 160 m
(E) 200 m

## [ENGAA, 2021S2Q19]

An electrical component is connected to a switch and a power supply which has a constant terminal potential difference $V$. The switch is initially open. At time $t=0$ the switch is closed. When the switch is closed, the current $I$ in the component increases with time $t$ as given by the equation

$$
I=k t^{2}
$$

where $k$ is a positive constant.
When the current reaches a value $I_{F}$ the component fails and the current falls instantly to zero. How much electrical energy has been transferred to the component by the time it fails?
(All quantities are in standard SI units.)
(A) $\frac{V k}{3}\left(\frac{I_{F}}{k}\right)^{\frac{3}{2}}$
(B) $V k\left(\frac{I_{F}}{k}\right)^{\frac{3}{2}}$
(C) $3 V k\left(\frac{I_{F}}{k}\right)^{\frac{3}{2}}$
(D) $\frac{V k}{3}\left(\frac{I_{F}}{K}\right)$
(E) $V k\left(\frac{I_{F}}{K}\right)$
(F) $3 V k\left(\frac{I_{F}}{K}\right)$
[ENGAA, 2021S2Q20]
A water trough has the shape of a prism, with a cross section that is a right-angled isosceles triangle.
One rectangular face and the two triangular ends of the trough are vertical, as shown.

[diagram not to scale]
The trough contains water of depth 0.60 m measured on the vertical rectangular face.
What is the force exerted by the water on one triangular end of the trough?
(density of water $=1000 \mathrm{~kg} \mathrm{~m}^{-3}$; gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 180 N
(B) 270 N
(C) 360 N
(D) 540 N
(E) 720 N
(F) 1080 N
(G) 6000 N
(H) 12000 N

## ENGAA 2022 S1



## TIME ALLOWED: 60 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.

## Part A Mathematics and Physics (20 questions)

Part B Advanced Mathematics and Advanced Physics (20 questions)
You are strongly advised to divide your time equally between the two parts: 30 minutes on Part A and 30 minutes on Part B. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 40 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

## Part A

[ENGAA, 2022S1Q1]
Which one of the following is a simplification of

$$
y\left(\frac{3 x^{\frac{1}{2}} Z}{y^{3}}\right)^{2}
$$

(A) $\frac{3 x z^{2}}{y^{4}}$
(B) $\frac{3 x z^{2}}{y^{5}}$
(C) $\frac{9 x^{\frac{1}{2}} z^{2}}{y^{5}}$
(D) $\frac{9 x z^{2}}{y^{4}}$
(E) $\frac{9 x z^{2}}{y^{5}}$
(F) $\frac{9 x^{\frac{5}{2} z^{2}}}{y^{5}}$
[ENGAA, 2022S1Q2]
There is a constant current in a conducting wire. A charge of 20 C passes through the wire in 1.5 minutes.

An 18 cm straight section of this wire lies in a uniform magnetic field. This section of wire is perpendicular to the direction of the field. The magnetic field strength is 0.15 T .

What is the magnitude of the magnetic force on this section of wire?
(A) 0.0060 N
(B) 0.36 N
(C) 0.60 N
(D) 0.81 N
(E) 36 N
(F) 49 N
(G) 81 N
(H) 4900 N
[ENGAA, 2022S1Q3]
Find the complete set of values of $x$ that satisfy the inequality

$$
\frac{3}{4}(5-x)-\frac{1}{2}(6-x)-x<0
$$

(A) $x<\frac{1}{3}$
(B) $x>\frac{1}{3}$
(C) $x<\frac{3}{5}$
(D) $x>\frac{3}{5}$
(E) $x<\frac{3}{4}$
(F) $x>\frac{3}{4}$
(G) $x<\frac{3}{2}$
(H) $x>\frac{3}{2}$

## [ENGAA, 2022S1Q4]

Two identical resistors are connected in parallel to a 6.0 V battery. The two resistors dissipate a total power of 0.15 W .
One of these resistors is removed from the circuit and connected to a 12 V battery.
How much charge passes through this resistor in 6.0 minutes?
(A) 0.025 C
(B) 0.050 C
(C) 0.15 C
(D) 0.30 C
(E) 0.75 C
(F) 1.5 C
(G) 9.0 C
(H) 18 C
[ENGAA, 2022S1Q5]
Rob keeps a record of what he earns each day.
On Monday, he earned 50\% less than he earned on Sunday.
On Tuesday, he earned $20 \%$ more than he earned on Monday.
On Wednesday, he earned $30 \%$ less than he earned on Tuesday.
On Wednesday, he earned $£ 84$.
How much did Rob earn on Sunday?
(A) $£ 15.12$
(B) $£ 35.28$
(C) $£ 117.60$
(D) $£ 200$
(E) $£ 210$
(F) $£ 300$
(G) $£ 1200$
[ENGAA, 2022S1Q6]
Ultrasound is used to find a crack inside a cuboid block of metal. An ultrasound probe is held in contact with the top surface of the metal block and perpendicular to the surface. A short pulse of ultrasound is sent into the metal block at time $t=0 \mathrm{~ms}$ and reflects from both the crack and the bottom surface of the metal block.


The times between the emission of the ultrasound pulse and the return of the reflections to the probe, and the strengths of the reflected pulses, are measured. The results are shown on the graph.
strength of reflected pulse


The speed of ultrasound in the metal is $5000 \mathrm{~m} \mathrm{~s}^{-1}$.
What is the distance between the bottom surface of the metal block and the crack?
(A) 0.2 m
(B) 0.3 m
(C) 0.4 m
(D) 0.5 m
(E) 0.6 m
(F) 1.0 m
[ENGAA, 2022S1Q7]
Which one of the following is a simplification of

$$
\frac{5 x^{2}-17 x-12}{25 x^{2}-9} \div \frac{x^{2}+x-12}{x^{2}-x-6}
$$

(A) $\frac{(x-4)(x+2)}{(x-3)(x+4)}$
(B) $\frac{(x-3)(x+2)}{(5 x-3)(x+3)}$
(C) $\frac{(x-4)(x+2)}{(5 x-3)(x+4)}$
(D) $\frac{(x-4)(x-3)}{(5 x-3)(x-6)}$
(E) $\frac{(x+2)}{(5 x+3)}$
(F) $\frac{(x+4)(x-6)}{(5 x+3)(x+2)}$
(G) $\frac{(x-3)(x+2)}{(5 x+3)(x+3)}$
[ENGAA, 2022S1Q8]
Power is supplied to an electric motor at 0.800 kW .
The motor has an efficiency of $60 \%$ and is switched on for half an hour.
How much energy is wasted during this time?
(A) 0.160 J
(B) 0.240 J
(C) 160 J
(D) 240 J
(E) 576 J
(F) 864 J
(G) $576,000 \mathrm{~J}$
(H) $864,000 \mathrm{~J}$
[ENGAA, 2022S1Q9]
A rectangle $P Q R S$ has length $(2 x-1) \mathrm{cm}$ and width $(x+1) \mathrm{cm}$ as shown on the diagram.
A larger rectangle is made by adding 3 cm to both the length and the width of $P Q R S$, as shown. The larger rectangle has an area of $360 \mathrm{~cm}^{2}$.

[diagram not to scale]
What is the ratio of $P Q$ to $P S$ ?
(A) $1: 2$
(B) $4: 7$
(C) $5: 8$
(D) $7: 11$
(E) $10: 17$
(F) $17: 31$
[ENGAA, 2022S1Q10]
The graph shows potential difference plotted against current for a filament lamp and a resistor.


The lamp and the resistor are connected in parallel with each other to a 6.0 V power supply and the current in the lamp, $I$, is recorded.
In a second circuit, the lamp and the resistor are now connected in series with each other to the same power supply, and the current in the resistor is 0.18 A . The potential difference across the lamp, $V$, is recorded.
What are the values of $I$ in the first circuit and $V$ in the second circuit?

|  | $I / \mathrm{A}$ | $V / \mathrm{V}$ |
| :---: | :---: | :---: |
| (A) | 0.25 | 1.6 |
| (B) | 0.25 | 3.0 |
| (C) | 0.25 | 4.4 |
| (D) | 0.35 | 1.6 |
| (E) | 0.35 | 3.0 |
| (F) | 0.35 | 4.4 |

[ENGAA, 2022S1Q11]
$P Q R S$ is a trapezium as shown.

[diagram not to scale]
$\tan R S Q=\frac{5}{8}$
What is the length of $P S$, in metres?
(A) 45
(B) 65
(C) 80
(D) 120
(E) $25+\frac{40 \sqrt{3}}{3}$
(F) $40+\frac{64 \sqrt{3}}{3}$
(G) $25+40 \sqrt{3}$
(H) $60+40 \sqrt{3}$
[ENGAA, 2022S1Q12]
A transverse wave on a string has a speed of $500 \mathrm{~m} \mathrm{~s}^{-1}$.
The horizontal distance between two points $P$ and Q on the wave is 4.0 m , as shown in the diagram.


At time $t=0 \mathrm{~ms}$, point X on the string is at its maximum displacement of 6.0 mm above equilibrium.
What is the displacement of point X at time $t=7.0 \mathrm{~ms}$ ?
(A) 6.0 mm above equilibrium
(B) between 0 mm and 6.0 mm above equilibrium
(C) 0 mm
(D) between 0 mm and 6.0 mm below equilibrium
(E) 6.0 mm below equilibrium
[ENGAA, 2022S1Q13]
A solid cylinder has radius $r \mathrm{~cm}$ and height $h \mathrm{~cm}$.
A cube has side length $3 r \mathrm{~cm}$.
The total surface area of the cylinder is equal to four times the total surface area of the cube.
Which of the following is an expression for $h$ in terms of $r$ ?
(A) $\left(\frac{18}{\pi}-2\right) r$
(B) $\left(\frac{18}{\pi}-1\right) r$
(C) $\frac{27 r}{\pi}$
(D) $\left(\frac{27}{\pi}-1\right) r$
(E) $\left(\frac{27}{4 \pi}-1\right) r$
(F) $\frac{108 r}{\pi}$
(G) $\left(\frac{108}{\pi}-1\right) r$
(H) $\left(\frac{108}{\pi}-\frac{1}{2}\right) r$
[ENGAA, 2022S1Q14]
A piece of metal of mass 50 g is at thermal equilibrium in a hot liquid at temperature $T$.
The metal is removed from the liquid and immediately placed in 100 g of water that is at $20^{\circ} \mathrm{C}$. The water is stirred and reaches a final temperature of $26^{\circ} \mathrm{C}$.

| material | specific heat capacity $/ \mathrm{J} \mathrm{kg}^{-1}{ }^{\circ} \mathrm{C}^{-1}$ |
| :---: | :---: |
| hot liquid | 2000 |
| metal | 350 |
| water | 4200 |

What is the temperature $T$ of the hot liquid?
(Assume that heat transfers to or from the surroundings are negligible.)
(A) $38^{\circ} \mathrm{C}$
(B) $51{ }^{\circ} \mathrm{C}$
(C) $150^{\circ} \mathrm{C}$
(D) $170^{\circ} \mathrm{C}$
(E) $480^{\circ} \mathrm{C}$
[ENGAA, 2022S1Q15]
The variables $x$ and $y$ are related by the equation:

$$
x=5-\frac{2 y^{3}+1}{1-2 y^{3}}
$$

Which of the following is a rearrangement to make $y$ the subject?
(A) $y=\sqrt[3]{\frac{x-4}{8 x-48}}$
(B) $y=\sqrt[3]{\frac{x-6}{8 x-32}}$
(C) $y=\sqrt[3]{\frac{x-2}{x-6}}$
(D) $y=\sqrt[3]{\frac{x-3}{x-4}}$
(E) $y=\sqrt[3]{\frac{x-4}{2 x-12}}$
(F) $y=\sqrt[3]{\frac{x-6}{2 x-8}}$
[ENGAA, 2022S1Q16]
A bar magnet is placed at position X close to one end of a coil and on the axis of the coil as shown.

The graph shows how the velocity of the magnet varies as it is then moved rapidly to position Y and back to position X .


The magnetic field of the bar magnet still affects the coil when the magnet is at position Y . Which graph represents how the induced voltage in the coil changes as the magnet moves?
(A)

(B)

(C)

(D)

(E)

(F)

[ENGAA, 2022S1Q17]
Three different numbers are chosen at random from $\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}$.
What is the probability that the three numbers form the three sides of a right-angled triangle?
(A) $\frac{1}{15}$
(B) $\frac{1}{10}$
(C) $\frac{3}{10}$
(D) $\frac{1}{3}$
(E) $\frac{2}{5}$
(F) $\frac{2}{3}$
(G) $\frac{4}{5}$
[ENGAA, 2022S1Q18]
A small slider of mass 30 g is at rest near the bottom of a frictionless slope and in contact with a light uncompressed spring as shown.

[diagram not to scale]
The spring is compressed by 5.0 cm and the slider remains in contact with it.
The spring is released and causes the slider to rise up the slope to a maximum vertical height of 20 cm .
The slider is replaced with one of mass 20 g .
The spring is now compressed by 15 cm , and the new slider remains in contact with it.
To what maximum vertical height does this new slider rise after it is released?
(the spring obeys Hooke's law; assume that air resistance is negligible)
(A) 40 cm
(B) 60 cm
(C) 90 cm
(D) 120 cm
(E) 180 cm
(F) 270 cm
[ENGAA, 2022S1Q19]
The point $(-1,5)$ is translated to the point $(3,2)$ by two successive translations.
The first translation is by the vector $\binom{3 p}{-4 p}$.
The second translation is by the vector $\binom{c}{-2 q}$.
What is the value of $p+q$ ?
(A) -14
(B) -7
(C) -5
(D) -1
(E) 1
(F) 5
(G) 7
(H) 14
[ENGAA, 2022S1Q20]
A tall, smooth cylinder contains air at atmospheric pressure of $1.00 \times 10^{5} \mathrm{~Pa}$. The density of the air in the cylinder is $1.20 \mathrm{~kg} \mathrm{~m}^{-3}$.
A heavy piston is now placed in the top of the cylinder and allowed to fall slowly downwards, compressing the air until the piston rests in equilibrium.
The mass of the piston is 50.0 kg and its cross-sectional area is $0.0200 \mathrm{~m}^{2}$.
What is the density of the air in the cylinder when the piston rests in equilibrium?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that the air behaves as an ideal gas and that the temperature remains constant)
(A) $0.960 \mathrm{~kg} \mathrm{~m}^{-3}$
(B) $1.20 \mathrm{~kg} \mathrm{~m}^{-3}$
(C) $1.25 \mathrm{~kg} \mathrm{~m}^{-3}$
(D) $1.28 \mathrm{~kg} \mathrm{~m}^{-3}$
(E) $1.50 \mathrm{~kg} \mathrm{~m}^{-3}$
(F) $4.80 \mathrm{~kg} \mathrm{~m}^{-3}$

## Part B

[ENGAA, 2022S1Q21]
Find the value of

$$
\int_{1}^{4} \frac{2 x^{2}-3}{x \sqrt{x}} \mathrm{~d} x
$$

(A) $\frac{19}{3}$
(B) $\frac{37}{3}$
(C) $\frac{53}{3}$
(D) $\frac{73}{4}$
(E) $\frac{81}{4}$
(F) $\frac{87}{4}$
[ENGAA, 2022S1Q22]
The diagram represents a stationary wave in a medium.


The transverse waves that are creating the stationary wave travel at a speed of $300 \mathrm{~m} \mathrm{~s}^{-1}$ through the medium.
What is the frequency of the transverse waves?
(A) 75 Hz
(B) 150 Hz
(C) 200 Hz
(D) 450 Hz
(E) 600 Hz
(F) 1200 Hz
[ENGAA, 2022S1Q23]
The diagram shows a semicircle of radius 5 units and a triangle.


The triangle has vertices at $(0,0),(10,0)$ and $(x, y)$.
$(x, y)$ is a point on the arc of the semicircle.
Which of the following is an expression in terms of $x$ for the area of this triangle?
(A) $5 \sqrt{10 x-x^{2}}$
(B) $5 x \sqrt{10-x}$
(C) $5 x \sqrt{10 x-x^{2}-20}$
(D) $15 x$
(E) $25 x$
(F) $5 x(10-x)$
[ENGAA, 2022S1Q24]
A stone is thrown vertically upwards from the surface of the Earth and reaches a maximum height $h$.
The same stone is thrown vertically upwards from the surface of the Moon, with the same initial speed.
What is the maximum height reached by the stone thrown on the Moon?
(gravitational field strength on the Earth $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; gravitational field strength on the Moon $=1.6 \mathrm{~N} \mathrm{~kg}^{-1}$; air resistance may be ignored)
(A) $\left(\frac{10}{1.6}\right) h$
(B) $\left(\frac{10}{1.6}\right)^{2} h$
(C) $\left(\frac{10}{3.2}\right) h$
(D) $\left(\frac{10}{3.2}\right)^{2} h$
(E) $\left(\frac{10}{8.4}\right) h$
(F) $\left(\frac{10}{8.4}\right)^{2} h$
(G) $\left(\frac{20}{1.6}\right) h$
(H) $\left(\frac{20}{1.6}\right)^{2} h$
[ENGAA, 2022S1Q25]
Four mathematically similar solids, $\mathrm{W}, \mathrm{X}, \mathrm{Y}$ and Z , have the following properties:

- The ratio of the lengths of W to the lengths of X is $1: 2$
- The ratio of the surface area of $X$ to the surface area of $Y$ is $2: 1$
- The ratio of the volume of Y to the volume of Z is $1: 2$

What is the order of the solids when arranged in increasing volume?
(A) W Y X Z
(B) W Y Z X
(C) WZYX
(D) $\mathrm{Y} W \mathrm{XZ}$
(E) $\mathrm{Y} W \mathrm{ZX}$
(F) $\mathrm{Y} Z \mathrm{WX}$
[ENGAA, 2022S1Q26]
A ray of light is incident on a boundary between a vacuum and medium $X$ at an angle $\theta$ as shown:


The incident ray is partially reflected and partially refracted. The angle between the reflected and refracted rays is $90^{\circ}$.
What is the refractive index of medium X?
(A) $\sin \theta$
(B) $\frac{1}{\sin \theta}$
(C) $\cos \theta$
(D) $\frac{1}{\cos \theta}$
(E) $\tan \theta$
(F) $\frac{1}{\tan \theta}$
[ENGAA, 2022S1Q27]
Which one of the following expressions is equal to

$$
\frac{(2+\sqrt{20})^{2}}{(1+\sqrt{5})^{3}}
$$

(A) $\sqrt{5}-1$
(B) $\frac{\sqrt{5}-1}{2}$
(C) $\frac{6(5 \sqrt{5}-1)}{31}$
(D) $\frac{3(5 \sqrt{5}-1)}{31}$
(E) $\frac{-22+10 \sqrt{2}+11 \sqrt{5}-4 \sqrt{10}}{2}$
(F) $\frac{-22+10 \sqrt{2}+11 \sqrt{5}-4 \sqrt{10}}{4}$
[ENGAA, 2022S1Q28]
An electric train is travelling along a straight horizontal track. It passes a point $Q$ on the track at time $t=0$.
The distance $x$ that it then travels away from $Q$ is given by the equation:

$$
x=a t+b t^{2}
$$

where $a$ and $b$ are constants.
Which of the following statements is/are correct?
1 The speed of the train increases with time at a constant rate.
2 The resultant force acting on the train increases with time.
3 The rate at which energy is transferred to the train increases with time.
(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2022S1Q29]
Evaluate

$$
\sum_{r=2}^{\infty}\left(\frac{1}{2^{r}}+\frac{1}{3^{r}}+\frac{1}{4^{r}}\right)
$$

(A) $\frac{21}{40}$
(B) $\frac{3}{4}$
(C) $\frac{13}{12}$
(D) $\frac{157}{120}$
(E) $\frac{11}{6}$
(F) $\frac{23}{12}$
[ENGAA, 2022S1Q30]
A length of wire has resistance $R$.
Another length of wire is made from the same material. This wire is twice as long as the first wire and has half the diameter.

Both wires have circular cross-sections.
The two wires are connected in parallel.
What is the total resistance of this combination?
(A) $\frac{2}{3} R$
(B) $\frac{8}{9} R$
(C) $2 R$
(D) $3 R$
(E) $8 R$
(F) $9 R$
[ENGAA, 2022S1Q31]
What is the complete set of real values of $x$ for which

$$
x^{2}\left(x^{2}+4\right)<21
$$

(A) $-\sqrt{3}<x<\sqrt{3}$
(B) $-\sqrt{7}<x<\sqrt{7}$
(C) $x<-\sqrt{3}$ or $x>\sqrt{3}$
(D) $x<-\sqrt{7}$ or $x>\sqrt{7}$
(E) $-\sqrt{7}<x<-\sqrt{3}$ or $\sqrt{3}<x<\sqrt{7}$
(F) $x<-\sqrt{7}$ or $-\sqrt{3}<x<\sqrt{3}$ or $x>\sqrt{7}$
[ENGAA, 2022S1Q32]
An empty bucket has a mass of 1.20 kg and an internal volume of $0.0150 \mathrm{~m}^{3}$. The bucket is used to lift water from a well.

The bucket is attached to a light, inextensible rope which winds onto a rotating cylinder of radius 0.200 m when a handle is turned.


To lift a bucket completely full of water at constant speed, it is necessary to apply a force of 250 N to the handle that acts along the tangent to the circle of radius 0.600 m , in which the handle moves.
The energy required to lift the bucket is wasted energy.
What is the efficiency of the system in lifting water from the well?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; density of water $=1000 \mathrm{~kg} \mathrm{~m}^{-3}$ )
(A) $2.00 \%$
(B) $2.16 \%$
(C) $20.0 \%$
(D) $21.6 \%$
(E) $60.0 \%$
[ENGAA, 2022S1Q33]
A sector of a circle has perimeter 24.
For what value of the radius does the sector have the maximum possible area?
(A) $3 \sqrt{2}$
(B) $2 \sqrt{6}$
(C) $3 \sqrt{6}$
(D) 6
(E) 12
(F) 18
(G) 36
[ENGAA, 2022S1Q34]
A uniform metre ruler has a mass of 100 g and is 1.00 m long.
A small object of mass 20 g is fixed at the 0 cm mark, and another small object of mass 80 g is fixed at the 100 cm mark.
The ruler is balanced on a pivot.
When the ruler is balanced, what distance is the pivot from the 0 cm mark?
(A) 15 cm
(B) 20 cm
(C) 25 cm
(D) 35 cm
(E) 65 cm
(F) 75 cm
(G) 80 cm
(H) 85 cm
[ENGAA, 2022S1Q35]
The curve $y=x^{2}-x-6$ intersects the $x$-axis at the points $A$ and $B$, and has a minimum at the point $C$.
The rectangle $A B D E$ has two of its vertices at $A$ and $B$.
The point $C$ lies on the edge $D E$, between $D$ and $E$.
What is the area of the rectangle $A B D E$ ?
(A) 6
(B) 6.25
(C) 30
(D) 31.25
(E) 35
(F) 36.75
(G) 42
(H) 43.75
[ENGAA, 2022S1Q36]
A circuit comprising two resistors and two batteries, with negligible internal resistance, is set up as shown in the diagram. The two junctions in the circuit are labelled X and Y .


What are the magnitude and direction of the current in the 3.0 V battery?

|  | magnitude of current/A | direction of current |
| ---: | :---: | :---: |
| (A) | 1.0 | from X to Y |
| (B) | 2.0 | from X to Y |
| (C) | 2.25 | from X to Y |
| (D) | 6.0 | from X to Y |
| (E) | 1.0 | from Y to X |
| (F) | 2.0 | from Y to X |
| (G) | 2.25 | from Y to X |
| (H) | 6.0 | from Y to X |

[ENGAA, 2022S1Q37]
A straight line passes through the points $(0,2 a)$ and $(a, 0)$, where $a$ is a positive constant. What is the perpendicular distance of the point $P(a, 2 a)$ from this line?
(A) $\frac{\sqrt{2}}{5} a$
(B) $\frac{\sqrt{5}}{2} a$
(C) $\frac{2 \sqrt{5}}{5} a$
(D) $\frac{4 \sqrt{5}}{5} a$
(E) $\frac{3 \sqrt{10}}{5} a$
[ENGAA, 2022S1Q38]
Two isolated spheres have masses $m$ and $2 m$. They are moving towards each other along the same straight line with speeds $4 v$ and $v$ respectively as shown:



The spheres collide with each other and coalesce.
What is the loss of kinetic energy during the collision?
(A) $\frac{1}{3} m v^{2}$
(B) $\frac{2}{3} m v^{2}$
(C) $\frac{25}{3} m v^{2}$
(D) $\frac{26}{3} m v^{2}$
(E) $\frac{2}{9} m v^{2}$
(F) $\frac{79}{9} m v^{2}$
(G) $9 m v^{2}$
[ENGAA, 2022S1Q39]
Find the sum of the real solutions to the equation

$$
2^{x}-(\sqrt{2})^{x+6}+12=0
$$

(A) 8
(B) 16
(C) $2^{\frac{4+\sqrt{2}}{2}}$
(D) $2^{\frac{6+\sqrt{2}}{2}}$
(E) $1+\frac{1}{2} \log _{2} 3$
(F) $4+2 \log _{2} 3$
[ENGAA, 2022S1Q40]
A steel cable has mass 64 kg and cross-sectional area $2.0 \times 10^{-4} \mathrm{~m}^{2}$.
The Young modulus of steel is $2.0 \times 10^{11} \mathrm{~Pa}$.
When the cable lies on horizontal ground its length is 40 m .
What is its extension when it is suspended freely from one end and hangs vertically? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that the cable obeys Hooke's law)
(A) 0 m
(B) $8.0 \times 10^{-7} \mathrm{~m}$
(C) $8.0 \times 10^{-6} \mathrm{~m}$
(D) $1.6 \times 10^{-5} \mathrm{~m}$
(E) $3.2 \times 10^{-5} \mathrm{~m}$
(F) $6.4 \times 10^{-5} \mathrm{~m}$
(G) $3.2 \times 10^{-4} \mathrm{~m}$
(H) $6.4 \times 10^{-4} \mathrm{~m}$

## ENGAA 2022 S2



## TIME ALLOWED: 60 MINUTES

This paper is Section 2 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.
[ENGAA, 2022S2Q1]
The diagram shows an object of mass 2.4 kg on a smooth horizontal surface.


A force $F$ acts on the object at an acute angle $\theta$ to the horizontal, where $\tan \theta=\frac{4}{3}$.
A force of 36 N acts on the object towards the right.
The object is in equilibrium.
What is the magnitude of the normal contact force exerted on the object by the surface?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 24 N
(B) 27 N
(C) 48 N
(D) 51 N
(E) 72 N
(F) 75 N
[ENGAA, 2022S2Q2]
The length of a spring when no force acts on it is $L$. The spring constant of the spring is $3.0 \times 10^{3} \mathrm{~N} \mathrm{~m}^{-1}$.

The spring is on the floor of an accelerating lift (elevator), and the spring supports a 30 kg mass.


The lift is accelerating downwards at $2.0 \mathrm{~m} \mathrm{~s}^{-2}$.
What is the difference between $L$ and the length of the spring when the lift is accelerating downwards?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; the spring obeys Hooke's law)
(A) 0 cm
(B) 2.0 cm
(C) 8.0 cm
(D) 10 cm
(E) 12 cm
[ENGAA, 2022S2Q3]
Electrical energy is transmitted at high voltage to a remote farm using an overhead power cable. Each of the two wires in the cable has a resistance of $2.5 \Omega$. The step-down transformer in the farm has a voltage ratio of 5.0. The transformer is ideal and $100 \%$ efficient. It supplies a power of 40 kW to a resistive load at the farm at a voltage of 250 V .
What is the rate at which electrical energy is transferred to thermal energy in the overhead cable?
(A) 1.28 kW
(B) 2.56 kW
(C) 5.12 kW
(D) 32 kW
(E) 64 kW
(F) 128 kW
[ENGAA, 2022S2Q4]
A wave is passing through a medium.
A particle of the medium has zero displacement from its equilibrium position at 0.12 s intervals, and at no other times.
The wavelength of the wave is greater than 10.0 m .
Two points are 5.0 m apart along the direction of travel of the wave.
The phase difference between the particles at the two points at the same instant is $\frac{\pi}{3}$ radians. What is the speed of the wave?
(A) $1.8 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $3.6 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $7.2 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $62.5 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $125 \mathrm{~m} \mathrm{~s}^{-1}$
(F) $250 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2022S2Q5]
Three light springs, $\mathrm{P}, \mathrm{Q}$ and R , are identical.
Springs P and Q are connected in series as shown. A downwards force $T$ is applied to the lower end.


Spring R is cut into four equal lengths, and the four pieces arranged symmetrically as shown. The two connecting bars have negligible mass. A downwards force $F$ is applied to the centre of the lower bar.


The total extensions of the two systems are equal. The springs obey Hooke's law.
Which expression gives $T$ in terms of $F$ ?
(A) $\frac{F}{16}$
(B) $\frac{F}{8}$
(C) $\frac{F}{4}$
(D) $\frac{F}{2}$
(E) $2 F$
(F) $4 F$
(G) $8 F$
(H) $16 F$
[ENGAA, 2022S2Q6]
A nylon cube resting on a horizontal surface has a volume of $64 \mathrm{~cm}^{3}$.
A force $F$ is applied vertically downwards on the top face of the cube so that it compresses the height by $x$.
The graph shows the variation of $F$ with $x$.


What is the Young modulus of the nylon?
(Assume that changes in horizontal cross-sectional area are negligible.)
(A) $7.7 \times 10^{3} \mathrm{~Pa}$
(B) $4.8 \times 10^{6} \mathrm{~Pa}$
(C) $9.6 \times 10^{6} \mathrm{~Pa}$
(D) $1.2 \times 10^{8} \mathrm{~Pa}$
(E) $1.5 \times 10^{9} \mathrm{~Pa}$
(F) $3.0 \times 10^{9} \mathrm{~Pa}$
(G) $1.9 \times 10^{12} \mathrm{~Pa}$

## [ENGAA, 2022S2Q7]

Five lampposts alongside a straight road are positioned at uniform intervals of 60 m .
A motorbike travelling at a constant velocity passes the first lamppost at time $t=0 \mathrm{~s}$. It passes the fifth lamppost at $t=20 \mathrm{~s}$.
A car travelling in the same direction as the motorbike is accelerating at $6.0 \mathrm{~m} \mathrm{~s}^{-2}$. At time $t=0$ s the car passes the first lamppost at a velocity of $3.0 \mathrm{~m} \mathrm{~s}^{-1}$.
At what time $t$ does the car overtake the motorbike?
(A) 1.5 s
(B) 2.0 s
(C) 2.5 s
(D) 3.0 s
(E) 3.5 s
(F) 4.0 s
(G) 5.0 s
[ENGAA, 2022S2Q8]
An electrical appliance has an input power $P$ which is a function of time $t$ during the first 10 seconds after it is switched on.
This function is

$$
P=3 t^{2}+4 t
$$

where $P$ is in watts and $t$ is in seconds.
The appliance is switched on at time $t=0$.
The appliance has a constant efficiency of $90 \%$.
What is the energy wasted by the appliance during the period $t=2.0 \mathrm{~s}$ to $t=3.0 \mathrm{~s}$ after it is switched on?
(A) 0.60 J
(B) 0.70 J
(C) 1.9 J
(D) 2.9 J
(E) 4.5 J
(F) 17 J
(G) 26 J
(H) 41 J
[ENGAA, 2022S2Q9]
A solid cylinder is made of transparent glass of refractive index $\frac{2}{\sqrt{3}}$. It is surrounded by air.
A ray of light travelling in air hits the cylinder at the centre of one circular face at a non-zero angle $\theta$ to the normal, and refracts as it enters the cylinder.
The ray then strikes the curved surface of the cylinder at an angle of incidence equal to the critical angle.
What is the value of $\theta$ ?
(A) $\sin ^{-1} \frac{\sqrt{3}}{4}$
(B) $\sin ^{-1} \frac{1}{\sqrt{3}}$
(C) $\sin ^{-1} \frac{2}{\sqrt{6}}$
(D) $\sin ^{-1} \frac{\sqrt{3}}{2}$
(E) $\sin ^{-1} 1$
[ENGAA, 2022S2Q10]
An object of mass 20 kg is acted on by a force that varies in magnitude during the time interval $t=0 \mathrm{~s}$ to $t=1.0 \mathrm{~s}$.
The force causes the object's displacement $x$ to change with time $t$ according to the relationship

$$
x=-t^{3}-3 t^{2}+4
$$

where $x$ is in metres and $t$ is in seconds.
What is the magnitude of the impulse on the object over this time interval?
(A) $2.8 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$
(B) $9.0 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$
(C) $55 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$
(D) $80 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$
(E) $100 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$
(F) $180 \mathrm{~kg} \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2022S2Q11]
Two small hard spheres of mass $m$ and $2 m$ are suspended side by side from light vertical strings of length $l$. The more massive sphere is raised so that its string is horizontal, and then released. It swings through $90^{\circ}$ and strikes the smaller sphere. The two spheres stick together, and rise to a maximum height $h$ as shown in the diagram.


Which expression gives the height $h$ in terms of $l$ ?
(Assume that air resistance is negligible.)
(A) $\frac{4 l}{27}$
(B) $\frac{8 l}{27}$
(C) $\frac{4 l}{9}$
(D) $\frac{2 l}{3}$
(E) $\frac{8 l}{9}$
(F) $l$
(G) $2 l$
[ENGAA, 2022S2Q12]
Three resistance wires $X, Y$ and $Z$, made from the same metal, are connected to each other and to a circular plastic ring as shown.

[diagram not to scale]
Wires $X$ and $Y$ each have twice the diameter of wire $Z$.
Wire $X$ is 12 cm long. Wire $Z$ is 15 cm long and is connected across a diameter of the ring. A power supply is connected to the two corners of the triangle that lie on the diameter.
What is the value of the ratio

$$
\frac{\text { current in } X}{\text { current in } Z} ?
$$

(A) $\frac{1}{5}$
(B) $\frac{7}{20}$
(C) $\frac{7}{10}$
(D) $\frac{5}{7}$
(E) $\frac{7}{5}$
(F) $\frac{10}{7}$
(G) $\frac{20}{7}$
(H) 5
[ENGAA, 2022S2Q13]
A light rope has cross-sectional area $6.0 \times 10^{-8} \mathrm{~m}^{2}$ and unstretched length 0.24 m .
The rope is fixed horizontally between two supports that are 0.24 m apart.
When a 1.0 kg mass is suspended from the middle of the rope, the vertical displacement of the middle of the rope from its original position is 0.050 m .
The rope obeys Hooke's law. Assume that changes in cross-sectional area are negligible.
What is the Young modulus of the material from which the rope is made?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) $5.2 \times 10^{8} \mathrm{~N} \mathrm{~m}^{-2}$
(B) $8.0 \times 10^{8} \mathrm{~N} \mathrm{~m}^{-2}$
(C) $1.0 \times 10^{9} \mathrm{~N} \mathrm{~m}^{-2}$
(D) $1.3 \times 10^{9} \mathrm{~N} \mathrm{~m}^{-2}$
(E) $2.0 \times 10^{9} \mathrm{~N} \mathrm{~m}^{-2}$
(F) $2.6 \times 10^{9} \mathrm{~N} \mathrm{~m}^{-2}$
(G) $5.2 \times 10^{9} \mathrm{~N} \mathrm{~m}^{-2}$
[ENGAA, 2022S2Q14]
The three graphs show the displacement, velocity and acceleration against time for an object moving in a straight line.
The time axis is shown to the same scale on all three graphs.




Which graph represents which quantity?

|  | graph $P$ | graph $Q$ | graph $R$ |
| :---: | :---: | :---: | :---: |
| (A) | acceleration | displacement | velocity |
| (B) | acceleration | velocity | displacement |
| (C) | displacement | acceleration | velocity |
| (D) | displacement | velocity | acceleration |
| (E) | velocity | acceleration | displacement |
| (F) | velocity | displacement | acceleration |

[ENGAA, 2022S2Q15]
A system of light springs that does not obey Hooke's law has an unstretched length of 2.0 m .
The extension $x$ of the system is related to the force $F$ applied to it by

$$
F=p x^{2}
$$

where $p$ is a constant.
A force of 2400 N increases the length of the system to 2.2 m .
How much work is done in increasing the length of the system from 3.0 m to 4.0 m ?
(A) 1.2 kJ
(B) 60 kJ
(C) 70 kJ
(D) 120 kJ
(E) 140 kJ
(F) 740 kJ
[ENGAA, 2022S2Q16]
The diagram shows a circuit that includes two batteries, each with negligible internal resistance.


What is the reading on the ammeter?
(A) 0.0029 A
(B) 0.0071 A
(C) 0.063 A
(D) 0.083 A
(E) 0.50 A
(F) 0.65 A
(G) 1.2 A
(H) 2.0 A
[ENGAA, 2022S2Q17]
The upper diagram shows the equilibrium positions of nine equally spaced particles in a medium.
The lower diagram shows the positions of the same nine particles when a longitudinal wave is travelling through the medium. The wave is shown at time $t=0$, travelling to the right.


The frequency of the wave is 0.5 Hz .
Which graph represents the displacements of the particles at a later time $t=0.5 \mathrm{~s}$ ?
(On the graphs, positive displacement values represent particle displacements to the right.)
(A)

(C)

(E)

(B)

(D)

(F)

(H)

[ENGAA, 2022S2Q18]
A power supply with constant emf and internal resistance $r$ is connected to an external resistor. The efficiency of the system is defined as

$$
\text { efficiency }=\frac{\text { power dissipated by external resistor }}{\text { total power supplied by cell }}
$$

Which graph shows how the efficiency varies with the resistance of the external resistor?
(A)

(B)

(C)

(D)

(E)

(F)

[ENGAA, 2022S2Q19]
A 10 kg projectile is launched from ground level at an angle of $60^{\circ}$ above the horizontal, with an initial speed of $12 \mathrm{~m} \mathrm{~s}^{-1}$. The horizontal component of its velocity is to the right.

At the point during its flight when the vertical component of its velocity is zero, the projectile splits into two pieces, $P$ and $Q$, each of mass 5 kg .

Immediately after the projectile splits, piece $P$ has velocity $14 \mathrm{~m} \mathrm{~s}^{-1}$ to the right.
What is the speed of piece $Q$ immediately before it hits the ground?
(Assume that air resistance is negligible, and that the ground is horizontal.)
(A) $2 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $\sqrt{31} \mathrm{~m} \mathrm{~s}^{-1}$
(C) $6 \sqrt{3} \mathrm{~m} \mathrm{~s}^{-1}$
(D) $4 \sqrt{7} \mathrm{~m} \mathrm{~s}^{-1}$
(E) $2 \sqrt{43} \mathrm{~m} \mathrm{~s}^{-1}$
(F) $4 \sqrt{13} \mathrm{~m} \mathrm{~s}^{-1}$
(G) $4 \sqrt{19} \mathrm{~m} \mathrm{~s}^{-1}$
(H) $2 \sqrt{127} \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2022S2Q20]
The density $\rho$ of a sphere varies from its centre to its surface according to the equation

$$
\rho=\rho_{0}\left(1-\frac{x}{2 R}\right)
$$

where $x$ is the distance from its centre, $R$ is its radius and $\rho_{0}$ is the density at its centre. What is the mass of the sphere?
(the surface area of a sphere of radius $x$ is equal to $4 \pi x^{2}$ )
(A) $\frac{2 \pi R^{3} \rho_{0}}{3}$
(B) $\frac{5 \pi R^{3} \rho_{0}}{6}$
(C) $\frac{8 \pi R^{3} \rho_{0}}{9}$
(D) $\pi R^{3} \rho_{0}$
(E) $\frac{29 \pi R^{3} \rho_{0}}{24}$
(F) $\frac{19 \pi R^{3} \rho_{0}}{15}$
(G) $\frac{4 \pi R^{3} \rho_{0}}{3}$
(H) $2 \pi R^{3} \rho_{0}$

## ENGAA 2023 S1



## TIME ALLOWED: 60 MINUTES

This paper is Section 1 of 2 .
A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains two parts, A and B, and you should attempt both parts.

## Part A Mathematics and Physics (20 questions)

Part B Advanced Mathematics and Advanced Physics (20 questions)
You are strongly advised to divide your time equally between the two parts: 30 minutes on Part A and 30 minutes on Part B. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 40 questions. Each question is worth one mark.

For each question, choose the one option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You must complete the answer sheet within the time limit.
You can use the question paper for rough working, but no extra paper is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

## Part A

[ENGAA, 2023S1Q1]
The surface area of a solid sphere of radius $R$ is equal to the total surface area of 10 solid closed cylinders of radius $r$ and height $4 r$.
Which of the following is an expression for $R$ in terms of $r$ ?
(The surface area of a sphere of radius $R$ is $4 \pi R^{2}$.)
(A) $R=5 r$
(B) $R=12 r$
(C) $R=2 \sqrt{5} r$
(D) $R=\frac{1}{2} \sqrt{10} r$
(E) $R=\sqrt{10} r$
(F) $R=\frac{3}{2} \sqrt{10} r$
(G) $R=\sqrt{15} r$
[ENGAA, 2023S1Q2]
A spaceship of mass 10000 kg is moving at $2.0 \mathrm{~m} \mathrm{~s}^{-1}$ relative to a space station.
The spaceship is captured by a robotic arm attached to the space station and brought to rest by a force of 1000 N .
How far will the spaceship move in its initial direction relative to the space station while the force is being applied?
(Assume that the acceleration of the space station is negligible.)
(A) 0.050 m
(B) 0.10 m
(C) 0.20 m
(D) 5.0 m
(E) 10 m
(F) 20 m
[ENGAA, 2023S1Q3]
Which of the following is a correct rearrangement of

$$
y=p-\frac{q-r}{s-x}
$$

to make $x$ the subject?
(A) $x=s-\frac{q-r}{p+y}$
(B) $x=\frac{q-r}{p+y}-s$
(C) $x=s-\frac{q-r}{p-y}$
(D) $x=\frac{q-r}{p-y}-s$
(E) $x=s-\frac{q-r}{y-p}$
(F) $x=\frac{q-r}{y-p}-s$
[ENGAA, 2023S1Q4]
A circuit is set up as shown. All three resistors are identical.
When the switch is open, the reading on the ammeter is 1.0 A and the power transferred from the battery is 1.0 W .


The switch is now closed.
What is the new reading on the ammeter and what is the new power transferred from the battery?

|  | ammeter reading / A | power transferred / W |
| :---: | :---: | :---: |
| (A) | 0.67 | 0.67 |
| (B) | 0.67 | 1.3 |
| (C) | 0.67 | 1.5 |
| (D) | 0.67 | 2.0 |
| (E) | 1.0 | 1.0 |
| (F) | 1.0 | 1.5 |
| (G) | 1.0 | 2.0 |
| (H) | 1.0 | 3.0 |

[ENGAA, 2023S1Q5]

[diagram not to scale]
$W X Y Z$ is a square of side length 1.
$W M: M X=1: 2$
$X N: N Y=3: 1$
$Y P: P Z=4: 1$
What is the area of triangle $M N P$ ?
(A) $\frac{1}{3}$
(B) $\frac{2}{5}$
(C) $\frac{9}{20}$
(D) $\frac{1}{30}$
(E) $\frac{19}{60}$
(F) $\frac{23}{60}$
[ENGAA, 2023S1Q6]
A spring is initially unstretched. A force $F$ is used to stretch the spring. The extension $x$ and the energy $E$ stored in the stretched spring are measured for different values of $F$.
The graph shows how the energy $E$, in J, varies with the extension squared, $x^{2}$, in $\mathrm{cm}^{2}$.


What is the magnitude of $F$ when the spring stores 0.015 J of energy?
(A) 0.30 N
(B) 0.60 N
(C) 1.2 N
(D) 1.5 N
(E) 2.4 N
(F) 3.0 N
(G) 30 N
(H) 60 N
[ENGAA, 2023S1Q7]
Given that

$$
\frac{27^{2(x-2)}}{9^{(2 x-3)}}=(81)^{\frac{3}{2}}
$$

what is the value of $x$ ?
(A) 0
(B) 2.5
(C) 3
(D) 6
(E) 7.5
(F) 9
(G) 10.5
(H) 12
[ENGAA, 2023S1Q8]
A solid, cylindrical metal bar has a uniform cross-sectional area of $12 \mathrm{~cm}^{2}$ and a volume of 180 $\mathrm{cm}^{3}$.
The bar rests on a horizontal surface on one of its circular faces.
The pressure on the surface due to the bar is $0.45 \mathrm{~N} \mathrm{~cm}^{-2}$.
What is the density of the metal, in $\mathrm{g} \mathrm{cm}^{-3}$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) $2.5 \mathrm{~g} \mathrm{~cm}^{-3}$
(B) $3.0 \mathrm{~g} \mathrm{~cm}^{-3}$
(C) $3.75 \mathrm{~g} \mathrm{~cm}^{-3}$
(D) $7.5 \mathrm{~g} \mathrm{~cm}^{-3}$
(E) $15 \mathrm{~g} \mathrm{~cm}^{-3}$
(F) $33 \mathrm{~g} \mathrm{~cm}^{-3}$
[ENGAA, 2023S1Q9]
Last year, the salary of the coach of a football club was $80 \%$ of the salary of the star player.
At the start of the new year, the coach received a $15 \%$ increase in salary and the star player received a $38 \%$ increase in salary.
What percentage of the star player's new salary is the coach's new salary?
(A) $46 \%$
(B) $57 \%$
(C) $61 \frac{3}{5} \%$
(D) $66 \frac{2}{3} \%$
(E) $77 \%$
(F) $83 \frac{1}{3} \%$
[ENGAA, 2023S1Q10]
Two samples of pure radioactive isotopes X and Y decay with half-lives of 2 days and 3 days, respectively.
Both X and Y decay in a single step into different stable isotopes.
Initially the number of atoms of X is twice the number of atoms of Y .
After how many days are the expected numbers of atoms of $X$ and $Y$ equal to each other?
(A) The expected numbers of atoms of X and Y are never equal.
(B) 2 days
(C) 3 days
(D) 4 days
(E) 6 days
(F) 12 days
[ENGAA, 2023S1Q11]
An athlete's training session consists of several complete repetitions of a three-part programme:

1. Walk 100 m at an average speed of $6 \mathrm{~km} \mathrm{~h}^{-1}$
2. Jog 200 m at an average speed of $10 \mathrm{~km} \mathrm{~h}^{-1}$
3. Run 100 m at an average speed of $20 \mathrm{~km} \mathrm{~h}^{-1}$

What is the athlete's average speed for the complete training session, in $\mathrm{km} \mathrm{h}^{-1}$ ?
(A) 7.2
(B) 9.6
(C) 11.5
(D) 12
(E) 14.4
[ENGAA, 2023S1Q12]
A large, flat, metal plate is coated on one side with a layer of thermally insulating material of the same thickness $a$ as the metal plate.
The uninsulated top surface of the metal plate is maintained at a constant temperature $T_{1}$.
The bottom surface of the insulating material is maintained at a constant, lower temperature $T_{2}$.
The system is in equilibrium.
The diagram shows this arrangement.


Which graph could show how the temperature varies with distance from the top surface of the metal plate to the bottom surface of the insulating material?
(A)

(B)

distance from top
of metal plate
(C)

(D)

(E)

$$
\text { temperature } T_{1}
$$

[ENGAA, 2023S1Q13]
Two objects $X$ and $Y$ are similar.
The surface area of object $Y$ is double the surface area of object $X$.
The volume of object $Y$ is $7 \sqrt{2} \mathrm{~cm}^{3}$ more than the volume of object $X$.
What is the volume of object $X$, in $\mathrm{cm}^{3}$ ?
(A) $14-7 \sqrt{2}$
(B) $14+7 \sqrt{2}$
(C) $\frac{42-7 \sqrt{2}}{17}$
(D) $\frac{42+7 \sqrt{2}}{17}$
(E) $\frac{7 \sqrt{2}}{3}$
(F) $7 \sqrt{2}$
(G) $4-\sqrt{2}$
(H) $4+\sqrt{2}$
[ENGAA, 2023S1Q14]
The voltage output of a power station is stepped up using a transformer before the power is transmitted to a distant town. The primary coil of this transformer has 300 turns and the secondary coil has 1500 turns.
In the town, a step-down transformer reduces the voltage supplied by the transmission cables to 33000 V for distribution within the town. The step-down transformer supplies a current of 1500 A.
The current in the transmission cables is 450 A and both transformers are ideal and $100 \%$ efficient.
What is the voltage output of the power station?
(Assume that the resistance of the transmission cables is negligible.)
(A) 1980 V
(B) 6600 V
(C) 22000 V
(D) 110000 V
(E) 550000 V
[ENGAA, 2023S1Q15]
The equation

$$
\left(\frac{a \times 10^{4}+2 a \times 10^{3}}{3 \times 10^{-1}}\right)^{2}=8 \times 10^{9}
$$

has two solutions for $a$.
What is the positive difference between these two solutions?
(A) 0
(B) $2 \sqrt{5}$
(C) $4 \sqrt{5}$
(D) $20 \sqrt{5}$
(E) $40 \sqrt{5}$
(F) $200 \sqrt{5}$
[ENGAA, 2023S1Q16]
A transverse wave with an amplitude of 3.0 cm travels along a stretched string. The wave has a frequency of 12 Hz and a wavelength of 0.25 m .
What is the average speed of a particle in the string as the string oscillates during a time of 2.0 s ?
(A) $36 \mathrm{~cm} \mathrm{~s}^{-1}$
(B) $72 \mathrm{~cm} \mathrm{~s}^{-1}$
(C) $125 \mathrm{~cm} \mathrm{~s}^{-1}$
(D) $144 \mathrm{~cm} \mathrm{~s}^{-1}$
(E) $300 \mathrm{~cm} \mathrm{~s}^{-1}$
[ENGAA, 2023S1Q17]
$X$ and $Y$ are the end-points of a line segment.
Point $P$ has coordinates $(-8,5)$.
$P$ lies on the line segment $X Y$ such that $X P: P Y$ is $1: 2$ and $\overrightarrow{X P}=\binom{4}{-3}$.
A point $Q$ is such that $\overrightarrow{Q Y}=\binom{7}{6}$.
What are the coordinates of point $Q$ ?
(A) $(7,5)$
(B) $(3,8)$
(C) $(1,-12)$
(D) $(-3,-10)$
(E) $(-7,-7)$
(F) $(-11,-4)$
[ENGAA, 2023S1Q18]
A battery and two resistors $X$ and $Y$ are connected in series.


The power transferred by the battery is 6 W .
The resistance of $X$ is $10 \Omega$.
The voltage across $Y$ is 4 V .
What is the current in the circuit?
(A) $\frac{2}{5} \mathrm{~A}$
(B) $\frac{3}{5} \mathrm{~A}$
(C) $\frac{3}{4} \mathrm{~A}$
(D) 1 A
(E) $\sqrt{\frac{3}{10}} \mathrm{~A}$
(F) $\sqrt{\frac{3}{5}} \mathrm{~A}$
[ENGAA, 2023S1Q19]
Find the maximum value of

$$
2^{\sin x} \times 3^{-\sin x}
$$

where $0^{\circ} \leq x \leq 360^{\circ}$.
(A) $\frac{2}{3}$
(B) 1
(C) $\frac{3}{2}$
(D) 2
(E) 3
(F) 6
[ENGAA, 2023S1Q20]
A diver at the bottom of a lake of depth $d$ fills a syringe with an ideal gas and seals the nozzle. The piston remains free to move. The volume of the gas in the syringe at the bottom of the lake is $90 \mathrm{~cm}^{3}$.
As the diver returns to the surface, the temperature of the gas does not change. At the surface of the lake the gas in the syringe is at atmospheric pressure and the volume of the gas is 720 $\mathrm{cm}^{3}$.
What is the volume of the gas in the syringe at a depth $\frac{d}{2}$ ?
(A) $160 \mathrm{~cm}^{3}$
(B) $180 \mathrm{~cm}^{3}$
(C) $206 \mathrm{~cm}^{3}$
(D) $225 \mathrm{~cm}^{3}$
(E) $288 \mathrm{~cm}^{3}$
(F) $315 \mathrm{~cm}^{3}$
(G) $360 \mathrm{~cm}^{3}$
(H) $405 \mathrm{~cm}^{3}$

## PART B

[ENGAA, 2023S1Q21]
$x^{2}-x-6$ is a factor of $x^{3}+a x^{2}+2 x+b$, where $a$ and $b$ are real constants.
What is the value of $a+b$ ?
(A) -39
(B) -21
(C) -3
(D) $-\frac{3}{5}$
(E) $\frac{3}{5}$
(F) 3
(G) 21
(H) 39
[ENGAA, 2023S1Q22]
Two pipes contain air at the same temperature and pressure.
A stationary sound wave is formed in the first pipe, which is closed at one end and open at the other end. The lowest frequency of stationary sound wave that can be formed in this pipe is 4000 Hz .
The second pipe has the same length as the first pipe, but is open at both ends.
What is the lowest frequency of stationary sound wave that can be formed in the second pipe?
(A) 1000 Hz
(B) 2000 Hz
(C) 4000 Hz
(D) 8000 Hz
(E) 16000 Hz
[ENGAA, 2023S1Q23]
An arithmetic progression has first term $a$ and common difference $d$.
The sum of the first 9 terms plus the sum of the first 10 terms is equal to the sum of the first 11 terms.
Which of the following is a correct expression for $a$ in terms of $d$ ?
(A) $a=-\frac{13}{4} d$
(B) $a=-\frac{15}{4} d$
(C) $a=-\frac{16}{3} d$
(D) $a=-\frac{19}{3} d$
(E) $a=-7 d$
(F) $a=-8 d$
[ENGAA, 2023S1Q24]
A cannon ball of mass 2.6 kg is fired horizontally from a cannon on the top of a cliff at a speed of $90 \mathrm{~m} \mathrm{~s}^{-1}$.
The height of the cliff above the horizontal ground below is 45 m .
What is the magnitude of the impulse that acts on the ball between leaving the cannon and reaching the ground?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$; assume that air resistance is negligible)
(A) 8.7 N s
(B) 13 Ns
(C) 52 Ns
(D) 78 Ns
(E) 117 N s
(F) 234 Ns
(G) 900 Ns
[ENGAA, 2023S1Q25]
The first three terms of a convergent geometric progression are:

$$
2 p, p-3, p-7
$$

What is the sum to infinity of this progression?
(A) -54
(B) -27
(C) $-13 \frac{1}{2}$
(D) -2
(E) 2
(F) $13 \frac{1}{2}$
(G) 27
(H) 54
[ENGAA, 2023S1Q26]
A child on a sledge is travelling straight down a snow-covered slope at a constant speed of 15 $\mathrm{m} \mathrm{s}^{-1}$.
The mass of the child and sledge together is 60 kg .
The angle of the slope to the horizontal is $30^{\circ}$.
What is the rate at which thermal energy is being produced due to friction forces? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 450 W
(B) 900 W
(C) 4500 W
(D) $4500 \sqrt{3} \mathrm{~W}$
(E) 6750 W
(F) 9000 W
(G) 13500 W
(H) $9000 \sqrt{3} \mathrm{~W}$
[ENGAA, 2023S1Q27]
The diagram shows a circle with centre $O$.


What is the value of $x$ ?
(A) $40^{\circ}$
(B) $50^{\circ}$
(C) $55^{\circ}$
(D) $70^{\circ}$
(E) $75^{\circ}$
(F) $80^{\circ}$
[ENGAA, 2023S1Q28]
The diagram shows a battery with no internal resistance and four identical resistors $P, Q, R$ and $S$.
Resistor $Q$ dissipates 4.0 W of power.


What is the total power supplied by the battery?
(A) 10 W
(B) 16 W
(C) 24 W
(D) 32 W
(E) 40 W
[ENGAA, 2023S1Q29]
The function

$$
f(x)=\sqrt{2} x^{2}-6 x+4
$$

can be written in the form

$$
f(x)=p(x+q)^{2}+r
$$

where $p, q$ and $r$ are constants.
What is the value of $p(r-q)$ ?
(A) 2
(B) 7
(C) $3-\frac{\sqrt{2}}{2}$
(D) $3-7 \sqrt{2}$
(E) $4-3 \sqrt{2}$
(F) $4 \sqrt{2}-6$
(G) $4 \sqrt{2}-12$
(H) $7 \sqrt{2}-18$
[ENGAA, 2023S1Q30]
Two identical, uniform, thin planks, each of mass 40 kg , are propped up against one another in equilibrium on a rough surface as shown. Each plank is at an angle of $30^{\circ}$ to the vertical.

[diagram not to scale]
What is the magnitude of the friction force at point $P$ ?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 100 N
(B) $\frac{200}{\sqrt{3}} \mathrm{~N}$
(C) 200 N
(D) $200 \sqrt{3} \mathrm{~N}$
(E) $\frac{400}{\sqrt{3}} \mathrm{~N}$
(F) 400 N
(G) $400 \sqrt{3} \mathrm{~N}$
[ENGAA, 2023S1Q31]
Which of the following is equal to

$$
\int_{-(3-\sqrt{5})}^{3-\sqrt{5}} \frac{x^{2}}{3-\sqrt{5}} \mathrm{~d} x ?
$$

(A) 0
(B) $\frac{2}{3}$
(C) $\frac{8}{3}$
(D) 4
(E) $\frac{28-12 \sqrt{5}}{3}$
(F) $28-12 \sqrt{5}$
[ENGAA, 2023S1Q32]
Three blocks $P, Q$ and $R$ are in contact on a horizontal surface as shown.

$\xrightarrow{20 \mathrm{~N}}$| P |  |  |
| :---: | :---: | :---: |
| 4.0 kg | Q <br> 2.0 kg | R <br> 2.0 kg |

The mass of $P$ is 4.0 kg and the masses of $Q$ and $R$ are each 2.0 kg .
A horizontal force of 20 N is applied to $P$ so that all three blocks move together.
The friction force between $P$ and the surface is 2.0 N , and the friction forces between $Q$ and $R$ and the surface are each 1.0 N .
What is the magnitude of the force that $R$ exerts on $Q$ ?
(A) 2.0 N
(B) 5.0 N
(C) 6.0 N
(D) 12 N
(E) 15 N
(F) 17 N
(G) 19 N
[ENGAA, 2023S1Q33]
A function $f$ is defined by

$$
f(x)=\frac{a}{x}+\frac{b}{x^{2}}
$$

where $a$ and $b$ are constants.
It is given that $f^{\prime}(1)=2$ and $f^{\prime \prime}(-1)=-2$.
What is the value of $a+b$ ?
(A) -5
(B) -4
(C) $-\frac{7}{5}$
(D) -1
(E) $\frac{9}{5}$
(F) 2
(G) $\frac{14}{5}$
(H) 3
[ENGAA, 2023S1Q34]
An object is released from rest at a height $H$ above the ground and falls freely in a uniform gravitational field. At time $t$ after being released, it has fallen a distance $s$ and is at a height $h$ above the ground, travelling at speed $v$.


The graph shows two quantities plotted.


Which of the rows show(s) a possible pair of quantities for the axes?
(Assume that air resistance is negligible.)

|  | $y$-axis | $x$-axis |
| :---: | :---: | :---: |
| 1 | $h$ | $t$ |
| 2 | kinetic energy | $h$ |
| 3 | gravitational potential energy | $s$ |

(A) none of them
(B) 1 only
(C) 2 only
(D) 3 only
(E) 1 and 2 only
(F) 1 and 3 only
(G) 2 and 3 only
(H) 1, 2 and 3
[ENGAA, 2023S1Q35]
Find the real value of $x$ that satisfies

$$
\log _{3}\left(x^{2}+3 x+2\right)=2+\log _{3}\left(x^{2}+2 x\right)
$$

(A) $\frac{1}{8}$
(B) $\frac{1}{7}$
(C) $\frac{1}{6}$
(D) $\frac{1}{2}$
(E) 2
(F) 7
[ENGAA, 2023S1Q36]
A 20 V battery of negligible internal resistance is connected in series with a $20 \Omega$ resistor and a cylindrical conductor. The current in the circuit is 0.20 A .
The cylindrical conductor is now removed, melted and re-formed into a new cylinder. After the cylinder has cooled to its original temperature, its length is 4 times greater than that of the original cylinder.
What is the resistance of the new cylinder?
(A) $5.0 \Omega$
(B) $20 \Omega$
(C) $320 \Omega$
(D) $400 \Omega$
(E) $1280 \Omega$
(F) $1600 \Omega$
[ENGAA, 2023S1Q37]
What is the constant term in the simplified binomial expansion of $\left(\frac{2}{x}+\frac{x}{4}\right)^{8}$ ?
(A) $\frac{7}{2}$
(B) $\frac{35}{8}$
(C) $\frac{1}{16}$
(D) $\frac{7}{256}$
(E) $\frac{35}{1024}$
(F) $\frac{1}{2048}$
[ENGAA, 2023S1Q38]
A sound wave can travel from a source at $P$ to a detector at $R$ directly or by reflecting at $Q$. The angle between $P R$ and $P Q$ is $30^{\circ}$ and the distance from $P$ to $R$ is $d$ as shown.


There is a phase difference of $\pi$ radians between the incident and reflected wave at $Q$.
Waves that reach $R$ via $Q$ are in phase with waves that reach $R$ directly from $P$.
Which expression gives the greatest wavelength of sound waves for which this is true?
(A) $2 d\left(\frac{2}{\sqrt{3}}-1\right)$
(B) $d\left(\frac{2}{\sqrt{3}}-1\right)$
(C) $d(2-\sqrt{3})$
(D) $\frac{4 d}{\sqrt{3}}$
(E) $\frac{2 d}{\sqrt{3}}$
[ENGAA, 2023S1Q39]
Given that

$$
x^{2}+y^{2}=1
$$

what is the greatest possible value of $2 x+3 y$ ?
(A) $\frac{7}{2}$
(B) 3
(C) $\frac{5 \sqrt{2}}{2}$
(D) $\sqrt{7}$
(E) $\sqrt{10}$
(F) $\frac{13 \sqrt{10}}{10}$
(G) $\sqrt{13}$
(H) $\frac{12 \sqrt{13}}{13}$
[ENGAA, 2023S1Q40]
Two springs have spring constants of $200 \mathrm{~N} \mathrm{~m}^{-1}$ and $600 \mathrm{~N} \mathrm{~m}^{-1}$, respectively.
The springs are joined in series, end-to-end, and stretched so that their combined extension is 0.80 m .

What is the total strain energy stored in the springs?
(A) 48 J
(B) 96 J
(C) 112 J
(D) 120 J
(E) 240 J
(F) 256 J
(G) 512 J

## ENGAA 2023 S2



## TIME ALLOWED: 60 MINUTES

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This paper contains 20 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

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Dictionaries and calculators are NOT permitted.
[ENGAA, 2023S2Q1]
A block of weight $W$ slides down a rough plane at a constant speed.
The plane is at an angle of $30^{\circ}$ to the horizontal.
The block is now pulled by a force of $3 W$ acting parallel to and up the plane. The block has constant acceleration.
Which expression gives the acceleration of the block?
(gravitational field strength $=g$ )
(A) $2 g$
(B) $\frac{5}{2} g$
(C) $3 g$
(D) $(3-\sqrt{3}) g$
(E) $\left(3-\frac{1}{\sqrt{3}}\right) g$
(F) $\left(3-\frac{\sqrt{3}}{2}\right) g$
(G) $\left(3-\frac{2}{\sqrt{3}}\right) g$
[ENGAA, 2023S2Q2]
The speed $v$ of an object moving in a straight line is related to time $t$ by the equation

$$
v=k t^{2}
$$

where $k$ is a constant.
At $t=10 \mathrm{~s}$ the speed of the object is $48 \mathrm{~m} \mathrm{~s}^{-1}$ and the resultant force on the object is 24 N .
What is the mass of the object?
(A) 0.15 kg
(B) 0.40 kg
(C) 1.2 kg
(D) 2.5 kg
(E) 6.7 kg
[ENGAA, 2023S2Q3]
Two waves $P$ and $Q$, which superpose, are shown in the diagram in a particular region at time $t=0$.


Both waves have period $T$ and are moving in the directions shown by the arrows. Wave $P$ has amplitude 2.0 cm and wave $Q$ has amplitude 1.0 cm .
Which diagram represents the resultant wave formed in the same region by waves $P$ and $Q$ at time $t=\frac{T}{2}$ ?
(A)

(C)

(E)

(G)

(B)

(D)

(F)

[ENGAA, 2023S2Q4]
Four identical springs are arranged as shown and suspended from a support.


The mass of the springs, rod and hook are negligible.
A load of weight 8.4 N is attached to the hook at the lower end of the springs and this causes a total extension of the system of 24 mm .
The arrangement is then changed to:


The load of 8.4 N is attached to the bottom of the lower spring.
What is the total extension of the system at equilibrium in the second arrangement?
(The springs obey Hooke's law.)
(A) 3 mm
(B) 12 mm
(C) 16 mm
(D) 24 mm
(E) 32 mm
(F) 48 mm
(G) 64 mm
[ENGAA, 2023S2Q5]
A student and a child are standing on trolleys $X$ and $Y$, respectively, which are close to each other but not touching. The trolleys are initially stationary on a straight, horizontal frictionless track. The student is initially holding a ball of mass 5.0 kg .
The total mass of the student, the ball and trolley $X$ is 80 kg .
The total mass of the child and trolley $Y$ is 20 kg .
The student on trolley $X$ throws the ball to the child on trolley $Y$. The ball travels at a horizontal speed of $12 \mathrm{~m} \mathrm{~s}^{-1}$ relative to the ground. The child then catches the ball.
What is the speed of separation of the trolleys after the child has caught the ball?
(Assume that air resistance is negligible.)
(A) $1.6 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $2.4 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $3.2 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $3.8 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $24 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2023S2Q6]
The variation of the acceleration with time of an object moving in a straight line is shown on the graph.
At time $=0 \mathrm{~s}$ the velocity of the object is $8.0 \mathrm{~m} \mathrm{~s}^{-1}$.


What is the maximum velocity of the object between time $=0 \mathrm{~s}$ and time $=6 \mathrm{~s}$ ?
(A) $5.0 \mathrm{~m} \mathrm{~s}^{-1}$
(B) $8.0 \mathrm{~m} \mathrm{~s}^{-1}$
(C) $12 \mathrm{~m} \mathrm{~s}^{-1}$
(D) $20 \mathrm{~m} \mathrm{~s}^{-1}$
(E) $32 \mathrm{~m} \mathrm{~s}^{-1}$
(F) $44 \mathrm{~m} \mathrm{~s}^{-1}$
[ENGAA, 2023S2Q7]
The diagram shows a circuit that includes a battery with an emf of 18 V and internal resistance $r$.


The three identical resistors in the external circuit each have resistance $R$.
The terminal potential difference across the battery is 16 V .
Which expression gives $R$ in terms of $r$ ?
(A) $R=\frac{10 r}{3}$
(B) $R=\frac{16 r}{3}$
(C) $R=6 r$
(D) $R=12 r$
(E) $R=\frac{27 r}{2}$
(F) $R=24 r$
(G) $R=\frac{51 r}{2}$
[ENGAA, 2023S2Q8]
Three identical bar magnets, each of mass $m$, and two identical trolleys, $X$ and $Y$, also each of mass $m$, are arranged with the bar magnets fixed to the trolleys as shown. The trolleys are held at rest a short distance apart on a smooth horizontal track.


The trolleys are released at the same time. They move towards each other and collide. Find the value of the ratio
kinetic energy of $X$ immediately before collision
kinetic energy of $Y$ immediately before collision.
(A) $\frac{4}{9}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) 1
(E) $\frac{3}{2}$
(F) 2
(G) $\frac{9}{4}$
[ENGAA, 2023S2Q9]
A uniform rod $X Y$ of length 3.0 m has a weight of 20 N . The rod is supported by two light wires, $P$ and $Q$, as shown. $P$ and $Q$ are attached 0.50 m from ends $X$ and $Y$, respectively.


A 40 N load is moved from end $X$ to end $Y$. The rod remains horizontal at all times.
Which graph shows the variation of the tension $T$ in wire $P$ with the position of the load as it is moved along the rod?
(A)

(B)

(C)

(E)

(D)

(F)

[ENGAA, 2023S2Q10]
A pipe of length $L$ open at both ends contains a stationary sound wave with 1 node, as shown in the diagram.


The frequency of the stationary wave in this pipe is $4 f$.
A second pipe is open at one end and closed at the other end. A stationary sound wave in this pipe contains one more node than the stationary wave shown in the diagram.
The frequency of the stationary wave in the second pipe is $f$.
The speed of sound is the same in both pipes.
What is the length of the second pipe?
(A) $4 L$
(B) $6 L$
(C) $8 L$
(D) 10 L
(E) $12 L$

## [ENGAA, 2023S2Q11]

The resistors in the following four circuits are identical.

1


2


3


4


The cells are identical and have no internal resistance. Each cell can supply the same total amount of energy at a constant voltage before becoming exhausted.
$t_{1}, t_{2}, t_{3}$ and $t_{4}$ are the lengths of time after which the cells in circuits $1,2,3$ and 4 , respectively, become exhausted.
Which comparison of $t_{1}, t_{2}, t_{3}$ and $t_{4}$ is correct?
(A) $t_{1}=t_{2}<t_{3}=t_{4}$
(B) $t_{1}=t_{3}<t_{2}=t_{4}$
(C) $t_{2}<t_{1}=t_{4}<t_{3}$
(D) $t_{2}=t_{4}<t_{1}=t_{3}$
(E) $t_{3}<t_{1}=t_{4}<t_{2}$
(F) $t_{3}=t_{4}<t_{1}=t_{2}$
[ENGAA, 2023S2Q12]
A particle of mass $m$ is accelerated from rest by a resultant force of varying magnitude that acts in a constant direction. The kinetic energy $E$ of the particle increases with time $t$ according to the equation

$$
E=k t
$$

where $k$ is a constant.
Which expression gives the resultant force on the particle at time $T$ ?
(A) $k$
(B) $2 m k$
(C) $\sqrt{2 m k T}$
(D) $\sqrt{\frac{m k}{2 T}}$
(E) $\sqrt{\frac{m k}{8 T}}$
(F) $\sqrt{\frac{2 m k}{T}}$
(G) $\sqrt{\frac{k}{2 m T}}$
[ENGAA, 2023S2Q13]
A light horizontal wire of cross-sectional area $A$ is fixed at two points a distance $2 L$ apart. The initial tension in the wire is zero.
An object of weight $W$ is fixed directly to the centre of the wire. The wire stretches so that the object rests in equilibrium at a vertical distance of $\frac{3 L}{4}$ below the original position of the wire.
What is the Young modulus of the wire?
(Assume that the wire does not exceed its limit of proportionality.)
(A) $\frac{2 W}{A}$
(B) $\frac{4 W}{A}$
(C) $\frac{5 W}{2 A}$
(D) $\frac{2 W}{3 A}$
(E) $\frac{10 W}{3 A}$
(F) $\frac{20 W}{3 A}$
(G) $\frac{5 W}{6 A}$
[ENGAA, 2023S2Q14]
A triangular ramp with angles to the horizontal of $60^{\circ}$ and $30^{\circ}$ is placed with its largest face horizontal. A block of mass 1.5 kg and a block of mass $m$ are joined by a light, inextensible string and placed on the ramp as shown.


The string passes over a light, frictionless pulley.
The maximum force of friction between the block of mass 1.5 kg and the surface of the ramp is 3.5 N .

The maximum force of friction between the block of mass $m$ and the surface of the ramp is 5.0 N.

What is the maximum value of $m$ that allows the blocks to remain stationary on the surfaces? (gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) 1.5 kg
(B) 1.65 kg
(C) 2.35 kg
(D) $\left(\frac{16 \sqrt{3}}{15}\right) \mathrm{kg}$
(E) $(0.60 \sqrt{3}) \mathrm{kg}$
(F) $(0.30+1.5 \sqrt{3}) \mathrm{kg}$
(G) $(1.7+1.5 \sqrt{3}) \mathrm{kg}$
[ENGAA, 2023S2Q15]
A sound wave travels through medium J, reaches a boundary, and then travels through medium K as shown. The thickness of each medium is $L$.


The wave travels a distance $q$ in medium J and a distance $y$ in medium K .
The horizontal distance travelled in medium Jis $p$. The horizontal distance travelled in medium K is $x$.
The wave travels at speed $v$ in medium J. The graph shows how the speed of the wave varies with time $t$ as it travels distances $q$ and $y$, and that the wave leaves medium K at $t=2.75 \mathrm{~ms}$.


What is the value of $\frac{x}{p}$ ?
(A) $\frac{3}{200}$
(B) $\frac{3}{40}$
(C) $\frac{8}{75}$
(D) $\frac{3}{8}$
(E) $\frac{8}{15}$
(F) $\frac{8}{3}$
(G) $\frac{200}{3}$
[ENGAA, 2023S2Q16]
The drag force $F$ acting on a sphere of radius $r$ falling at constant speed $v$ though air is given by

$$
F=k r v
$$

where $k$ is a constant.
For a sphere of uniform density and mass $m$ falling at a constant speed, the drag force heats the surrounding air at a constant rate $P$.
Another sphere of the same material but with mass $8 m$ falls through the air at a different constant speed.
What is the rate at which the drag force on the heavier sphere heats the surrounding air?
(A) $2 P$
(B) $4 P$
(C) $8 P$
(D) $16 P$
(E) $32 P$
(F) $64 P$
[ENGAA, 2023S2Q17]
A projectile is launched from an inclined plane.
The graphs show the variation of the horizontal and vertical components of the velocity of the projectile with time from when it is launched until it hits the plane at time $T$.


What is the angle of the plane to the horizontal?
(gravitational field strength $=10 \mathrm{~N} \mathrm{~kg}^{-1}$ )
(A) $\tan ^{-1} \frac{1}{32}$
(B) $\tan ^{-1} \frac{1}{8}$
(C) $\tan ^{-1} \frac{1}{4}$
(D) $\tan ^{-1} \frac{5}{16}$
(E) $\tan ^{-1} \frac{1}{3}$
(F) $\tan ^{-1} \frac{4}{3}$
[ENGAA, 2023S2Q18]
A tennis ball of mass 0.060 kg travels horizontally and strikes a vertical wall at $30 \mathrm{~m} \mathrm{~s}^{-1}$. It leaves the wall in the opposite direction at $20 \mathrm{~m} \mathrm{~s}^{-1}$.
The graph shows how the resultant horizontal force acting on the ball varies with time during this collision.


What is the duration of the collision?
(A) $\frac{1}{200} \mathrm{~s}$
(B) $\frac{1}{150} \mathrm{~s}$
(C) $\frac{1}{100} \mathrm{~s}$
(D) $\frac{1}{40} \mathrm{~s}$
(E) $\frac{1}{30} \mathrm{~s}$
(F) $\frac{1}{20} \mathrm{~s}$
[ENGAA, 2023S2Q19]
A battery with an emf of 8.0 V and internal resistance $R$ and another battery with an emf of 4.0 V and internal resistance $2.0 \Omega$ are connected to a cell with an emf of 2.0 V and internal resistance $4.0 \Omega$ in the circuit shown.
The current in the 2.0 V cell is 0.50 A in the direction shown in the diagram.


What is the resistance $R$ ?
(A) $1.6 \Omega$
(B) $2.7 \Omega$
(C) $3.2 \Omega$
(D) $8.0 \Omega$
(E) $16 \Omega$

## [ENGAA, 2023S2Q20]

A model for how the resistivity $\rho$ of damp soil varies with depth $x$ from the surface is given by

$$
\rho=\rho_{0}\left(1-\frac{k x^{2}}{h^{2}}\right)
$$

where $h$ is the maximum depth, and $k$ and $\rho_{0}$ are other constants.
What is the resistance of a vertical column of damp soil of cross-sectional area $A$ and depth $h$ ?
(A) $\frac{\rho_{0} h}{A}$
(B) $\frac{\rho_{0} h}{A}(1-k)$
(C) $\frac{\rho_{0} h}{2 A}(2-k)$
(D) $\frac{\rho_{0} h}{A}(1-3 k)$
(E) $\frac{\rho_{0} h}{A}\left(1-\frac{k}{3}\right)$
(F) $\frac{\rho_{0}}{A}\left(1-\frac{k h}{3}\right)$
(G) $\frac{\rho_{0} h^{2}}{A}\left(\frac{1}{2}-\frac{k}{4}\right)$

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[^0]:    Calculator model

